The significance of cross-order terms in interface mobilities for structure-borne sound source characterization

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For the characterization of structure-borne sound sources and the description of the associated transmission process, the source descriptor and coupling function were introduced. The concept of source descriptor and coupling function can be reformulated by incorporating the interface mobilities. The applicability for source-receiver assemblies with a multi-point or continuous connection is thereby granted. The accuracy of the results and consequently the validity of this approach, however, depends on the significance of the so-called cross-order terms. Such cross-order terms consist of force orders and cross-order interface mobilities. In recent theoretical and experimental work, the influence of cross-order interface mobilities as well as the distribution of force-orders have been investigated. Based on this knowledge, the significance of the cross-order terms is assessed in the present contribution.

1 Introduction

For the design of vibrational sources and minimization of the transmitted power, an approach for structure-borne sound source characterization is required which provides the engineer with physical insight and absolute source data. The vibration amplitude at the source-receiver interface and the active power fed to the receiver constitute the central quantities required and can be obtained from the complex power $\underline{Q}$ [1]. Derived from the definition of complex power and being independent of the receiver properties, see Eq. (1), the source descriptor [2] forms a consistent basis for studies of vibrational sources.

$$\underline{Q} = \underline{S} \cdot \underline{C}_f, \quad \underline{S} = \frac{1}{2}\frac{|\underline{v}_{FS}|^2}{\underline{Y}_S^*}, \quad \underline{C}_f = \frac{\underline{Y}_S^* \underline{Y}_R}{|\underline{Y}_S + \underline{Y}_R|^2} \quad (1)$$

Initially, the concept of source descriptor and coupling function is valid for the single-point and single-component case only. By reformulating the source descriptor concept in terms of interface mobilities [3], source-receiver assemblies with multi-point or continuous connections, see Fig. 1, can be investigated. The interface mobility approach offers a scheme where the physical source is subdivided into a series of theoretical source orders. By treating each order separately, the single-point and single-component case formally is retained. Thus, physical transparency is gained, which is essential for planning and low-noise design. The simplifications introduced with the interface mobility approach, however, require further physical underpinning in order to verify their validity [4]. In this work, the simplification regarding the validity of the concept of interface mobilities is investigated as specified in the following section.

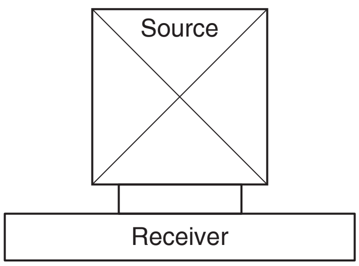


Figure 1: Source-receiver installation with a continuous interface.

2 Interface mobility approach

For multi-point connections between sources and receivers, a single continuous interface can be formed which passes all contact points. Consequently, the field variables, e.g. forces and velocities, are continuous and strictly periodic along the interface. By means of a spatial Fourier decomposition, the velocity $\underline{v}(s)$ can be treated in terms of its interface orders $\underline{\hat{v}}_p$,

$$\underline{\hat{v}}_p = \frac{1}{C}\int_0^C \underline{v}(s) e^{-jk_p s}\,\mathrm{d}s, \quad k_p = \frac{2p\pi}{C}, \quad (2)$$

with

$$\underline{v}(s) = \sum_{p=-\infty}^{\infty} \underline{\hat{v}}_p e^{jk_p s}, \quad p \in \mathbb{Z}, \quad (3)$$

where C is the interface circumference. Furthermore, the force orders are obtained as,

$$\underline{\hat{F}}_q = \frac{1}{C}\int_0^C \underline{F}(s_0) e^{-jk_q s_0}\,\mathrm{d}s_0, \quad k_q = \frac{2q\pi}{C}, \quad (4)$$

with

$$\underline{F}(s_0) = \sum_{q=-\infty}^{\infty} \underline{\hat{F}}_q e^{jk_q s_0}, \quad q \in \mathbb{Z}. \quad (5)$$

An illustration of an interface order is presented in Fig. 2. The zero order is constant along the interface

