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Benchmarking for acoustic simulation software

Alfonso Molares and Manuel A. Sobreira-Seoane

University of Vigo, E.T.S.I de Telecomunicación, Rúa Maxwell s/n, 36310 Vigo, Spain
amolares@gts.tsc.uvigo.es

The validation of simulation software is not a common practice in industry. Validation studies of commercial implementations are rarely provided by vendors which are reluctant to reveal product weaknesses. On the other hand, in open academic implementations, validation studies rather than marketing are a key factor to their being used at all. They require, however, a high level of user knowledge, and long calculation times can make them impractical for common industrial purposes.
The aim of this study is to help solve this issue by setting a simple benchmark for measuring the accuracy and performance of several simulation software packages for sound field calculations. The acoustic benchmark will be presented by applying it to two widely used commercial implementations of the finite element method and an open-source implementation of the boundary element method.
The validation is set against an analytical formula and against experimental results. In order to study the balance between accuracy and computational cost, the results are expressed in terms of relative errors versus CPU time.

1 Introduction

In the last two decades finite element method (FEM) software implementations of have been used extensively in industry for structural analysis. Consequently, those applications have been developed so that industry accepts that simulation software is highly accurate for resolving a broad range of structural problems.

This is not exactly the case for acoustic simulation software packages which are still few and relatively young. Some specific phenomena exist in acoustics that are foreign to structural analysis and not easily implemented, such as the structural-fluid coupled problem, the modelling of porous media or propagation in unbounded media.

Most of these specific problems have been solved in the last two decades, and new approaches and improvements are still coming. Unfortunately, these solutions are not always implemented in commercial software packages, or are sometimes used outside the limits of their application. Industry users are not familiar with these limits, and calculations are made with no prior knowledge of how much accuracy should be expected.

This can be solved by a simple convergence study, which is common practice in academic environments. Using an analytical formula or an experimental result as a reference, a comparison can be made to determine the accuracy of the numerical calculation. Of course, this validation will be compromised by the accuracy of the reference, and for that reason, suitable test cases should be chosen. There are a few publications, such as [1, 2, 3], that present validation studies of acoustic simulation software for non-academic applications.

Here, we will present a validation study which we think should be a common practise in industry, a practise that would increase the reliability of numerical calculation in acoustics. We also want to point out the difficulties in comparing several software implementations.

Validation studies are necessary to defend the results provided by software tools. They are useful not only for software developers or even users, but for the general consumer; since they will ultimately trust these products, or choose not to.

2 The Validation of Acoustic Simulation Software

In a perfect and simple world, validation studies could be performed using both an analytical formula and an experimental result as a reference. However, in the real world, analytical formulae exist only for the simplest and most ideal scenarios, and when measurements are performed, the theoretical assumptions implicit in the formulae are often not fulfilled. On the other hand, a validation based only on complex and realistic models that can be readily measured, may become obscure without the corresponding analytical formula. Even in a simple experiment there may be many factors that cannot be inserted into the simulation without complications (for instance, non-uniformities in the materials, the modal damping ratio, non-linearities of the system, electrical noise, contact noise, etc). Complex models are also difficult to debug, due to the several error sources that can interfere in the final results. For all those reasons, it is convenient to come to an agreement between theory and technique.

With that aim, a simple scenario will be posed, where an analytical formula is available and where measurements can be performed fulfilling the conditions of the theoretical formulation. The model for this scenario is a monopole on an spherical baffle (see figure 1), which has been used as reference in other works [4, 5]. The main advantage of the monopole-on-sphere model is that it can be easily built and measured if an anechoic chamber is available.

Modern engineering products are composed of many different components, provided by many different manufacturers. It is quite normal that each manufacturer is required to provide a prediction of the component acoustic response. To do that, the component should be simulated under free-field conditions. As we know, it is possible to calculate that response by numerically, but not directly by means of a standard FEM simulation. Sommerfeld's radiation condition, whose derivation is outlined in Pierce [6], must be included in the numerical formulation, and that can be achieved with different techniques, such as the infinite element method [7, 8], absorbing boundary conditions [9, 10], perfect matched layers [11, 12] or the boundary element method [13, 14].

