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Resonance frequencies of the multilayered piezotransducers

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Piezoelectric transducers used in high power ultrasonic applications are composed of piezoelectric ceramics, metallic blocks and a central bolt that pre-stresses the assembly. The characterization of these transducers has been done by using numerical methods but, in many cases, simplified one-dimensional models are enough to a good physical interpretation of the problem. In this work it was used Mason´s equivalent electric circuit for one-dimensional modeling of composed transducers. It was employed Thévenin´s equivalent circuit to simplify the problem and evaluate the effects of the central bolt on the resonances. Some transducers were mounted and their resonance frequencies were measured. The results show a good fitting between the experimental and the calculated data. By using the presented model, with bolt included, it was verified that other new longitudinal vibration modes occur. This fact enhances the possibility of analysis of the behavior of transducers through the frequency spectrum.

1 Introduction

The piezoelectric transducers used in industrial, military and medical applications work upon high power [1, 2]. Usually, these transducers develop high strains and are composed of piezoelectric ceramics and metallic blocks. There are many configurations for these transducers depending on the application which they are used [3]. A simplified configuration is composed of two piezoelectric ceramics with annular shape (piezoceramics rings), matched with metallic pieces in the ends. These metallic pieces also have a central hole. The assembly is mechanically pre-stressed by a central bolt that provides an enough mechanical bias to avoid possible fracture of the ceramics on expansion semicycle [1, 4].

A pulsed or alternating voltage is applied to the electric terminals of the piezoelectric ceramics. Usually, this signal must be in or near to the resonance frequency. The electric energy supplied is partially converted into mechanical energy by piezoelectric effect.

The analysis of the performance of these transducers uses some investigation tools. These tools can be simpler as one-dimensional modeling or more complex by using advanced numerical techniques. The numerical methods, such as finite elements method (FEM), have been pointed out due to high power for simulating transducers with different shapes [5, 6]. However, this method needs large computational memory and, due to numerical characteristic, sometimes the physical meaning is lost.

Analytical models do not have similar power due to the difficulty in solving equations of coupled vibration modes. The scientific literature has shown the treatment of some problems using two and three dimensional models that are restricted to transducers of one element [7 - 11]. Even so, usually, the study of simplified cases is very close to the real operating condition because the transducers used on high power have longitudinal dimensions larger than radial ones. Thus, the first resonance corresponds to a longitudinal vibration mode.

Moreover, one-dimensional models can be dealt as equivalent electric circuits. The modeling of equivalent electric circuits for piezoelectric transducer [12, 13] is very common in the literature. It provides a systematic and easy interpretation of the physical concepts when only the longitudinal vibration is assumed.

In the multilayered transducers [14 - 21] each piece of the transducer has an equivalent electric circuit associated. The complete transducer is modeled as a electric network composed of several unities, each one correspondent to a piece of the transducer. The whole circuit can be analyzed by using the basic laws of circuit theory in order to obtain the resonances, currents and voltages.

The one dimensional modeling of Langevin transducer has been the target of many recent studies. Arnold and Mühlen studied the effects of the mechanical pre-stressing on the resonances of the transducer [4, 22] and the length of the metallic adjacent blocks [23]. Shuyu investigated the effects of the mechanical loads on the resonances too [24]. Güney and Eskinat modeled the concentrators used as tools in the technological applications [25].

In the multilayered transducers, some blocks can be simplified by using Thevenin´s electric circuit. Thévenin´s electric circuit was used in the computational simulation of piezoelectric transducer on transmission and reception mode, but constrained to the mechanical equivalent [26]. Here, Thévenin´s equivalent circuit of the piezoelectric ceramic was obtained. The other components of the transducer were converted into equivalent electric components. The circuits of the passive elements were matched to the output of the Thévenin´s equivalent circuit. This procedure leads to a simpler circuit and facilitates the comparison between calculated and experimental data. Besides, the experimental data were obtained from the usual proceeding for impedance determination.

