Underwater low frequency sound field simulation with the digital waveguide mesh method

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Abstract Digital waveguide mesh method has been widely used to model musical instruments and simulate room acoustics. In this paper, digital waveguide mesh method was applied to calculate the acoustic vector fields in the Pekeris waveguide. After introducing a boundary condition treatment method for the ocean bottom in the digital waveguide mesh, the acoustic pressure and particle velocity fields are calculated in spatial and temporal dimensions. Using these calculation results, the waveforms for the received signals and distribution of acoustic intensity in the underwater sound channel can also be obtained. Numerical simulation shows that the digital waveguide mesh method can be used to simulate the low frequency two-dimensional acoustic vector fields in shallow water and this method can be easily applied to three-dimensional acoustic vector fields calculation.

1 Introduction

Recent years, the vector sensor is widely used in underwater acoustics research [1, 2, 3, 4]. Several methods have been suggested for underwater acoustic vector fields modeling. The university of Miami parabolic equation code (UMPE) was modified in order to efficiently calculate acoustic particle velocity [5]. The acoustic power flow in ideal waveguide was also analyzed according to normal mode theory [6]. The digital waveguide (DWG) mesh is a method for simulating wave propagation in multiple dimensions. The digital waveguide (DWG) mesh was first introduced for acoustical instrument modeling inherently includes the diffraction and interference effects into the model [7, 8, 9, 10]. It is a promising method especially for modeling small rooms and low frequencies, where the geometrical methods typically fail. In this paper, this method was applied to modelling the acoustic vector fields in the Pekeris waveguide. A boundary condition treatment method is introduced for modelling changes in wave propagation media such as the ocean bottom. Using digital waveguide mesh method, the acoustic pressure and particle velocity fields are calculated in spatial and temporal dimensions and the waveforms of received signals and distribution of acoustic intensity in the underwater sound channel are also obtained. The predictions of the digital waveguide (DWG) mesh method are compared with the results calculated with normal mode theory. Numerical simulation shows that the digital waveguide mesh method can be used to simulate the low frequency two-dimensional acoustic vector fields in shallow water and this method is easily applied to the calculation of three-dimensional acoustic fields.

Section 2 provides the basic theory of underwater sound propagation and normal mode method. In section 3, the acoustic vector fields calculation with digital waveguide mesh method is discussed. Numerical examples are given in section4, in order to illustrate the accuracy and capability of this approach.

2 Basic theory of wave propagation

In classic acoustic theory, the propagation of sound in the ocean is described by the Euler equation, continuity equation and the state equation obtained by the linearization of the hydrodynamic equations of the ideal liquid [11]. The Euler equation is

$$\frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho_0}\nabla p \tag{1}$$

The continuity equation is

$$\frac{\partial \rho}{\partial t} + \nabla(\rho_0 \vec{v}) = 0 \tag{2}$$

Where $\vec{v}$ is the particle velocity, p is the sound pressure, ρ_0 is the density of the medium, and t is time. The state equation is

$$\frac{1}{c^2}\frac{\partial p}{\partial t} = -\frac{\partial \rho}{\partial t} \tag{3}$$

According to these equations, the wave equation for sound propagation in a homogeneous medium can be derived as

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \tag{4}$$

For the harmonic sound propagation problem, the wave equation reduces to the Helmholtz equation

$$\nabla^2 p + k^2 p = 0 \tag{5}$$

where $k = \omega/c$ is the wavenumber. In the layered medium, the sound pressure can be evaluated by contour integration [12]

$$p(r,z) = \oint \sum \frac{Z_n(z)Z_n(z_0)}{\xi^2 - \xi_n^2} H_0^{(1)}(\xi r)\xi d\xi + \text{branch-cut integral} \tag{6}$$

where Z_n is the mode function and ξ_n is the eigenvalue. The contour integral represents the trapped modes that propagate through the water column. The branch-cut integral is associated with the continuous mode spectrum and describes the near-field conditions. In the normal mode theory, the branch-cut integral is neglected, and the sound pressure can be obtained:

$$p(r,z) = g(r,\rho)\sum \frac{Z_n(z)Z_n(z_0)}{\sqrt{\xi_n}} \times \exp[i(\xi_n r - \pi/4)]\exp(-\delta_n r) \tag{7}$$

where $g(r,\rho)$ is a general function of range and water density δ_n is the attenuation coefficient.

