Global synthesis of superdirective frequency-invariant beam patterns

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Frequency-invariant beam patterns are often required by systems using an array of sensors to process broadband signals. If the spatial aperture is shorter than the involved wavelengths, the use of a superdirective beam pattern is essential to attain an efficient system. In this context, robustness to array imperfections is a crucial feature. In the literature, only a few approaches have been proposed to design a robust, superdirective, frequency-invariant beamformer, based on a filter and sum architecture: in all of them, the frequency invariance is achieved imposing an *a priori* desired beam pattern. However the choice of a suitable desired beam pattern is not trivial and depends on the specific design case: an improper selection of the desired beam pattern can produce unsatisfactory performances. We propose a new method of global synthesis, computationally inexpensive, allowing to design a robust broadband beam pattern with an optimal trade-off between the frequency invariance and the directivity, without the need of imposing *a priori* a desired beam pattern. The results show that the synthesized beam patterns have a directivity, a frequency-invariance, and a robustness that are very similar to or better than those of the beam patterns obtained by the literature methods.

1. Introduction

Systems using sensor arrays are very often involved in processing broadband signals. In some interesting cases, it is important that the performance of the array processor should be adequately constant over the entire frequency band of the signal. In the context of audio signal processing, especially speech, microphone arrays can be used to pick up a sound emitted by a distant source, while suppressing noise and reverberation coming from other directions [1,2,3]. Since the signal of interest may come from a direction different from the array looking one, it is extremely important to obtain a frequency invariant beam pattern (FIBP), in order to avoid signal distortions. However in some applications (i. e. hearing aids) the array design is restricted by the dimension of the array itself [4], hence solutions requiring short apertures are mandatory. In the recent decades, some papers have described the general structure of a broadband filter-and-sum beamformer and have proposed methods to optimise the beamformer in such a way that an FIBP may be obtained [1,5-7]. However, although many different operating conditions and array geometries have been successfully applied till now the case in which the array aperture D is shorter than the wavelengths has rarely been considered. In such a case, the generation of a superdirective beam pattern, achieved by synthesizing specific apodization functions [8], is essential. In this context, robustness to array imperfections and random errors is a very crucial feature. The inadequate robustness of many solutions inhibits their practical application, despite the nice characteristics of the nominal beam patterns.

To the best of our knowledge, only very recently have a couple of approaches been proposed [9, 10] that can be used to synthesize the FIR filters necessary to produce an FIBP by using a superdirective array, and that are also robust enough to deal, with errors in the array characteristics. While in [10] the robustness of the solution should by assessed *a posteriori,* the procedures described in [9] take into account the statistics of the gain and phase errors of the array elements during the synthesis phase by optimizing, for instance, the mean performance of the broadband beamforming, in a least-squares sense, for all the feasible element characteristics. The common approach adopted in [9,10] to obtain a FIBP is based on the minimization of a cost function expressing the adherence between the actual broadband beam pattern (ABP), depending on the FIR filter's coefficient, and a desired frequency invariant beam pattern. The desired beam pattern (DBP) is obtained replicating over the frequency range of interest a narrowband beam pattern function of the signal direction of arrival (DOA). However such narrowband beam pattern has to be set *a priori* as an input parameter and no method is provided to find a suitable one. This can be regarded as a serious drawback of the considered approaches, since the choice of a proper DBP is extremely important in order to obtain a good result and at the same time not trivial, since it depends on many system characteristics. As an example, a smooth DBP, with a large main lobe, will probably produce a frequency flat but poorly directive ABP; on the contrary, an ambitious, highly directive DBP will not be followed by the ADP at all the frequencies of interest, so failing in producing a frequency invariant behavior. Moreover, being equal the directivities DBPs with different shapes will be followed more or less easily by the ABP. As a consequence the choice of the DBP implies a time-consuming trial-and-error process which doesn't assure to find the optimal solution.

In this paper we propose a new method that allows to design a robust, superdirective, broadband beam pattern without the need of choosing an *a priori* DBP: the key idea is to perform a global optimization, synthesizing at the same time the FIR filter coefficients producing the ABP and the values of the DBP. The optimization criterion for the DBP values consists in maximizing the DBP directivity while maintaining a fair adherence by the ABP.