In this study, Thévenin´s electric circuit of a multilayered transducer is used for investigating the influence of the central bolt on resonances. The goal is to investigate the influence of this element on fundamental resonances and electromechanical coupling factor of these vibration modes. Thus, the investigation methodology employed two loads which were matched to Thévenin´s electric circuit, one neglecting the bolt and another including it.

2 Mason´s model and Thévenin´s equivalent circuit

The main representation of equivalent electric circuit in Langevin transducers is Mason´s model [11, 12]. Figure 1 shows a representation of the equivalent electric circuit of a piezoelectric ceramic. In this model the piezoelectric element has three ports, two mechanical, represented by F_1-F_0 and F_2-F_0, and one electrical given by V. C_0 is the capacitance of the ceramic without strain. The ideal electromechanical transformer ($1{:}n$) converts electric into mechanic energy. Z_1 and Z_2 are given by Eq. (1) and Eq. (2)

$$Z_1 = jZ_0 \tan\left(\frac{\beta l}{2}\right) \quad (1)$$

$$Z_2 = -jZ_0 \csc(\beta l) \quad (2)$$

where Z_0 is the characteristic mechanical impedance of the piezoelectric ceramic; β is the propagation constant; l is the thickness of the piezoelectric ceramic.

The electric circuit shown in Fig. (1) can be simplified by using Thévenin equivalent circuit. As the transducers are symmetrical the center is mechanically clamped, so that, one of the mechanical port is opened.

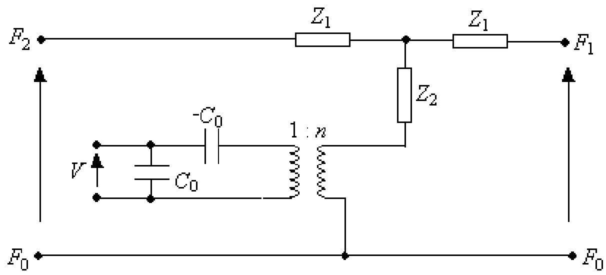

Fig. 1. Mason´s circuit model of a piezoelectric ceramic.

The stress in the port F_0-F_1 is identified as the voltage to be calculated from Thévenin equivalent. This stress is equal to V_2. By converting V_2 into electric parameter and regarding that this procedure is done with secondary circuit clamped, it can be determined $V_{Th} = V$. The mechanical impedance seen by F_0-F_1 port is calculated with the source V short-circuited. The impedance obtained is given by Eq. (3)

$$Z_{Thm} = Z_1 + Z_2 - \frac{n^2}{j\omega C_0} \quad (3)$$

where $n = h_{33}C_0$ and h_{33} is the piezoelectric coefficient of the piezoelectric ceramic.

The mechanical impedance of Eq. (3) is converted into the electrical impedance by using the impedance ratio of the transformer. This results in Eq. (4).

$$Z_{Th} = \frac{Z_1 + Z_2}{n^2} - \frac{1}{j\omega C_0} \quad (4)$$

The resulting Thévenin´s equivalent circuit is added to the intrinsic capacitance C_0. Due to symmetric distribution of the piezoelectric ceramics and the consequent clamped center, the final circuit has two parts, that is, each side of the transducer has its own equivalent electric circuit and these circuits are connected electrically in parallel. Fig. (2) shows this circuit. Each side of the transducer can be loaded by metallic pieces according to the application to be destined.

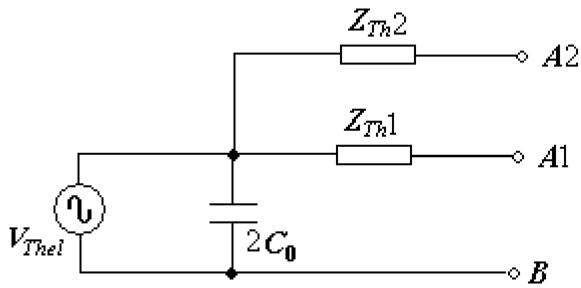

Fig. 2. Complete Thévenin´s equivalent circuit. B is the common terminal. A_1 and A_2 are the points where mechanical parts converted into electric ones are matched.