It should be noticed that the Euler equation has indicated the relation between the sound pressure p and particle velocity $\vec{v}$. It can be rewritten as

$$\vec{v} = -\frac{1}{\rho_0}\int \nabla p dt \tag{8}$$

According to Eq.(8), the particle velocity fields $\vec{v}$ can be obtained if the sound pressure fields have been calculated.

3 Theory of digital waveguide mesh

Digital waveguide and wave equations

In cylindrical coordinates (r,θ,z), according to the digital waveguide theory, the set of partial differential equations describing a lossless, source-free parallel-plate transmission line in (2+1) dimension is the following equations [13]:

$$l_r \frac{\partial i_r}{\partial t} + \frac{\partial u}{\partial r} = 0 \tag{9-1}$$

$$l_r \frac{\partial i_z}{\partial t} + \frac{\partial u}{\partial z} = 0 \tag{9-2}$$

$$l_\theta \frac{\partial i_\theta}{\partial t} + \frac{\partial u}{\partial \theta} = 0 \tag{9-3}$$

$$c_u \frac{\partial u}{\partial t} + \frac{\partial i_r}{\partial r} + \frac{\partial i_z}{\partial z} + \lambda \frac{\partial i_\theta}{\partial \theta} = 0 \tag{9-4}$$

where i and u are the current in and voltage across the lines, l and c are the inductance and capacitance per unit length, and $l_r = l/r$, $l_\theta = rl\lambda$, $c_u = rc$, $\lambda = \Delta\theta/\Delta r$. If we assume that l and c are constant, then the set of equations can be reduced to a single second-order equation in the voltage alone.

$$\nabla^2 u - \frac{1}{\gamma^2}\frac{\partial^2 u}{\partial t^2} = 0 \tag{10}$$

where $\gamma = 1/\sqrt{lc}$. For a two dimensional sound problem, the centered difference scheme can be applied to Eq.(9) with the interleaved grid as shown in Fig.1 [13]. Then the ccurrent i and voltage u can be calculated in spatial and temporal dimensions.

Comparing Eq.(9) with the equations in section 2, it can be found that the current i is equivalent to particle velocity and the voltage u is equivalent to sound pressure. So digital waveguide mesh method can be used to simulate the acoustic vector field for underwater sound propagation.

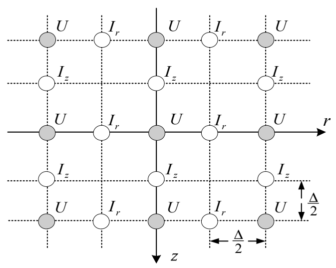

Fig.1 Interleaved computational grid for the (2+1)D

3.1 Boundary conditions

When the digital waveguide mesh method is applied to model underwater acoustic vector fields, the boundary conditions are needed for modeling changes in wave propagation media at ocean bottom. Several boundary condition treatment methods have been implemented in digital waveguide mesh. For underwater sound propagation, one can get the following relationships from boundary conditions at a horizontal interface as:

$$p^{(1)} = p^{(2)} \tag{11-1}$$

$$v_z^{(1)} = v_z^{(2)} \tag{11-2}$$

where $p^{(1)}$ and $v^{(1)}$ represent the sound pressure and particle velocity in the water column, $p^{(2)}$ and $v^{(2)}$ represent the sound pressure and particle velocity in the ocean bottom. From Eq.(11), the relation between the current i and voltage u in the digital waveguide mesh can derived as

$$u^{(1)} = u^{(2)} \tag{12-1}$$

$$i_z^{(1)} = i_z^{(2)} \tag{12-2}$$

Then the acoustic vector fields in water column and ocean bottom can be calculated using Eq.(9) separately with the interleaved grid for the ocean bottom boundary as shown in Fig.2.

