Two Dimensional Wavelet Coefficient Statistics for Sea Bottom Classification

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The analysis of texture of side scan sonar images plays a key role for the interpretation of the geology of the seafloor. In this paper we examine the classification of different seafloors based on the analysis of texture of side scan sonar images. For this purpose, we apply the Two Dimensional Discrete Wavelet Transform on side scan sonar images obtained from three different seafloor types, namely, sand ripples, rocks and sands, and then we examine the statistics of the corresponding wavelet coefficients. The observed probability density functions (PDFs) of the coefficients are modelled using the Symmetric Alpha Stable distribution. The parameters of the fitted model are used to classify the side scan sonar images according to well known cluster analysis techniques. The preliminary results are promising.

1 Introduction

A growing need for fast and accurate mapping of the top sea floor sediments for economic, scientific and military purposes has led to the development of various acoustic survey methods. One of the most widely used acoustic methods is by using the side scan sonar [1]. Usually human intervention is important in segmenting and classifying a side scan sonar image. Perhaps, this is the most reliable and accurate method of image segmentation and classification, since the human visual system is very sophisticated and proper for this task. However, modern side scan sonar acquisition methods create a huge amount of image data for which manual analysis would be prohibitively expensive and time-consuming. Therefore, the automatic classification of side scan sonar images is an important task in various sea bottom surveys.

Typical stages of sea bottom mapping include segmentation, classification and characterization (Fig. 1).

Segmentation involves the partitioning of a sonar image into homogenous and meaningful regions. Once the image is segmented into regions of different textures, the next step is to classify these regions using real-world sediments (classes). Finally, the characterization of the classified regions is achieved by finding important parameters, such as geotechnical properties, that concern these sediments. This paper focuses on the classification stage.

Various methods have been proposed for the analysis of textures of side scan sonar images. All of them depend on the extraction of certain features that reduce the dimensionality of the problem and form a characteristic signature of the image that can be used for the classification procedure. In the present work we apply the 2D Discrete Wavelet Transform to decompose side scan sonar images and then we analyze the statistical behavior of the wavelet coefficients by employing Symmetric Alpha Stable models. We have chosen side scan sonar images containing three different types of sea bottoms, namely, sand ripples, rocks and sand. These bottoms are frequently encountered in sea bottom surveys and it is important to distinguish between them since they bear different geotechnical properties.

The paper is organized as follows: Section 2 contains a brief overview of the various algorithms used to describe image texture in side scan sonar images and gives the mathematical background that is used later. In Section 3 we describe the data that we have used. In Section 4 we present the statistical analysis of the wavelet coefficients and the clustering results. Finally in Section 5 the main conclusions of this work are drawn.

2 Background

During the years, a variety of methods have been proposed for deriving features for the mathematical description of texture in side scan sonar images. These include the use of linear transformations, statistical features from the image pixels and by using a model-based approach.

By applying linear transformations to an image (e.g. Fourier transform, Wavelet transform, Gabor, etc) we get new features that can be useful in describing texture. After the transformation of an image with properly chosen decomposition functions, a new data set is derived that is sometimes smaller than the image itself. This is done in order to decrease the dimensionality of the problem and “expose” the image texture.

One of the first transform-based techniques used for classification of sonar images relates to the Fourier Transform of one dimensional signals coming from side scan sonar [2]. More techniques followed mainly based on the two dimensional Fourier and Wavelet Transforms [3, 4].

In the statistical approach the pixel statistics of an image are considered. The simplest statistical analysis is done by employing first-order statistics of the gray-level histogram of an image [5]. The gray levels are directly related to the backscattering amplitude of local sediments.

One of the most important statistical methods is the Gray Scale Co-occurrence Matrix method [6] that uses the second-order statistics of an image. Here, the joined gray-level histogram of several neighbor pixels is taken into account. This technique was successfully introduced in side scan sonar images in order to classify different sedimentary sea bottoms [7].

Statistical methods sometimes fail if used alone since they are susceptible to noise for example because amplitude changes due to variations in local bathymetry which is irrelevant to sediment texture.

The model-based methods rely on fitting an analytic function to the texture in order to capture some texture characteristics. Usually this function is based on a two-dimensional stochastic process. Markov Random Fields (MRF) have been successfully used in this direction [8]. Also, fractal approaches to the problem have been suggested [9].
2.1 Two Dimensional Wavelet Transform

In this work the classification of side scan sonar images is based on the observation that the texture of these images can be characterized by the statistical behavior of their wavelet subband coefficients.