The mechanical loads must be converted into electrical ones and matched in the ports A_1-B and A_2-B. These mechanical load are passive elements, that is, non-piezoelectric, thus they do not have electric ports. An equivalent electric circuit for a non-piezoelectric piece is shown in Fig. 3. Obviously, there are not electric ports. Eq. (5) and Eq. (6) describe the components of the electric circuit of the passive elements.

Analogously, Z_{1a} and Z_{2a} are parts of the mechanical impedance of these pieces.

$$Z_{1a} = jZ_{0a} \tan\left(\frac{\beta_a l_a}{2}\right) \quad (5)$$

$$Z_{2a} = -jZ_{0a} \csc(\beta_a l_a) \quad (6)$$

where Z_{0a} is the characteristic mechanical impedance of the passive element; β_a is the propagation constant of the passive element; l_a is the thickness of the passive element.

The characteristic impedances are given by Eq. (7)

$$Z_0 = A\rho c \quad (7)$$

where A is the cross section of the component of the transducer, ρ is the density and c is the propagation velocity.

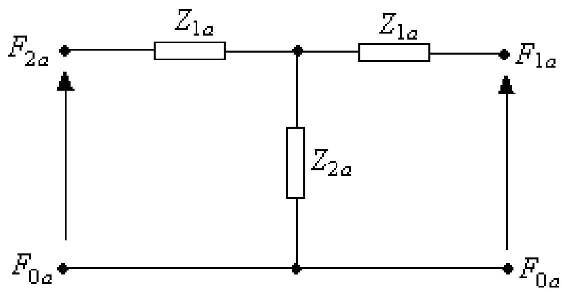

Fig. 3. The circuit model of a passive element.

The equivalent electric circuit of the bolt is similarly described by Eq. (5) and Eq. (6). They are denoted by index b.

It was adopted in the modeling that the components of the transducer are lossless and longitudinal dimensions are larger than radial ones.

A schematic representation of Langevin transducer plugged to the excitation source is shown in Fig (4).

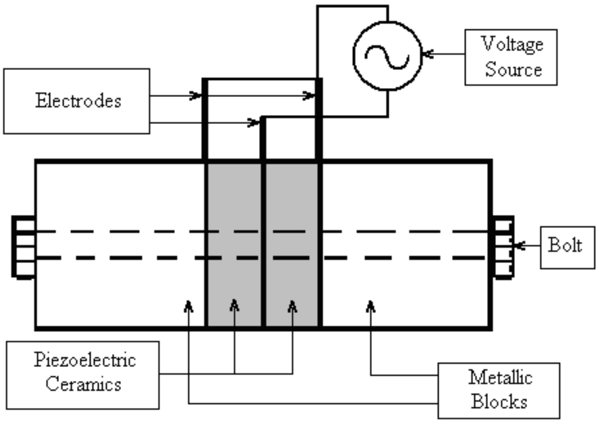

Fig. 4. Schematic representation of the transducer used in the experiments.

Two kinds of load configurations were used for comparison with experimental data. In both cases, it was assumed that the center is perfectly clamped.

In the first configuration, named EB circuit, the bolt was neglected. The load used in this circuit is shown in Fig. (3) with one port short-circuited.

In the second configuration, named IB circuit, the bolt is included. The load network of this configuration is shown in Fig. (5). Port F_{0b}-F_{2b} is left open-circuited because it corresponds to the clamped center of the transducer. The other port of this circuit is plugged to the output of the Thevenin´s equivalent circuit of Fig. (2).

Eq. (8) and Eq. (9) present the expressions of the impedances matched to Thevenin´s equivalent electric circuit. Eq. (8) is the impedance from electric circuit shown in Fig. (3).

$$Z_L = jZ_{0a}\tan(\beta_a l_a) \tag{8}$$

Eq. (9) corresponds to the circuit from Figure 5.

$$Z_L = Z_{1a} + \frac{Z_{2a}(Z_{1a}+Z_b)}{Z_{2a}+Z_{1a}+Z_b} \tag{9}$$

where

$$Z_b = -jZ_{0b}\cot(\beta_b l_b) \tag{10}$$

The expressions for Z_L must be converted into electric impedances through of dividing by n^2.