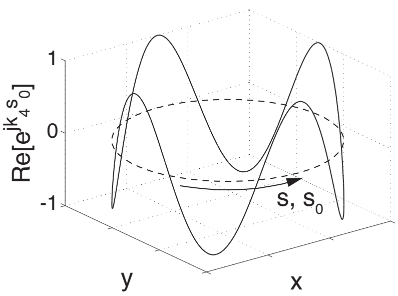


Figure 2: Schematic illustration of a source order (——) along a circular line interface (– – –).

and describes the in-phase motion of the structure when dealing with velocity.
By similarly expanding the point and transfer mobilities, the interface mobilities are written as,

$$\underline{\hat{Y}}_{pq} = \frac{1}{C^2}\int_0^C \int_0^C \underline{Y}(s|s_0) e^{-jk_p s} e^{-jk_q s_0}\,\mathrm{d}s\,\mathrm{d}s_0, \quad (6)$$

with

$$\underline{Y}(s|s_0) = \sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty} \underline{\hat{Y}}_{pq}(k_p, k_q) e^{jk_p s} e^{jk_q s_0}. \quad (7)$$

In the interface-order domain, the relationship $\underline{Y} = \underline{v}/\underline{F}$ is found to be given by [3],

$$\hat{\underline{v}}_p = C \sum_{q=-\infty}^{\infty} \hat{\underline{Y}}_{p-q} \hat{\underline{F}}_q. \quad (8)$$

The pth velocity order consists of two parts, i.e. the contribution of a force of the same order $\hat{\underline{F}}_p$ coupled through the interface mobility $\hat{\underline{Y}}_{p-p}$ and the contribution of all forces of orders different than p, i.e. $\hat{\underline{Y}}_{p-q}\hat{\underline{F}}_q$ with $p \neq q$. The first part is termed the equal-order term and includes the equal-order interface mobility $\hat{\underline{Y}}_{p-p}$. The second part comprises all cross-order terms with the cross-order interface mobilities.
With the sum in Eq. (8), the derivation of all quantities relevant for source characterization is prevented. Hence, it is suggested to neglect the cross-order terms, which yields a relaxed version of Eq. (8), $\hat{\underline{v}}_p = C\hat{\underline{Y}}_{p-p}\hat{\underline{F}}_p$. Without a coupling between different orders, each order can be treated separately.
For the derivation of the quantities relevant for source characterization, the activity of the source can be described by the source free velocity $\underline{v}_{FS}$ [5]. The source descriptor and coupling function for the source order q can be written as [3],

$$\underline{S}_q = \frac{1}{2} \frac{|\hat{\underline{v}}_{q,FS}|^2}{\hat{\underline{Y}}^*_{q-q,S}}, \quad (9)$$

$$\underline{C}_{f,q} = \frac{\hat{\underline{Y}}^*_{q-q,S} \hat{\underline{Y}}_{q-q,R}}{|\hat{\underline{Y}}_{q-q,S} + \hat{\underline{Y}}_{q-q,R}|^2}, \quad (10)$$

with the field variables defined as indicated in Fig. 3. The power transmitted to the receiver is obtained by

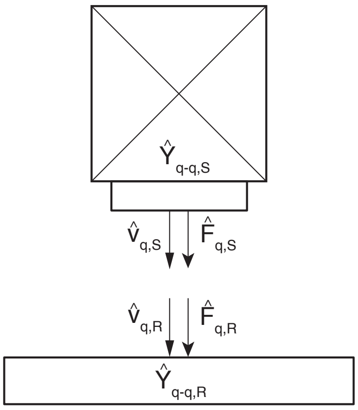


Figure 3: Indication of the field variables for the translatory component of excitation and response.

taking the real part of the product of all source descriptor and coupling function orders,

$$W = \frac{1}{2} \sum_{q=-\infty}^{\infty} \frac{|\hat{\underline{v}}_{q,FS}|^2}{|\hat{\underline{Y}}_{q-q,S} + \hat{\underline{Y}}_{q-q,R}|^2} \mathrm{Re}[\hat{\underline{Y}}_{q-q,R}]. \quad (11)$$