Research on the human texture perception suggests that the human eye uses some kind of multi-scale linear decomposition in order to analyze an image [10]. Such linear multi-resolution decomposition can be achieved with the help of the Two-Dimensional Wavelet Transform. The equation for the Two Dimensional Continuous Wavelet Transform (2D CWT) case is [11]:

\[ C(s, p_x, p_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \psi_{s, p_x, p_y}(x, y) \, dx \, dy \]  

(1)

Where \( C(s, p_x, p_y) \) is the decomposition coefficient of image \( f(x, y) \) at scale \( s \) and position \( p \) by using the wavelet \( \psi_{s, p_x, p_y} \).

The wavelet itself is given by:

\[ \psi_{s, p_x, p_y}(x, y) = \frac{1}{\sqrt{s}} \psi\left(\frac{x-p_x}{s}, \frac{y-p_y}{s}\right) \]  

(2)

Where \( \psi \) is the mother wavelet from which all the wavelets used in the decomposition described by Eq.(1) are derived from.

The 2D-Discrete Wavelet Transform (2D DWT) allows the implementation of the CWT in computers and it requires two basic procedures [11]: (1) the cascading of high and low-pass digital filters and (2) the subsampling of each filter output. The decomposition of an image is realized by applying 1D Wavelet Transforms separately along the horizontal and vertical dimensions of the image. Therefore, for the realization of a 2-D Wavelet Transform of one stage we need (a) Low and High -pass filters, two of which are applied along the rows and the rest along the columns of the image to be transformed and (b) sub-sampling in every filter output [11].

2.2 The Symmetric Alpha Stable (SaS) distribution

Alpha Stable distributions [12] are a family of statistical distributions that are defined by four parameters \( \alpha, \beta, \gamma \) and \( \delta \). In particular \( \alpha \) and \( \beta \) parameters define the shape and \( \gamma \) and \( \delta \) are related to the scale and position of the distribution. Parameter \( \alpha \) (also called the characteristic exponent) defines the shape of the tails of the distribution. It lies in the range \([0, 2]\). For \( \alpha=2 \) the distribution reduces to a Gaussian distribution with variance \( \sigma = 2\gamma^2 \) and mean \( \delta \) and the parameter \( \beta \) has no effect. The parameter \( \beta \) (also called index of skewness) is related to the symmetry of the distribution and lies in the range \([-1, 1]\). When \( \beta=0 \) the distribution is symmetrical around the location parameter \( \delta \) (-\(\infty < \delta < +\infty \)). Such distributions are called Symmetric Alpha Stable (SaS). A SaS distribution is defined by its characteristic function [13]:

\[ \phi(t) = \exp( i \delta t - \gamma^a |t|^\alpha ) \]  

(3)

\( a, \gamma \) and \( \delta \) being the SaS distribution parameters. In our case the observed distributions are symmetric around zero so \( \beta = \delta = 0 \) (Fig. 2).

The symmetric stable densities possess many features of the Gaussian distribution. They are smooth, unimodal, symmetric with respect to the median and bell–shaped. However, the main characteristic of a non–Gaussian stable probability density function is that its tails are heavier than those of the normal density. As a result the stable law is regarded suitable for modeling signals and noise of impulsive nature. The SaS distributions have also proven to be efficient in describing the content of many texture images [14] and acoustic signals [15]. Also, at present work, it is demonstrated that this family of distributions is also appropriate for the classification of side scan sonar images. The classification is achieved by modeling the Probability Density Function (PDF) of the image 2D-DWT coefficients using the SaS distribution. It is demonstrated that the parameters \( \alpha \) and \( \gamma \) of the resulting SaS depend on the image texture.