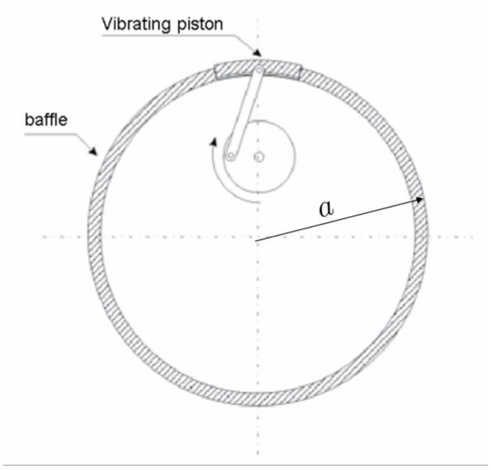

Figure 1: The monopole-on-sphere model

3 The monopole-on-sphere model

The monopole-on-sphere model consists of a small vibrating spherical cap on a rigid sphere (see figure 1). Under free-field conditions the complex sound pressure at any point in the outside of the sphere is given by the expression,

$$p(r) = -j\frac{\rho c u_0}{2}\sum_{n=0}^{\infty} P_n(\cos\theta)\left[P_{n-1}(\cos\alpha - P_{n+1}(\cos\alpha)\right]\frac{h_n(kr)}{h_n'(ka)}, \quad (1)$$

where $\alpha = \arcsin(b/a)$, a is the radius of the sphere, b is the hole radius, r is the distance to the evaluation point, k is the wavenumber, u_0 is the velocity of the small vibrating piston, c is the sound wave speed in air, h_n is the first kind spherical Hankel function of order n and P_n are the Legendre functions. A complete derivation of the expression (1) can be found in Junger [15]. In order to achieve a maximum evaluation frequency of 10kHz the infinite summation in equation (1) has been truncated at $N = 366$ terms, following the truncation rule $N \approx 2k$ showed in Ihlenburg [16].

The monopole-on-sphere model can be built with a high degree of accuracy by means of a thick plastic sphere with a small hole on it. If a loudspeaker is placed inside the plastic sphere and in front of the hole, the sound will travel outwards through it. In this way, the hole behaves as a small vibrating piston. Figure 2 is a photograph of the monopole-on-sphere model built for the study [4] and performed at the Technical University of Denmark. The sphere radius was $a = 135$ mm and the hole radius $b = 10$ mm. The data from that study [4] have been provided by the authors to perform the experimental validation for benchmarking.

4 Tested implementations

Three implementations are tested. Two of them are commercial software packages based on the finite element method and the other is an open-source implementation of the boundary element method. The characteristics of the tested implementations are:

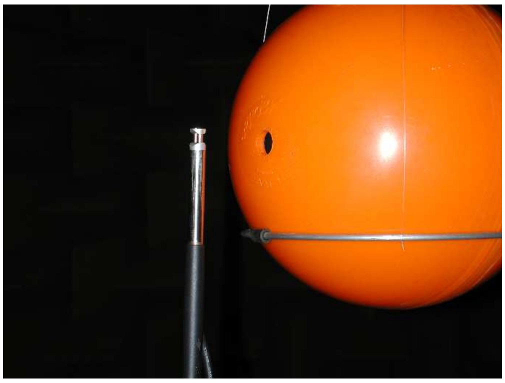

Figure 2: The monopole-on-sphere model build with a thick plastic sphere. Photograph provided by Finn Jacobsen from the Technical University of Denmark.

- **MD Nastran** is an FEM software package for solving structural and structural-acoustic problems. It makes use of the infinite element method (IEM) for modelling free-field conditions.
- **OpenBEM** is an open-source BEM implementation for sound field calculations over the MATLAB platform, which was developed by Peter M. Juhl and Vicente Cutanda at the University of Southern Denmark.
- **COMSOL Multiphysics** is an FEM software package for solving all kinds of physical problems. Both absorbing boundary conditions (ABC) and perfect matched layers (PML) are available, although only ABC will be tested in this study. The underlying technology is also MATLAB.

A sequence of numerical solutions based on uniform partitions of the spatial domain was performed (see figure 3). For a given resolution and same material constants, the input of all implementations is the same mesh. The minimal differences are set for the different elements used in FE and BE methods (BE methods do not need the solid elements, just the definition of shell elements along the boundary). The relative error and computational cost of the different implementations are therefore directly comparable.