The obtained solution is intrinsically robust since, as in [9], the statistics of the errors affecting the microphones are taken into account during the synthesis phase.

In addition, the optimization procedure is computationally inexpensive, since both the FIR filter coefficients and the values of the DBP can be found in a closed form.

The proposed method has been applied to a very short microphone array and the obtained results have been quantitatively compared to the ones obtained with the method exposed in [9] by means of a set of metrics including the directivity, the white noise gain and the frequency distortion. We can anticipate that the performances achieved by the proposed method are comparable and, in some respect, better than the ones obtained by the literature method.

This paper is organized as follows. Section II describes the broadband beam pattern and the metrics to assess its performance and robustness. The method for the synthesis of a broadband superdirective beam pattern is presented in Section III. Section IV reports the results obtained on a very short microphone array over a bandwidth of about 3 octaves and compares them with those yielded by a literature method. Finally, in Section V, some conclusions are drawn.

2. Frequency-invariant filter-and-sum beamforming

In filter-and-sum beamforming, tapped delay line architectures, where each array element (i.e., a sensor) feeds a transversal filter and the filter outputs are summed up to produce the beam signal, are typically exploited to design a broadband spatial filter [6]. Let us consider a linear, equispaced array composed of N omnidirectional, point-like sensors, each connected to an FIR filter composed of L taps. The far-field beamformer response, i.e., the beam pattern, is a function of the DOA and of the frequency, and can be expressed [6] as follows:

$$BP(\theta, f; \mathbf{w}) = = \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} \mathrm{w}_{n,l} \exp\left[-j2\pi f\left(\frac{n\, d \sin\theta}{c} + l\, T_c\right)\right] \quad (1)$$

where f is the frequency, θ is the arrival angle belonging to the interval [-90°, 90°], c is the speed of the acoustic waves in the medium, T_c is the sampling interval of the FIR filters, d is the inter-element spacing, and $\mathrm{w}_{n,l}$ represents the l-th tap coefficient of the n-th filter. The L coefficients of the N FIR filters are independently adjustable, and can be arranged in a row vector **w** of length $M=NL$. Similarly the complex exponentials, accounting for the delays introduced by plane wave propagation and filter delay lines, can be arranged in a row vector $\mathbf{g}(\theta, f)$ of length $M=NL$. In this way the expression (1) can be written as a dot product:

$$BP(\theta, f, \mathbf{w}) = \mathbf{w} \cdot \mathbf{g}^{\mathrm{T}}(\theta, f) \quad (2)$$

2.1 Beamformer performance analysis

The beamformer performance can be derived from the array gain and the white noise gain. The array gain indicates the improvement in the signal-to-noise ratio (SNR) provided by the array for ambient noise. For an isotropic noise field and plane waves, the array gain is called "directivity" [8,11,12]. The white noise gain indicates the improvement in the SNR provided by the array for sensor self-noise, assumed to be white [11,12]. The inverse of the white noise gain is called "sensitivity factor" [11] and corresponds to the sensitivity of the array beam pattern to array imperfections (e.g., element position errors and element response errors). Consequently, an excessive decrease in the white noise gain value cannot be accepted.
The mathematical formulations of the directivity and the white noise gain versus frequency, for a broadside linear array can be found in [10].
In order to evaluate the degree of frequency flatness we have devised an additional parameter called "frequency distortion" (FD) defined as follows:

$$FD(\theta) = \frac{\sqrt{\dfrac{\int_{f_{\min}}^{f_{\max}} \left[\left|BP(f,\theta)\right| - \overline{BP}(\theta)\right]^2 df}{f_{\max} - f_{\min}}}}{\overline{BP}(\theta)} \quad (3)$$

with:

$$\overline{BP}(\theta) = \frac{\int_{f_{\min}}^{f_{\max}} \left|BP(f,\theta)\right| df}{f_{\max} - f_{\min}} \quad (4)$$

where $f_{\max}$ and $f_{\min}$ are respectively the highest and lowest frequency of interest. The FD express, for each DOA, the standard deviation of the BP modulus, calculated over the frequency range of interest, normalized by the mean value of the BP modulus. Given a DOA, an higher value of the FD implies a less uniform frequency behavior.