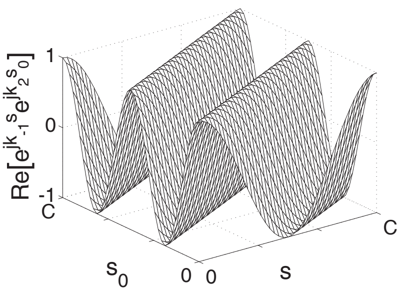
Furthermore, the vibration amplitude along the interface is found to be given by,

$$\underline{v}(s) = \sum_{q=-\infty}^{\infty} \hat{\underline{v}}_{q,FS} \left(1 - \frac{\hat{\underline{Y}}_{q-q,S}}{\hat{\underline{Y}}_{q-q,S} + \hat{\underline{Y}}_{q-q,R}}\right) e^{jk_q s}. \quad (12)$$

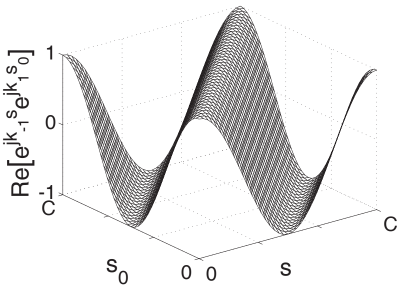
The interface mobility approach for source characterization defined in such a way, elegantly resolves the inherent problems of the source descriptor concept involving multi-point installations. However, the physical admissibility of neglecting the cross-order terms remains to be clarified. In the next sections, the two components of the cross-order terms are investigated separately followed by a complete analysis for the case of a laboratory source-receiver installation.

3 Cross-order interface mobilities

In the interface-order domain, the cross-order interface mobilities describe the coupling between different orders of force and velocity. The physical meaning of the cross-order interface mobilities in the spatial domain will be outlined in the following.
Analogue to the shape of source orders, see Fig. 2, the shape functions of interface mobilities can be visualized as shown in Fig. 4, cf. [3]. The superposition of all inter-



(a) $\hat{\underline{Y}}_{-12}$



(b) $\hat{\underline{Y}}_{-11}$

Figure 4: Interface mobility shape functions.

face mobilities reproduces the ordinary mobility matrix. Hence, the interface mobility shape functions can directly be compared with the shape of ordinary mobility matrices. In ordinary mobility matrices, point mobilities are situated along the main diagonal, where $s = s_0$, and transfer mobilities are found in the off-diagonal elements.
The group of cross-order interface mobilities varies in all directions along the coordinates s and s_0, as exemplified in Fig. 4(a). With a superposition of cross-order interface mobilities, therefore, any variation of an ordinary mobility shape function can be reproduced. The equal-order interface mobilities, however, only vary along the co-diagonal and lines parallel thereto, see Fig. 4(b). It

can be assumed that the equal-order interface mobilities fully reproduce the variations of ordinary mobility matrices along the co-diagonal and parallel lines. This would imply, that the cross-order interface mobilities describe the variation along the main diagonal and lines parallel thereto. Along such lines, the distance between excitation and response positions is constant. Hence, the variations along the main diagonal and parallel lines describe the dependence of an ordinary mobility $\underline{Y}(s|s_0)$ with $|s-s_0| = \text{const.}$ on the position along the interface. In order to verify the above assumption about the physical meaning of the cross-order interface mobilities, an ordinary mobility with the excitation located at s_e and response position at s_r can be averaged along the interface,

$$\overline{\underline{Y}}(s+s_r|s+s_e) = \frac{1}{C}\int_0^C \underline{Y}(s+s_r|s+s_e)\,\mathrm{d}s. \quad (13)$$