Both sides of the transducer are loaded with mechanical blocks. From the point of view of the electric circuit analysis there are two networks connected in parallel.

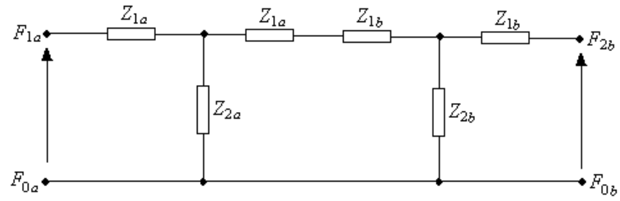

Fig. 5. Network of mechanical load used in the IB model.

The electromechanical coupling factor is designed by k. There are some variations in the definition of the electromechanical coupling factor [27]. Here, the transducers are used on transmission mode, so k is defined as the square of the rate of stored mechanical energy by total energy supplied by exciting source. This definition can be expressed in a more direct way by using the resonance and anti-resonance frequencies. Eq. (11) shows this simplified definition.

$$k^2 = 1 - \left(\frac{f_r}{f_a}\right)^2 \tag{11}$$

where f_r is the resonance frequency and f_a is the anti-resonance frequency.

3 Methodology

Four transducers were mounted, all of them similar to that schematized in Fig. (4). These transducers, named T1, T2, T3 and T4 are described in Table I.

	Side 1	Side 2
T1	Aluminum – 33mm	Aluminum – 33mm
T2	Aluminum – 37mm	Aluminum – 37mm
T3	Aluminum – 41mm	Aluminum – 41mm
T4	Steel – 54mm	Aluminum – 41mm

Table I. Description of the materials and dimensions of the metallic pieces used in the experiments.

The physical parameters of the piezoelectric ceramics [12] are: the elastic coefficient with electric displacement null, c_{33}^D=13.9×10^{10} N/m^2; the piezoelectric coefficient, h_{33}= 14.8×10^8 N/C; the electric coefficient with clamped surface is ε_{33}^S=11×10^{-9} F/m; the thickness of the ceramic is l_c = 6.3 *mm* and the internal and external radius of the ring pieces are r_{ic} = 6.3 *mm* and r_{ec} = 19.0 *mm*. The cross section of the ceramics is 1009.4 mm^2. The total intrinsic capacitance is 3.7 nF.

The aluminum pieces are annular shaped and have the same radial dimensions of the ceramics. The velocity of wave propagation and the density of the aluminum are 6420 *m/s* and 2700 kg/m^3, respectively. The Young modulus is 1.11 × 10^{11} N/m^2.

The density and the velocity of the wave propagation in the bolt and in the steel piece of the transducer T4 are 7700 *kg/m*^{3} and 5610 *m/s*, respectively. Young modulus is 2.42× 10^{11} *N/m*^{2}. The radius of the bolt is r_p=3.5 *mm*.

The transducers were mechanically pre-stressed with 30 MPa to guarantee a good matching of the parts [22]. Resonance and anti-resonance frequencies were determined using the resonance method [28]. These frequencies correspond to minimum and maximum electric impedance modulus of the transducers, respectively. By using the measured frequencies, the electromechanical coupling factor was calculated from Eq. (11).

4 Results

Table II shows the results obtained experimentally and calculated from IB and EB models. Table III shows the electromechanical coupling factors. In these Tables, the X means that the resonance (or anti-resonance) was not found.