3 Proposed approach

The proposed global synthesis procedure is based on the minimization of a cost function expressing both the adherence between the ADP and DBP and the DBP directivity.
Let P be the odd number of points used in discretizing the direction-of-arrival axis, from -90° to 90°, Q the number of points used in discretizing the frequency axis over the desired bandwidth, $BP_{pq}(\mathbf{w})$ the value of the broadband beam pattern in θ_p and f_q, computed by Eq. (1), applying the tap coefficients contained in the vector **w**, and BPd_p the value of the desired beam pattern calculated in θ_p for an arbitrary frequency (since the DBP is supposed to be frequency invariant it doesn't depend on the index q).
Without loss of generality let be $\theta_{(P+1)/2}$=0° the steering direction.
A cost function well tailored to our aim is the following:

$$J(\mathbf{w}, \mathbf{BP}d) = \sum_{p=1}^{P} \sum_{q=1}^{Q} \left|BP_{pq}(\mathbf{w}) - BPd_p\right|^2 + K\left|BPd_p\right|^2 \quad (5)$$

Such cost function is made up of two terms: the first accounts for the adherence between ABP and DBP, in a least squares sense, for all the frequencies and directions of interest and the second express the DBP energy. The relative weight of the two terms can be tuned by the parameter K. This cost function has to be minimized in respect to the FIR filter's coefficient **w** and the values of the DBP, contained in the vector $\mathbf{BP}d$, calculated for every discretized direction except the steering one.

$$\mathbf{BP}d = \left[BPd_1 \quad BPd_2 \quad \cdots \quad BPd_{\frac{P-1}{2}} \quad BPd_{\frac{P+3}{2}} \quad \cdots \quad BPd_P \right] \quad (6)$$

The DBP at the steering direction is kept fixed at the normalized value 1:

$$BPd_{\frac{P+1}{2}} = 1 \quad (7)$$

For each discretized frequency, considering the constraint expressed in Eq.(7) and the definition of directivity reported in [10], the minimization of the DBP energy is equivalent to the maximization of the DBP directivity (calculated in a discretized way, over a grid of directions).
Therefore the minimization process produces both the DBP which assures the best trade off between directivity and adherence to the ABP, and the filter's coefficients which assure the best adherence to the optimized DBP.

Unlike many other synthesis methods the adherence between DBP and ABP is intended not only in modulus but also in phase: in order to avoid phase distortions on the acquired signals the phase of the obtained beam pattern should be linear over frequency for each DOA. To this end the values of the DBP BPd_p have been forced to be real. In this way, for each DOA, the DBP has phase either equal to 0° or 180°: the first case implies no phase-distortion and no delay on the acquired signal, while the second case implies only a change on the sign of the acquired signal without modifying its envelope.
Using Eq(2) and assuming the vectors **w** and $\mathbf{BP}d$ to be real valued we can develop Eq(5) obtaining:

$$J(\mathbf{w},\mathbf{BP}\,d)=\sum_{p=1}^{P}\sum_{q=1}^{Q}\mathbf{w}\cdot\mathbf{g}_{p,q}^{\mathrm{T}}\mathbf{g}_{p,q}^{*}\cdot\mathbf{w}^{\mathrm{T}}+ \\ +BPd_p^{\,2}-2BPd_p\cdot\mathbf{w}\cdot\mathrm{Re}\left\{\mathbf{g}_{p,q}^{\mathrm{T}}\right\}+K\cdot BPd_p^{\,2} \tag{8}$$

where $\mathbf{g}_{p,q}=\mathbf{g}(\theta_p,f_q)$ and the superscripts $*$ and $\mathbf{T}$ denote respectively the complex conjugate and the transposed.
The cost function can be now written using a matrix formalism defining :

$$\mathbf{v}=[\mathbf{w}\quad \mathbf{BP}d] \tag{9}$$

$$\mathbf{G}=\sum_{p=1}^{P}\sum_{q=1}^{Q}\mathbf{g}_{p,q}^{\mathrm{T}}\mathbf{g}_{p,q}^{*} \tag{10}$$

$$\overline{\mathbf{g}}_{p}^{\mathrm{T}}=-2\sum_{q=1}^{Q}\mathrm{Re}\left\{\mathbf{g}_{p,q}^{\mathrm{T}}\right\} \tag{11}$$