5 Error calculation

The error of a numerical prediction may be calculated by means of the discrete L^2-norm [16],

$$e = ||p^{num} - p^{ana}||_{L_2} = \frac{1}{M}\left(\sum_{j=1}^{M}|p_j^{num} - p_j^{ana}|^2\right)^{1/2}, \quad (2)$$

where p^{num} and p^{ana} are the numerical and the analytical solutions, respectively. However, the best indicator of the accuracy of the solution is the relative error, which is given by the following expression,

$$e_r = \frac{||p^{num} - p^{ana}||_{L_2}}{||p^{ana}||_{L_2}}. \quad (3)$$

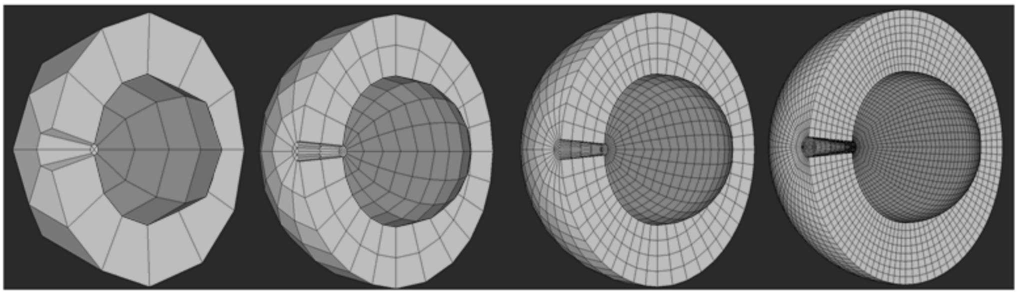

Figure 3: Section of the four meshes used to perform the benchmarking. From left to right: meshes of 53 , 401, 3073 and 24577 elements.

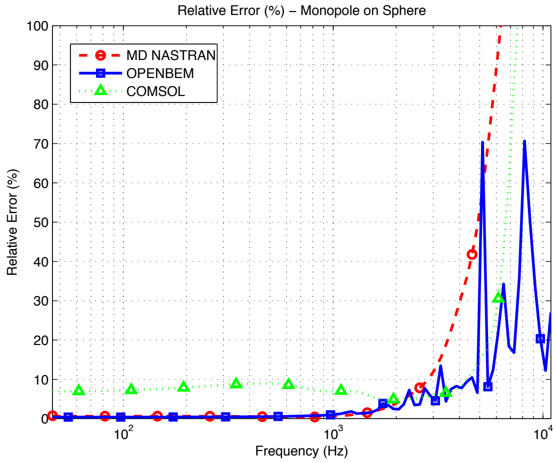

Figure 4: Relative error against frequency for the 24577 elements mesh

6 Validation

The validation was done at $M = 100$ evaluation points, which were placed in a circumference around the sphere at one meter from its centre.

The comparison between the results of the three implementations for the fourth mesh, which has a size of $h = 3$ cm, can be seen in figure 4. COMSOL presents a general error, even at low frequencies, of about 7% which is probably due to the use of the ABC. ABC perform well for plain or spherical wave fronts, but in this case we got a de-centered spherical sound field. A free-field boundary with its centre in the monopole and not in the centre of the sphere would perform better. We also expect that using ABC instead of PML would result in a much better agreement. Future development of benchmarking should resolve the misuse of ABC.

MD Nastran, which uses IEM, and OpenBEM show a very good agreement for low and middle frequency range. The zone where error skyrocketed starts at almost the same point (about 2kHz) although the increment for OpenBEM is more abrupt, with several maxima and minima.

Following the *"rule of thumb"* of 6 nodes per wavelength, the limit frequency is found to agree with those at 2kHz.

7 Convergence

The three implementations were executed on the same machine, a Dual Core Opteron 270 (3Ghz) with 8GB of RAM. During the calculations, no other relevant process or applications were launched. The CPU time consumed was obtained from the reports of the applications themselves, and is preferred as x-axis instead of the mesh size. This is due to the unknown relation between mesh size and computing time, since BEM or IEM matrixes are usually fuller than in bounded FEM problems, implying an increment of the calculation time.

Representation of the relative error against the CPU-time can be seen in the figure 5.