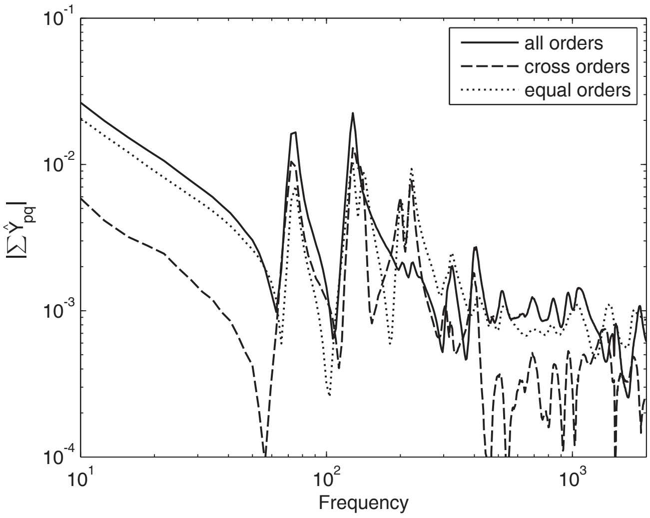
Upon substitution of the series expansion of $\underline{Y}(s+s_r|s+s_e)$, see Eq. (7), the integral can be solved and only equal-order interface mobilities remain,

$$\overline{\underline{Y}}(s+s_r|s+s_e) = \sum_{q=-\infty}^{\infty} \hat{\underline{Y}}_{q-q} e^{jk_q(s_r+s_e)}. \quad (14)$$

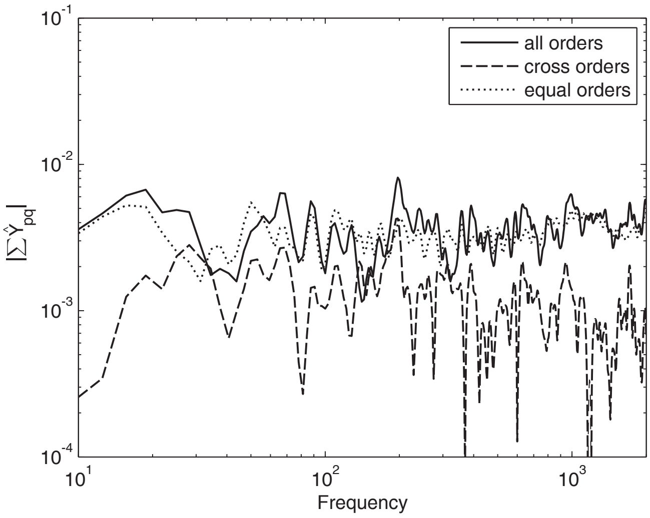
Hence, the dependence of ordinary mobilities on the location along the interface is the physical characteristic described by the cross-order interface mobilities in the spatial domain.

For plate-like structures, there are two attributes which promote cross-order interface mobilities. In presence of boundaries or discontinuities, ordinary mobilities become sensitive to the location of excitation and response positions along the interface. Furthermore, non-circular interface geometries amplify the importance of cross-order interface mobilities. This is due to the fact that the actual distance between excitation and response positions varies when moving along the interface with $|s-s_0| = \text{const.}$, see [3].

Regarding the influence of cross-order interface mobilities, in Fig. 5, the superpositions of the different terms are compared for the laboratory source and receiver structures described in Sec. 5. As seen in Fig. 5, cross-order interface mobilities can be of importance, especially at intermediate frequencies. For an exact representation of dips and peaks, the cross-order interface mobilities are required. However, the superposition of all equal-order interface mobilities manages to capture the overall magnitude and trends [3, 6]. For engineering practice, therefore, this constitutes an indication that the cross-order terms possibly are of subordinate significance.



(a) Source mobilities



(b) Receiver mobilities

Figure 5: Superposition of interface mobilities.

4 Force-order distribution

The distribution of force orders can be decisive for the admissibility of neglecting the cross-order terms. Consider the case where a force order $\hat{\underline{F}}_p$ is substantially smaller than another force order $\hat{\underline{F}}_q$ and the cross-order interface mobility $\hat{\underline{Y}}_{p-q}$ is of similar magnitude as the equal-order interface mobility $\hat{\underline{Y}}_{p-p}$. Here, neglecting the cross-order terms is likely to result in a poor estimate of the source descriptor and coupling function of order p. This is due to the fact, that the cross-order term $\hat{\underline{Y}}_{p-q}\hat{\underline{F}}_q$ is larger than the equal-order term $\hat{\underline{Y}}_{p-p}\hat{\underline{F}}_p$, see Eq. (8).