Using Eq.(11) we can define:

$$\mathbf{A}^{\mathrm{T}}=\begin{bmatrix}\overline{\mathbf{g}}_1^{\mathrm{T}} & \overline{\mathbf{g}}_2^{\mathrm{T}} & \cdots & \overline{\mathbf{g}}_{\frac{P-1}{2}}^{\mathrm{T}} & \overline{\mathbf{g}}_{\frac{P+3}{2}}^{\mathrm{T}} & \cdots & \overline{\mathbf{g}}_P^{\mathrm{T}}\end{bmatrix} \tag{12}$$

$$\mathbf{D}=(K+1)\cdot\mathbf{1}_{P-1} \tag{13}$$

where $\mathbf{1}_{P-1}$ is a (P-1)x(P-1) identity matrix;

$$\mathbf{r}^{\mathrm{T}}=\begin{bmatrix}\overline{\mathbf{g}}_{\frac{P+1}{2}}^{\mathrm{T}}\cdot BPd_{\frac{P+1}{2}}\\ \mathbf{zeros}_{P-1}\end{bmatrix} \tag{14}$$

where $\mathbf{zeros}_{P-1}$ is a P-1 column vector whose elements are equal to zero.

$$d=\sum_{q=1}^{Q}(K+1)BPd_{\frac{P+1}{2}}^{2} \tag{15}$$

Using Eq(10), Eq.(12) and Eq(13) the (NL+P-1)x(NL+P-1) matrix **M** can be defined. as

$$\mathbf{M}=\begin{bmatrix}\mathbf{G} & \dfrac{\mathbf{A}^{\mathrm{T}}}{2}\\ \dfrac{\mathbf{A}}{2} & \mathbf{D}\end{bmatrix} \tag{16}$$

Using Eq.(9), Eq.(14) Eq.(15) and Eq.(16) the cost function Eq.(8) becomes:

$$J(\mathbf{w},\mathbf{BP}d)=\mathbf{v}\cdot\mathbf{M}\cdot\mathbf{v}^{\mathrm{T}}+\mathbf{v}\cdot\mathbf{r}^{\mathrm{T}}+d \tag{17}$$

The function in Eq(17) has only one global minimum whose argument $\mathbf{v}_{\mathrm{opt}}$ is is given by:

$$\mathbf{v}_{\mathbf{opt}}=-\frac{1}{2}\mathbf{M}^{-1}\mathbf{r}^{\mathrm{T}} \tag{18}$$

The first NL components of $\mathbf{v}_{\mathrm{opt}}$ are the optimized FIR filters coefficients while the last P-*1*ones are the values of the optimized DBP for all the DOAs except the steering one.

3.1 Robust beamformer synthesis

The design method based on the cost function presented in section 3 lies on the hypothesis that the microphones characteristics are perfectly known. However, using small-size microphone arrays,. the resulting beamformers are known to be highly sensitive to errors in the array characteristics, especially the microphone gain and phase. To overcome this drawback the strategy presented in [9] has been adopted .The idea is to optimise the mean performance, i. e. the weighted sum of the cost functions for all feasible microphone characteristics using the microphone characteristics PDFs as weights. To this end a total cost function $J^{tot}(\mathbf{w},\mathbf{BP}d)$ can be defined as

$$J^{tot}(\mathbf{w},\mathbf{BP}d)=\int_{A_0}\cdots\int_{A_{N-1}}J(\mathbf{w},\mathbf{BP}d,A_0,\ldots,A_{N-1})\cdot f_A(A_0) \\ \ldots f_A(A_{N-1})dA_0\cdots dA_{N-1} \tag{19}$$