![Fig. 2 SaS distributions for various \( \alpha \) values (\( \gamma=1, \delta = 0 \)).](image)

3 The data

We used images from two different data sets. The first one was provided by NATO Underwater Research Centre (NURC) and the other one by L3 Communications-Klein Associates Inc. Both data sets were ground truthed. All selected images depict homogeneous regions of a single texture in grayscale. The size of each image is \(128 \times 128\) pixels. For our tests we used 16 images from each class (sand ripples, rocks and sand). Figure 3 depicts typical images of sand ripples (columns 1 and 2), rocks (column 3) and sand (column 4) from both data sets.
4 Statistical analysis of the wavelet coefficients

Several different wavelets were tested and it was found that the Daubechies family gave the best results for our purposes. Among all, we mainly used the first (db1) and fourth (db4) members of the family. The wavelet coefficients under consideration are these of the third decomposition level. Figure 4 depicts three typical PDFs of the wavelet coefficients of images from our data set. In all different wavelets were tested and it was found that the Daubechies family gave the best results for our purposes. Among all, we mainly used the first (db1) and fourth (db4) members of the family. The wavelet coefficients under consideration are these of the third decomposition level. Figure 4 depicts three typical PDFs of the wavelet coefficients of images from our data set. In all three diagrams the points correspond to the observed PDF of the coefficients, while the solid line is a Gaussian approximation. These diagrams clearly indicate that the PDF can be modeled as Gaussian only for images of pure sand. For sand ripples and rock images, the resulting PDFs exhibit heavier tails than the normal distribution, indicating that the SaS is a more suitable model for these seabottom classes.

Figures 5 and 6 depict the results of the SaS modeling. The axes in the figures are the $\alpha$ and $\gamma$ parameters of the SaS distribution that models the PDF of the wavelet coefficients of the images. Thus, each point in the two diagrams represents a single image, while its coordinates are the corresponding $(\alpha, \gamma)$ parameters. The Koutrouvelis method [16, 17] was used in order to estimate these parameters. It is clear that the three seafloor types under consideration form three different clusters. The cluster centroids (star symbols) are calculated using the k-Means algorithm [18].

![Typical images from our data sets.](image1)

![PDFs of decomposition coefficients for three different types of bottoms (sand ripples, rocks and sand respectively).](image2)

![SaS modeling of the third level wavelet coefficients of side scan sonar images from the Klein data set using the db4 wavelet (squares for sand ripples, triangles for rocks & circles for sand).](image3)
From figures 5 and 6 we can draw the following conclusions:

- Images of pure sand concentrate near $\alpha=2$ which corresponds to the Gaussian case. This is in accordance with the observed PDF of Fig. 3. The pure sand cluster is well formed with a small spreading in comparison to the other two seafloor types.

- Images with sand ripples exhibit significant variations in the $(\alpha, \gamma)$ plane. This is due to the fact that sand ripples images come with variable texture (Fig. 3) which depends mainly on the wavelength and the orientation of the sand waves. As expected, the wavelet transform is sensitive to all these characteristics and this results in a large spread of the corresponding points on the $(\alpha, \gamma)$ plane. This could potentially increase the probability of error in an automated classification system but on the other hand, the observed spreading could give information about the characteristics of the sand ripples.

- Rock images wavelet statistics lie in between sand and sand ripples statistics. If the sonar image is not clear enough, the probability of an erroneous decision may be significant. This is due to the fact that the tails of the resulting PDF will not be heavy enough and rock could be classified as sand.

5 Conclusions and future work

In this paper we have attempted to classify side scan sonar images based on the statistical behavior of wavelet decomposition coefficients. The images that we analyzed contained characteristic textures from three common sea bottoms namely sand ripples, rocks and pure sand.

The suggested method of side scan sonar image classification is outlined as follows:

1. Image segmentation: The image is segmented according to different textures. This is accomplished by using well-known segmentation algorithms such as the GLCM method.

2. Calculation of the wavelet coefficients of the single texture images up to an optimum level of decomposition.

3. Calculation of the PDF of the wavelet decomposition coefficients.

4. Estimation of the $(\alpha, \gamma)$ parameters of the closest SaS to the observed PDF.

Although more analysis is required, the initial results suggest that the use of Symmetric Alpha Stable statistical models can be a promising tool in classifying side scan sonar images.

For the further development of the above scheme our investigations will move towards the following directions:

- The use of linear combination of SaS models (SaS mixtures [19]) for better classification results.

- The use of advanced clustering algorithms (such as spectral clustering) for the definition of the clusters.

- The development of a systematic approach to the wavelet decomposition level selection.

- The development of a mathematical framework for the computation of the probabilities of error in the classification procedure.

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References