If all force orders are equal, the cross-order terms reduce to the cross-order interface mobilities, see Eq. (8). With such a uniform force-order distribution, the frequency characteristics outlined in Sec. 3 apply for the importance of the cross-order terms.

In recent work [7], the distribution of force orders of vibrational source installations has been investigated. From experimental analyses, it was found that the force orders vary in magnitude by up to a factor ten at intermediate and high frequencies. At low frequencies variations up to two orders of magnitude were observed.

The forces at source-receiver interfaces have previously been studied in conjunction with the effective mobilities [8], where the assumption of equal magnitude of the contact forces was introduced. From experimental studies, the variations in magnitude of the contact forces were observed to range up to a factor ten [9] and 100 [10].

The relative phases of the contact forces have been investigated in [11]. Only in the mass-controlled region of the source and if the internal source mechanisms are coherent, the relative phases between two contact forces can be discretized at either 0 or $\pm\pi$. For all other cases, the relative phases will fluctuate with frequency.
In [7], Monte Carlo simulations were conducted for the distribution of force orders with the contact forces having the assumed magnitude and phase relations from the preceeding paragraph. For the case where the relative phases of the contact forces are discretized, large variations in magnitude can be observed for the force orders. If the relative phases of the contact forces fluctuate with frequency, the force orders vary by less than one order of magnitude. As the mass-controlled region is located at low frequencies, these theoretical results are corroborated by those from the experimental analyses in [7].
The comparably small variation of the force orders at frequencies where the source is not mass controlled suggests the uniform force-order distribution to be a valid approximation. It can furthermore be surmised, that the larger variation at low frequencies could be compensated by the predominance of the equal-order interface mobility of order zero, see [3, 6].

5 Experimental comparison

For an experimental investigation of the significance of the cross-order terms, a laboratory source-receiver installation with circular line interface is constructed. A circular interface geometry does not promote cross-order interface mobilities [3]. By choosing a circular interface, therefore, the importance of cross-order terms due to the presence of discontinuities can be assessed. Furthermore, a continuous interface is necessary in order to obtain high-order data [3]. With the inclusion of high-order terms, the quality of the results solely depends on the influence of the cross-order terms.
The laboratory source is shown in Fig. 6, where it is freely suspended for measurements of the free velocity and the mobilities. It is constructed of PVC with a thickness of 10 mm. The source structure is tapered at the interface, yielding a line connection at which moment excitation is minimized. Hence, only the force and velocity components perpendicular to the plate are considered. The receiving structure is a simply-supported chipboard plate of 8 mm thickness as analyzed in [3]. The interface is sampled at 24 points, yielding the highest orders of ± 12. The interface mobilities of both source and receiver structures are plotted in Fig. 5.
With the free velocity of the source and the mobilities of both source and receiver structure, the free velocity orders and interface mobilities are readily obtained by means of Fast Fourier Transforms. The transmitted power in terms of interface mobilities is then calculated by Eq. (11). For comparison, the transmitted power also is calculated by the matrix formulation as shown in the following equation.

$$W = \frac{1}{2}\mathrm{Re}\left[\underline{\mathbf{v}}_{FS}^{T}\left(\underline{\mathbf{Y}}_{S}+\underline{\mathbf{Y}}_{R}\right)^{-1T}\underline{\mathbf{Y}}_{R}^{T}\left(\underline{\mathbf{Y}}_{S}+\underline{\mathbf{Y}}_{R}\right)^{-1*}\underline{\mathbf{v}}_{FS}^{*}\right] \tag{15}$$