where $J(\mathbf{w},\mathbf{BP}d,A_0,\ldots,A_{N-1})$ is the cost function for a specific microphone characteristic set $\{A_0,\ldots,A_{N-1}\}$ and $f_A(A)$ is the probability density function of the stochastic variable $A=a\exp(-j\gamma)$ i. e. the joint PDF of the stochastic variables a (gain) and γ (phase) related to a single microphone. The cost function $J(\mathbf{w},\mathbf{BP}d,A_0,\ldots,A_{N-1})$ is the same defined in Eq.(5) except that the ABP is calculated multiplying each coefficient w_{nl} of the n-th FIR filter by the variable A_n related to the n-th microphone, for n=1…N. Regarding $f_A(A)$ we assume the same hypothesis stated in [9] i. e. : $f_A(A)$ is independent of frequency and direction of arrival; all the microphone characteristics A_n, n=0,…,N-1 are described by the same PDF $f_A(A)$; a and γ are independent stochastic variables such that the joint PDF is separable, i. e., $f_A(A)=f_a(a)f_\gamma(\gamma)$ where $f_a(a)$ is the PDF of the gain a and $f_\gamma(\gamma)$ is the PDF of the phase γ.; $f_\gamma(\gamma)$ is a symmetric function.
With the previous hypotheses the vectors **w** and $\mathbf{BP}d$ can be extracted from the multiple integrals and, after some passages analogous to the ones reported in [9], the total cost function in Eq(19) becomes:

$$J^{tot}(\mathbf{w},\mathbf{BP}d)=\mathbf{v}\cdot\widetilde{\mathbf{M}}\cdot\mathbf{v}^{\mathrm{T}}+\mathbf{v}\cdot\widetilde{\mathbf{r}}^{\mathrm{T}}+d \tag{20}$$

with:

$$\widetilde{\mathbf{M}}=\begin{bmatrix}\widetilde{\mathbf{G}} & \dfrac{\widetilde{\mathbf{A}}^{\mathrm{T}}}{2}\\ \dfrac{\widetilde{\mathbf{A}}}{2} & \mathbf{D}\end{bmatrix} \tag{21}$$

$$\widetilde{\mathbf{r}}^{\mathrm{T}}=\mu_a\mu_\gamma\mathbf{r}^{\mathrm{T}} \tag{22}$$

where:

$$\widetilde{\mathbf{A}}=\mu_a\mu_\gamma\mathbf{A} \tag{23}$$

$$\widetilde{\mathbf{G}}=\begin{bmatrix}\sigma_a^2\mathbf{1}_L & \mu_a^2\sigma_\gamma\mathbf{1}_L & \cdots & \mu_a^2\sigma_\gamma\mathbf{1}_L\\ \mu_a^2\sigma_\gamma\mathbf{1}_L & \sigma_a^2\mathbf{1}_L & \cdots & \mu_a^2\sigma_\gamma\mathbf{1}_L\\ \vdots & \vdots & & \vdots\\ \mu_a^2\sigma_\gamma\mathbf{1}_L & \mu_a^2\sigma_\gamma\mathbf{1}_L & \cdots & \sigma_a^2\mathbf{1}_L\end{bmatrix}\otimes\mathbf{G} \tag{24}$$

where $\otimes$ denotes the element wise multiplication.

Finally:

$$\mu_a = \int_a a f_a(a) da \quad (25)$$

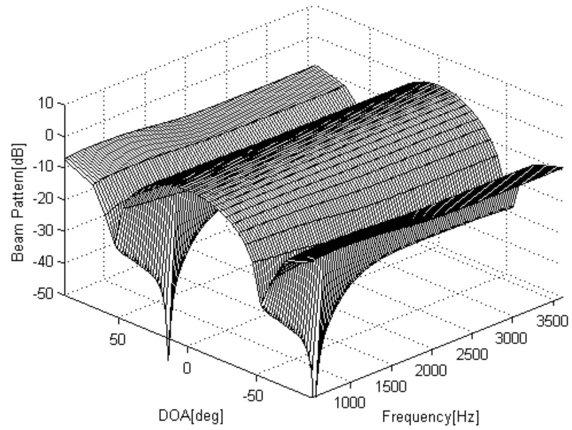
$$\mu_\gamma = \int_\gamma \cos(\gamma) f_\gamma(\gamma) d\gamma \quad (26)$$