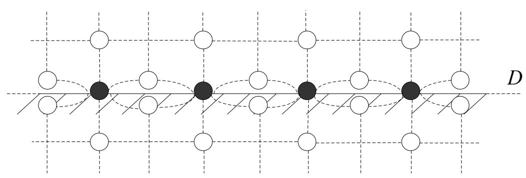

Fig.2 Interleaved computational grid on ocean bottom

We define grid function $I_{r,i,j}(n)$, $I_{z,i,j}(n)$, and $U_{i,j}(n)$, which run over half-integer values of i, j and n, that is,

$$i,j,n = \ldots -1, -\frac{1}{2}, \frac{1}{2}, 1\ldots$$

Then the difference scheme can be obtained as:

$$I^{(1)}_{r,i+\frac{1}{2},j}(n+\frac{1}{2}) - I^{(1)}_{r,i+\frac{1}{2},j}(n-\frac{1}{2}) + \frac{1}{v_0 \bar{l}^{(1)}_{i+\frac{1}{2},j}}(U^{(D)}_{i+1,j}(n) - U^{(D)}_{i,j}(n)) = 0 \tag{13-1}$$

$$I^{(2)}_{r,i+\frac{1}{2},j}(n+\frac{1}{2}) - I^{(2)}_{r,i+\frac{1}{2},j}(n-\frac{1}{2}) + \frac{1}{v_0 \bar{l}^{(2)}_{i+\frac{1}{2},j}}(U^{(D)}_{i+1,j}(n) - U^{(D)}_{i,j}(n)) = 0 \tag{13-2}$$

$$I^{(1)}_{z,i,j-\frac{1}{2}}(n+\frac{1}{2}) - I^{(1)}_{z,i,j-\frac{1}{2}}(n-\frac{1}{2}) + \frac{1}{v_0 \bar{l}^{(1)}_{i,j-\frac{1}{2}}}(U^{(D)}_{i,j}(n) - U^{(1)}_{i,j-1}(n)) = 0 \tag{13-3}$$

$$I^{(2)}_{z,i,j+\frac{1}{2}}(n+\frac{1}{2}) - I^{(2)}_{z,i,j+\frac{1}{2}}(n-\frac{1}{2}) + \frac{1}{v_0 \bar{l}^{(2)}_{i,j+\frac{1}{2}}}(U^{(2)}_{i,j+1}(n) - U^{(D)}_{i,j}(n)) = 0 \quad (13\text{-}4)$$

$$U^{(D)}_{i,j}(n) - U^{(D)}_{i,j}(n-1) + \frac{1}{v_0(\bar{c}^{(1)}_{i,j} + \bar{c}^{(2)}_{i,j})}(I^{(1)}_{r,i+\frac{1}{2},j}(n-\frac{1}{2}) - I^{(1)}_{r,i-\frac{1}{2},j}(n-\frac{1}{2}) + I^{(2)}_{r,i+\frac{1}{2},j}(n-\frac{1}{2}) - I^{(2)}_{r,i-\frac{1}{2},j}(n-\frac{1}{2})) + \frac{2}{v_0(\bar{c}^{(1)}_{i,j} + \bar{c}^{(2)}_{i,j})}(I^{(2)}_{z,i,j+\frac{1}{2}}(n-\frac{1}{2}) - I^{(1)}_{z,i,j-\frac{1}{2}}(n-\frac{1}{2})) = 0 \quad (13\text{-}5)$$

where $U^{(D)}_{i,j}$ represent the sound pressure on the ocean bottom boundary, and

$$\bar{l}^{(k)}_{i,j} = l^{(k)}(i\Delta, j\Delta) + O(\Delta^2)$$

$$\bar{c}^{(k)}_{i,j} = c^{(k)}(i\Delta, j\Delta) + O(\Delta^2)$$

$$v_0 = \Delta / T$$

where k stands for either of 1 or 2, represent the parameter in the water column or in the ocean bottom.

4 Numerical examples

In this section, we present solutions that demonstrate the accuracy and capability of the digital waveguide mesh method for acoustic vector field modeling. We compare the solution with the results obtained using normal mode method or fast-field processing method (FFP).

As shown Fig.3, example A involves a 100Hz source at $z = 4$ m in 50m depth water column with an absolutely rigid bottom. The sound speed of the water column is 1500m/s.