		Experimental		EB model		IB model	
		f_r (kHz)	f_a (kHz)	f_r (kHz)	f_a (kHz)	f_r (kHz)	f_a (kHz)
T1	1	40.67	42.00	40.98	42.08	39.57	40.38
	2	74.70	75.00	X	X	73.42	73.92
T2	1	38.07	38.86	37.23	38.14	35.93	36.58
	2	63.47	63.90	X	X	66.73	67.14
T3	1	34.84	34.88	34.07	34.87	32.91	33.47
	2	60.21	60.43	X	X	61.10	61.46
T4	1	26.60	27.00	21.00	21.36	21.26	21.56
	2	35.20	36.40	34.07	34.47	32.91	33.21
	3	44.50	44.70	X	X	46.03	46.13
	4	61.15	61.50	X	X	61.09	61.24
	5	67.50	68.25	64.27	64.97	64.86	65.51

Table II. Resonance and anti-resonance frequencies from experimental proceeding and calculated from EB and IB models.

		Experimental	EB model	IB model
T1	1	0.25	0.2277	0.1986
	2	0.09	X	0.1164
T2	1	0.20	0.2165	0.1881
	2	0.11	X	0.1093
T3	1	0.05	0.2135	0.1810
	2	0.08	X	0.1068
T4	1	0.17	0.1807	0.1667
	2	0.25	0.1523	0.1344
	3	0.09	X	0.0660
	4	0.10	X	0.0701
	5	0.15	0.1468	0.1406

Table III. Values of electromechanical coupling factor of the piezoelectric transducers used in the experiments.

5 Discussion and Conclusion

This present work takes as reference the Thévenin´s equivalent electric circuit for a piezoelectric ceramic. This circuit is loaded with two sets of metallic blocks, creating a multilayered configuration that can be studied in one-dimensional models, once they have longitudinal dimension larger than radial. The literature shows that well fitted results to experimental data are obtained in these cases [15 – 21].

The proceeding for handling the problem is easier than for solving differential equations because of electric circuits methods employed. In addition, the results obtained from the calculus can be compared with experimental data from conventional electric measurements.

The frequency spectra of multilayered transducers are very complex. Many peaks representing resonance and anti-resonance frequencies are observed during measurements proceeding. These frequencies are due to the different vibration modes of the transducers. In addition, the level of mechanical bias contributes to shift the resonances of the transducers [4, 23] and, so that, it introduces errors in the comparison of experimental data and calculated ones.

All cases were dealt as taking into account that the central bolt has length equal to total length of the transducer, that is, length of piezoelectric ceramics plus length of metallic blocks. However, in the transducer T4, in the side of the steel block, the course of bolt was until 45 mm. This difference was neglected in the calculations.

The obtained results show that in the symmetrical transducers, the resonances denoted by 1 are fitted with both data from models. However, the seconds resonances obtained experimentally are only determined with data from IB model. A similar situation arises with the transducer T4 for the resonances 1, 2 and 5. This allowed one to conclude that model IB can be used for enhancing the description of the frequencies spectra of the sandwiched transducers mechanically pre-stressed by a central bolt.

The values of electromechanical coupling factor are well fitted for T1 and T2 transducers. The significant deviate in the first resonance of the T3 is attributed to a neighbor resonance, detected around 37 kHz, which could be the true resonance of a longitudinal mode.

The frequency spectra of the transducers present many peaks of resonance. Some of these peaks were grouped and made difficult the determination of the exact resonance. The criterion adopted in this work takes into account the lowest and the highest value of the impedance of the researched group for obtaining the resonances and anti-resonances frequencies.

It is supposed that effect of grouping of the resonances contributes for decreasing of the electromechanical coupling factor.

It was observed that the electromechanical coupling factor decreased with the inclusion of the thicker loads [27]. Furthermore, the inclusion of the central bolt also caused an additional little decreasing of the electromechanical coupling factor.

The results show that IB model enhances the use of one-dimensional model of the equivalent electric circuit, because it adds a new resonance to the frequency spectrum of the transducer relative to vibration mode in the longitudinal axis. Besides, the central bolt reduces the electromechanical coupling factor because it represents a load for the transducers, but it is important to avoid the breakdown of the piezoelectric ceramics [1].

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