$$\sigma_a^2 = \int_a a^2 f_a(a) da \quad (27)$$

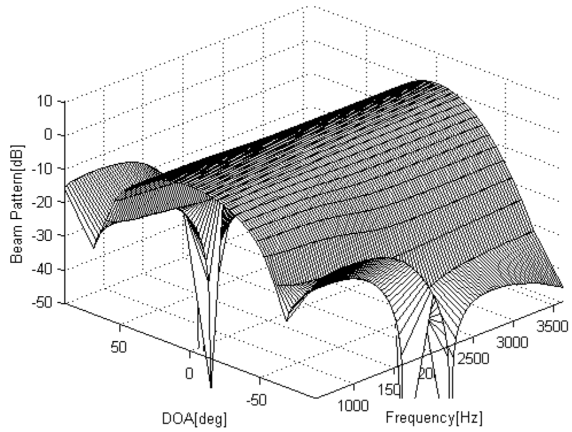
$$\sigma_\gamma = \int_{\gamma 1} \int_{\gamma 2} (\cos(\gamma_1)\cos(\gamma_2) + \sin(\gamma_1)\sin(\gamma_2)) f_\gamma(\gamma_1) f_\gamma(\gamma_2) d\gamma_1 d\gamma_2 \quad (28)$$

4. Results and discussion

The proposed array is linear, and made up of 8 equally spaced point-like omnidirectional microphones with a spatial aperture, *D*, equal to 12 cm and an inter-element spacing, *d*, equal to 1.714 cm.
In our design, each microphone feeds a 34^{th}-order FIR filter (i.e., having $L = 35$ taps) with a sampling frequency equal to 8 kHz. The frequency interval considered for the design of the FIBP ranges from 513 to 3591 Hz (i.e., about 3 octaves) and is discretized by using $Q = 100$ equally spaced points. The direction-of-arrival angle, θ, ranges between – 90° and 90°; it is discretized by using $P = 31$ points that are equally spaced in the domain of sinθ. It is very important to note that the ratio D/λ is below unity up to 2835 Hz and above unity at higher frequencies.
The PDFs of the microphone gain and phase are assumed to be gaussian functions with a mean respectively equal to 1 and 0rad and a standard deviation respectively equal to 0.01 and 0.01rad.
The parameter *K* in the cost function has been set at 0.05.
For the sake of comparison the FIR filter coefficient have been synthesized also by the method reported in [9], employing the weighted least-squares cost function jointly with the mean performance criterion of robustness. It is worth noting that the cost function adopted is equal to the first term of the cost function defined in Eq.(5) considering the DBP as a fixed parameter. The same array and the same PDFs for microphone gain and phase have been assumed. Since this method requires an *a priori* fixed DBP, different DBPs have been tried retaining the one that yielded the best performance.
The broadband beam pattern obtained by the proposed method (PM) and by the literature method (LM) are shown respectively in Fig.1(a) and Fig.1(b).
While the *BP* produced by the PM shows a fair frequency invariance far almost all the DOAs, the *BP* produced by the LM exhibits a consistent non uniform frequency behavior in the DOA ranges [60° 90°] and [-90° -60°].
Concerning the obtained directivities, shown in Fig.2(a), the PM allows an higher directivity below 2230 Hz and a slightly lower one above this frequency in, comparison with LM. It is important to note that the PM allows a more uniform directivity over frequency and a better performance over almost all the critical frequency range for which $D<\lambda$.
The white noise gain in Fig.2(b), which measures the robustness to array imperfections, is higher for the *BP* produced by the PM in the frequency range [1108Hz 2957Hz], lower in the range [2957Hz 3591Hz] and slightly lower in the range [513Hz 1108Hz].