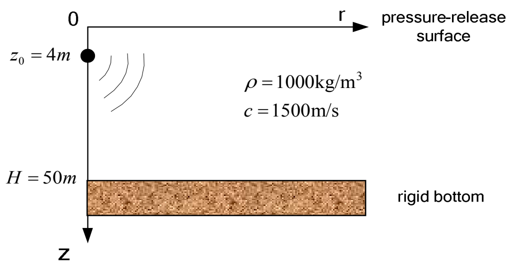

Fig.3 Environment parameters for example A

The calculated temporal waveform of the for the sound pressure and particle velocity signal received at range 1000, on the depth 10m is shown in Fig.4. The transmission loss of the sound pressure, horizontal particle velocity and vertical particle velocity calculated using digital waveguide mesh method with grid spacing $\Delta_r = \Delta_z = 1.0m$ are compared with the results obtained using normal mode method in Fig.5, Fig.6 and Fig.7 separately. The results with grid spacing $\Delta_r = \Delta_z = 0.5m$ are also shown in Fig.8, Fig.9 and Fig.20. It can be seen that the results are agree very well if the grid spacing is small enough in digital waveguide mesh.

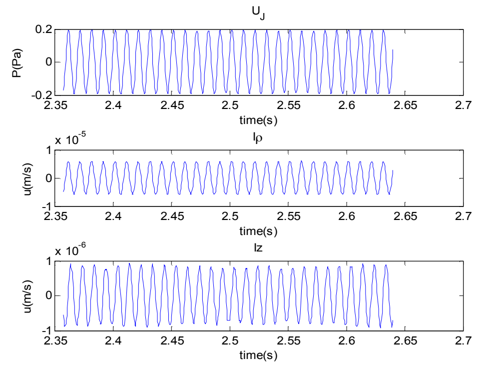

Fig.4 Calculated temporal waveform of the signal for sound pressure and particle velocity