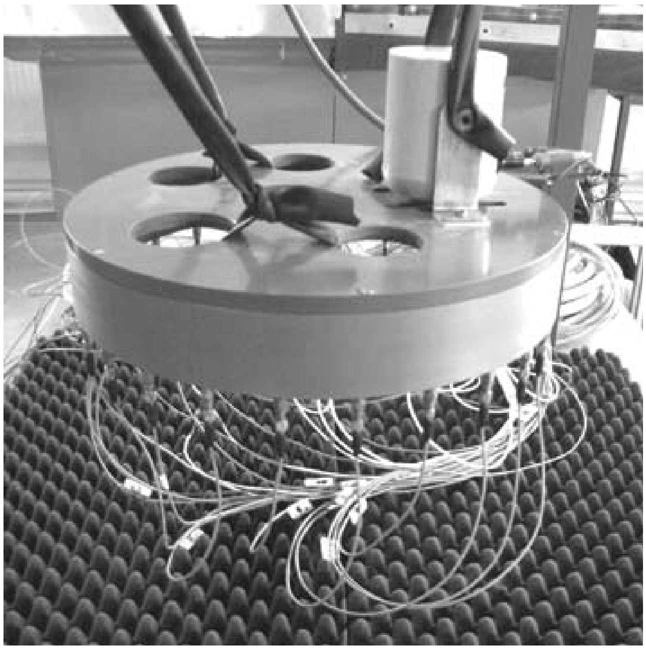


Figure 6: Freely suspended laboratory source.

Owing to the applicability of FFT algorithms, the calculation of the transmitted power proved to be faster with the interface mobility approach than with the matrix formulation. The power spectra obtained from Eqs. (11) and (15) are plotted in Fig. 7.

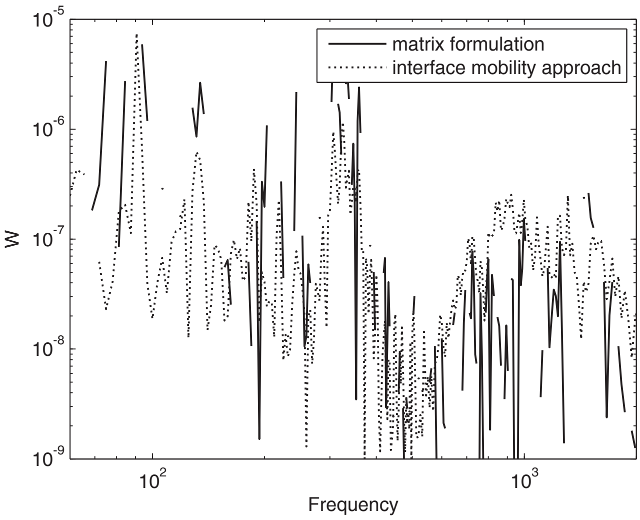


Figure 7: Transmitted power to the receiving structure.

The transmitted power calculated by the interface mobility approach shows negative values only at a few frequencies below 100 Hz. In contrast, the matrix formulation exhibits numerous gaps in the power spectrum presented in Fig. 7. The matrix inversions in Eq. (15) lead to an increased sensitivity of the matrix formulation to measurement deficiencies. With the interface mobility approach, the matrix inversion is circumvented resulting in a superior numerical stability.
The power spectrum obtained from the interface mobility approach features both over- and underestimations of that calculated by the matrix formulation. Due to the uncertainties in the prediction from Eq. (15), it is not possible to determine whether the discrepancies be-

tween the two curves result from the missing cross-order terms or not.
In an overall sense, the predictions for the transmitted power plotted in Fig. 7 show the same trends. This is an indication of the admissibility of neglecting the cross-order terms. However, more data is required in order to substantiate the applicability of the concept of interface mobilities.

6 Concluding remarks

The applicability of interface mobilities for the characterization of vibrational sources depends on the admissibility of neglecting the cross-order terms. The two components of the cross-order terms, i.e. cross-order interface mobilities and force orders, have been investigated separately. The resultant characteristics suggest that neglecting the cross-order terms results in useful estimates for engineering practice.
For an experimental investigation, a laboratory source is constructed, suitably designed for studies of the importance of cross-order terms. The transmitted power is predicted by the interface mobility approach and the matrix formulation. From comparisons of both predictions, similar overall trends are observed, constituting an indication of the admissibility of neglecting the cross-order terms. Additionally, a markedly improved numerical stability is observed by the application of the interface mobilities.
In order to substantiate the applicability of the interface mobility approach for structure-borne sound source characterization, however, more measured data is required. In particular, source-receiver installations which promote cross-order terms have to be studied in order to clarify the range of validity of neglecting the cross-order terms.

Acknowledgments

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