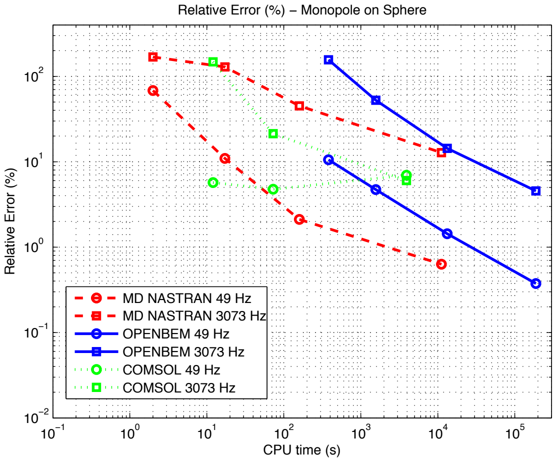

Figure 5: Relative error against CPU-time

The convergence of COMSOL, whose analysis was limited by MATLAB use of memory (notice that just three meshes are presented since the fourth one was not processed due to the "not enough memory" error), is quite strange, presenting a variable convergence slope and even a slight increment of the error from second to third mesh. We have no other explanation for this behaviour except the modelling of the ABC conditions and the de-centered definition.

The results of MD Nastran presents a first convergence slope of almost $-9/10$ ($e_r \leq Ct^{-9/10}$), for low frequencies and low resolution meshes, but as soon as frequency or resolution increases the slope decays to $-3/8$ ($e_r \leq Ct^{-3/8}$), perhaps due to the influence of the IEM.

The convergence of OpenBEM is quite clear, presenting a constant slope of $-1/2$ ($e_r \leq Ct^{-1/2}$). It must be noted, however, that although the convergence rate is constant, the CPU-time consumed is considerably larger. OpenBEM takes at least 20 times the MD Nastran CPU-time to solve the same problem with the same level of accuracy. For example, the calculation of the last mesh took around two days and a half in OpenBEM, and less than three hours in MD Nastran.

8 Comparison against measurements

The frequency response calculated on the fourth mesh was compared with the measurements provided by the Technical University of Denmark. The result can be seen in the figure 6.

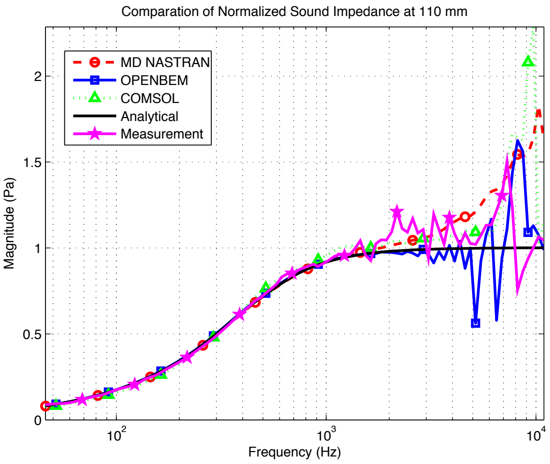

Figure 6: Comparison with the measurements at 110 mm from the hole. The experimental data have been provided for Finn Jacobsen from the Technical University of Denmark

The calculations give a good approximation up to the 2kHz barrier, according to the six nodes per wavelength rule of thumb. Notice that even the 7% error in COMSOL is barely perceptible in the frame of a frequency response. Coincidentally the deviation of measurement from the analytical result, probably due to experimental uncertainties, starts at almost the same point, near 2kHz. Therefore, all the implementations tested can give us a good estimation of the measurement.

9 Conclusions

Validation and convergence studies are quite common in academic environments, and we think they should be common practice in industry to check for good use of numerical tools. By means of simple test cases, the accuracy of the simulation tools can be assessed, and may serve as an objective way of evaluating the strengths and weakness of specific numerical simulation packages.

We must admit to several shortcomings in this this first approach to benchmarking. Specifically,

1. Meshes should present a more regular configuration and a more homogeneous size. Since the four meshes were created by successive refinements of a primary mesh, the final elements were not very regular, which may affect accuracy.
2. More meshes are needed to obtain a better picture of the curve for relative error versus CPU-time.
3. For testing absorbing boundary conditions, the free-field boundary should be centred in the hole, and not in the centre of the sphere.
4. Perfect matched layers should also be included in the benchmarking.
5. The convergence study should give the slope with respect to the frequency to get a better characterisation of the 6 nodes per wavelength rule of thumb.

It is also necessary to relate the convergence slope with the slope in classic FEM simulations, and to give users practical comments about the estimated CPU-time for each application.

Acknowledgments

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