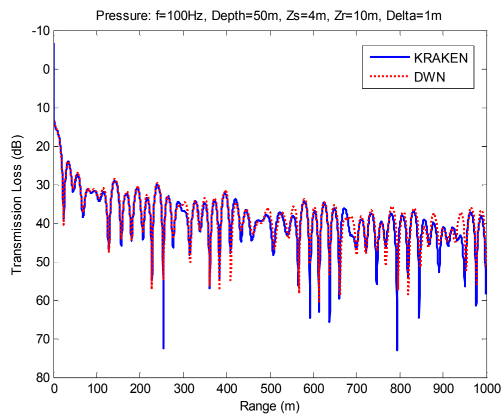

Fig.5 Transmission loss for sound pressure

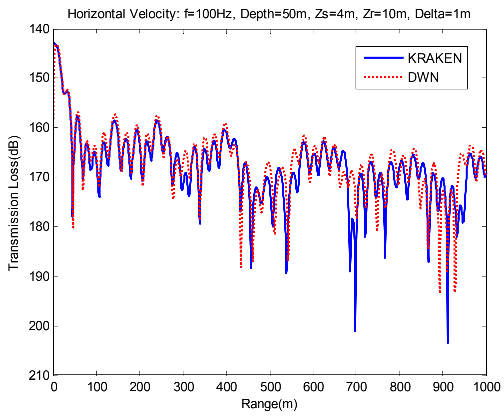

Fig.6 Transmission loss for horizontal particle velocity

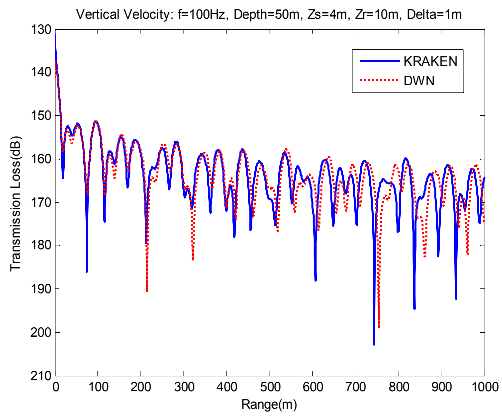

Fig.7 Transmission loss for vertical particle velocity

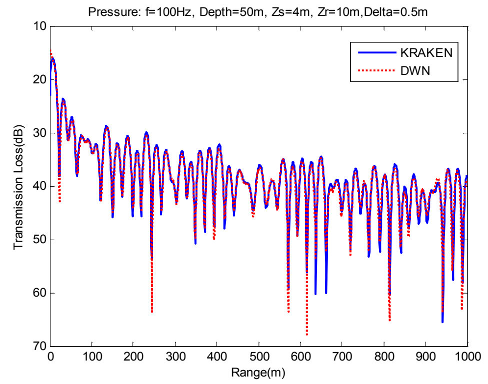

Fig.8 Transmission loss for sound pressure

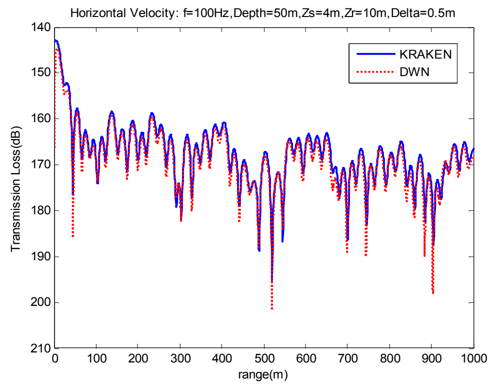

Fig.9 Transmission loss for horizontal particle velocity

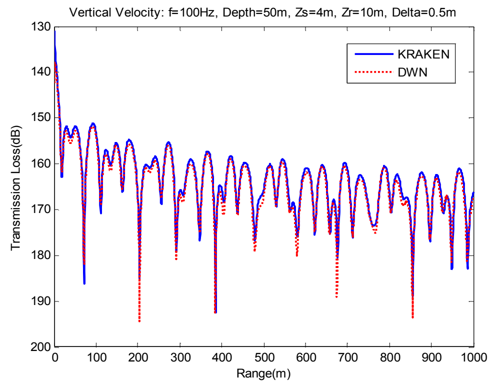

Fig.10 Transmission loss for vertical particle velocity

Example B involves a 50Hz source at $z = 25$ m. The depth of the water column is 100m and the thickness of the sediment layer is also 100m. The lower boundary of the sediment is selected as an absolutely soft boundary condition. The environment parameters are shown in Fig.11.

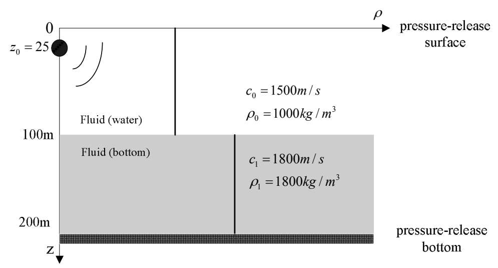

Fig.11 Environment parameters for example B

The transmission loss for the acoustic vector fields at depth 30m are calculated with program KRAKEN, FFP and DWG method [14]. The results of KRAKEN and DGW method are shown in Fig.12, Fig.13 and Fig.14. The comparison between the results of DWG method and FFP method are given in Fig.15, Fig.16 and Fig.17.