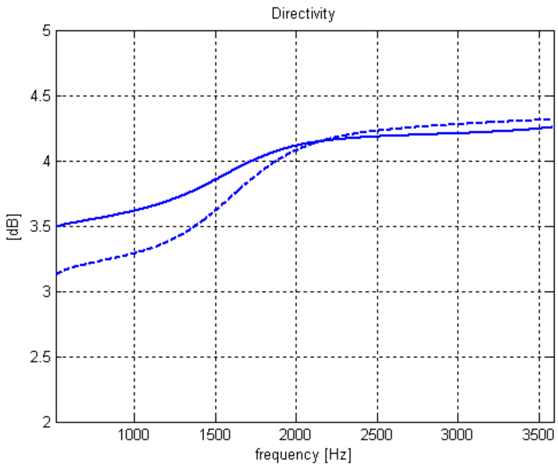
(a)



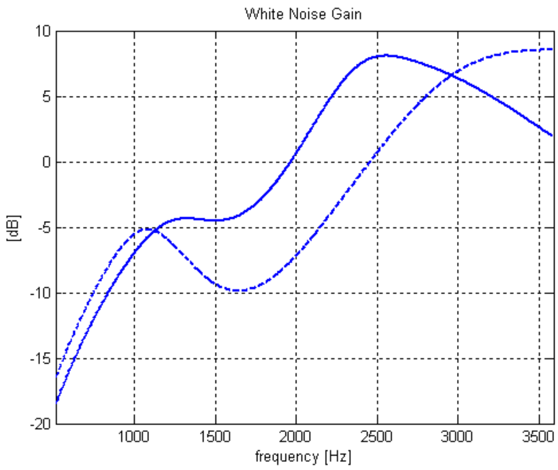
(b)

Fig.1 Broadband beam pattern for an eight-microphone linear array with an aperture *D*=0.12 m obtained by the FIR filters synthesized by the proposed method (a) and by the literature method (b)

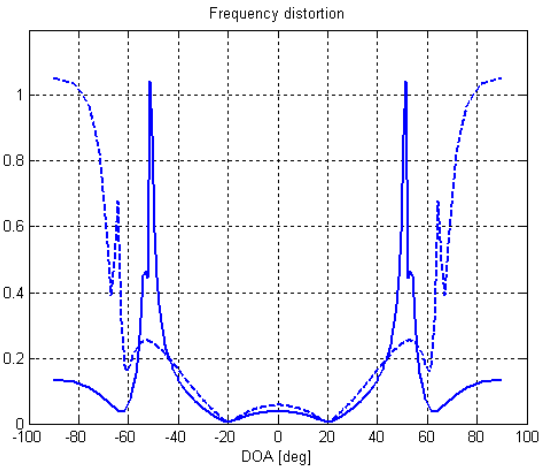
Globally the *BP* robustnesses yielded by the two methods are comparable. The negative values of the white noise gain at the lower frequencies are not so harmful as to prevent a real application.
Finally, a comparison between the frequency distortions in Fig.2(c) bears out the conclusions about the frequency invariance already drawn by the *BP* analysis: the *FD* value obtained by the PM is lower for all the considered directions except the limited ranges [42° 56°] and [-56° -42°]; moreover while the *FD* obtained by the PM is kept, almost everywhere, under the reasonable value of 0.5, the *FD* obtained with the LM exceeds this value in a relevant range of directions, roughly in [65° 90°] and [-90° 65°].
It is possible to conclude that the solution generated by the proposed method is characterized by an higher and more uniform directivity, a considerably higher frequency invariance and a similar robustness to the array imperfections, in comparison with the solution generated by the literature method.
Other comparisons have been carried out, trying to obtain *BPs* with a different trade-off between directivity and frequency invariance, simply tuning the *K* parameter in the PM or changing the DBP in the LM. In all the examined cases the PM yielded *BPs* with similar, and more often better, overall performances.



(a)



(b)



(c)

Fig.2 Performances for the broadband beam patterns obtained by the proposed method (solid line) and by the literature method (dashed line): (a) directivity versus frequency, (b) white noise gain versus frequency, (c) frequency distortion versus direction of arrival.

5. Conclusion

In this paper, an efficient method to design a robust broadband beamformer producing a frequency-invariant beam pattern for superdirective arrays is proposed. Unlike the other methods reported in literature, the difficult and time-consuming operation of choosing a desired beam pattern is avoided by performing a global optimisation of both the desired and the actual beam pattern. This strategy leads to results that are comparable or better, in terms of frequency invariance, directivity and robustness of the beamformer, than that obtained by the literature methods.

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