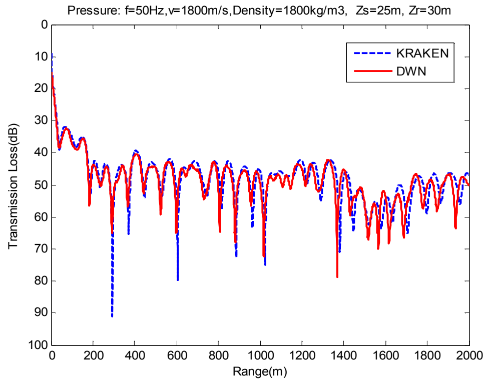

Fig.12 Transmission loss for sound pressure

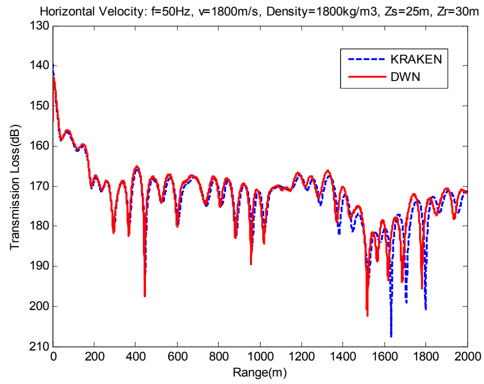

Fig.13 Transmission loss for horizontal particle velocity

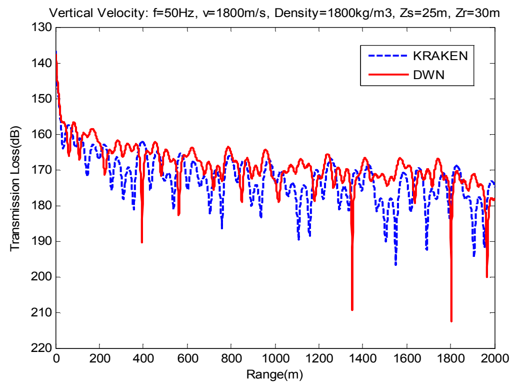

Fig.14 Transmission loss for vertical particle velocity

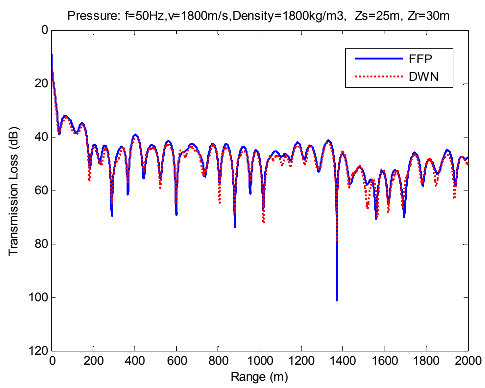

Fig.15 Transmission loss for sound pressure

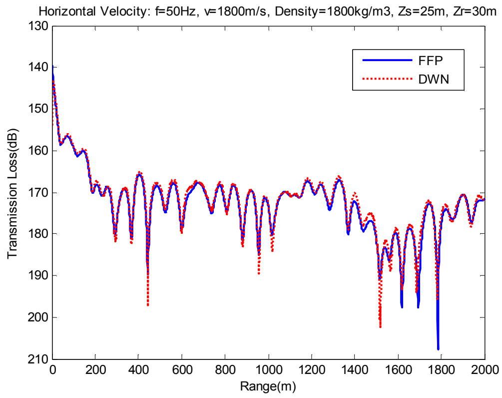

Fig.16 Transmission loss for horizontal particle velocity

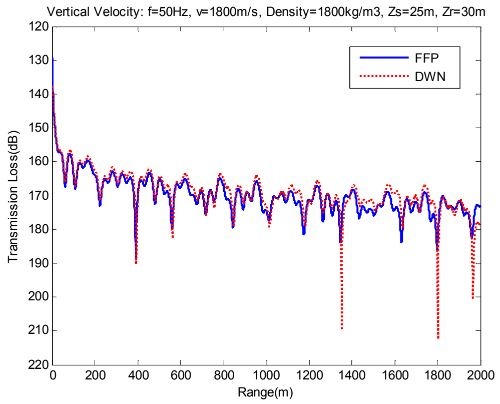

Fig.17 Transmission loss for vertical particle velocity

From these figures, it can be found that the error of transmission loss for vertical particle velocity is greater than the errors for sound pressure and horizontal particle velocity. It is shown that the results obtained with DWG method and FFP method are agreed very well and the difference between the DWG method and normal mode results is greater. As the Fast-field model has considered the contribution of branch-cut integral, the results of FFP method is more accurate than the results obtained with KRAKEN method in the near field. So it can be deduced that the DWG method is accurate in the near field acoustic vector field modeling.

5 Conclusion

In this paper, the digital waveguide mesh method is used to model the acoustic vector fields for underwater sound propagation. A boundary structure is introduced for modeling the ocean bottom conditions for the digital waveguide mesh calculation. Compared with the results of normal mode method and fast-field model, it is shown that the digital waveguide mesh method gives remarkably accurate results for near-field acoustic vector fields modeling in spatial and temporal dimensions. Using the calculated sound pressure and particle velocity results, the waveforms for the received signals and distribution of acoustic intensity in the underwater sound channel can also be obtained. Numerical simulation shows that the digital waveguide mesh method can be used to simulate the low frequency two-dimensional acoustic vector fields in shallow water and this method can be easily extended to three-dimensional acoustic vector fields calculation.

Acknowledgments

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