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Stationary structures in acoustically active nonequilibrium media with one relaxation process

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In the present paper it is investigated the solutions of a general acoustical equation, describing in the second order perturbation theory a nonlinear evolution of wide spectrum acoustical disturbances in nonequilibrium media with one relaxation process. Stationary structures of general equation, the conditions of their establishment and all their parameters are found analytically and numerically. In acoustically active media it is predicted the existence of the stationary solitary pulse. It is considered 1-D relaxing gas dynamics system of equations with simple Landau-Teller model of relaxation. The possible stationary profiles are shown in nonequilibrium degree- stationary wave speed bifurcation diagram. The boundaries of this diagram are obtained in analytical forms. The field of weak shock wave instability is shown in this bifurcation diagram. Unstable shock wave disintegrates into the sequence of solitary pulses described by the general acoustical equation.

1 Introduction

For the last years, a large number of experiments have been demonstrating the unusual shock wave modification in nonequilibrium media. Chemical active mixtures with irreversible reactions, vibrationally excited gases, nonisothermal plasmas are examples of acoustically active nonequilibrium media. In such media it is possible the existence of stationary nonlinear structures that are different from the step-wise shock wave structures. In particularly, the shock wave amplification, the shock front splitting and the precursor generation were observed in weakly ionized gases [1-7]. One of the reasons of these structure changes can be connected with the new acoustical properties of nonequilibrium media. Acoustics of thermodynamically nonequilibrium media differs significantly from the acoustics of equilibrium media [8]. In the nonequilibrium media, the second (bulk) viscosity coefficient ξ and sound dispersion can be negative: $\xi < 0$ and $c_0 > c_\infty$. Here, c_0 and c_∞ are the equilibrium (low-frequency) and frozen (high frequency) sound velocities, respectively. The media possessing negative viscosity can be acoustically active. The acoustical increment in these media has simple form

$$\alpha = \frac{\omega^2[\xi(\omega)+\mu]}{2\rho_0 c_s^3(\omega)}$$

where $\mu = 4\eta/3 + \chi(1/C_V - 1/C_P)$, η, χ are the shear viscosity and the thermal conductivity coefficients; c_S is the sound speed, ρ_0 is density, C_V, C_p are specific heats at constant volume and pressure respectively. The general condition of acoustically instability is $\xi(\omega) + \mu < 0$.

Moreover, the low-frequency coefficient of gas dynamic nonlinearity $\Psi_0 \neq (\gamma_0 + 1)/2$. Besides, Ψ_0 is a complicated function on a nonequilibrium degree. The frozen coefficient of gas dynamic nonlinearity has the usual form

$$\Psi_\infty = (\gamma_\infty + 1)/2$$

These new acoustical properties of nonequilibrium media should be taken into account in studies of different gasdynamic phenomena.

In the present work, we investigate the qualitative influence of the nonequilibrium on the shock wave structure in nonequilibrium vibrationally excited gas with the simplest exponential relaxation law:

$$\frac{dE}{dt} = -\frac{E - E_e}{\tau} + Q \qquad (1)$$

Here, E is the energy of the vibrational degrees of freedom of the molecules, E_e is its equilibrium value, τ is the vibrational relaxation time, and Q is the energy source sustaining thermal nonequilibrium in the system (in particular, electric pumping in discharge).

In the first part of the present paper it is investigated the solutions of a general acoustical equation, describing in the second order perturbation theory a nonlinear evolution of wide spectrum acoustical disturbance. Its low- and high-frequency limits correspond to Kuramoto-Sivashinsky equation and the Burgers equation with a source, respectively. Stationary structures of general equation, the conditions of their establishment and all their parameters are found analytically and numerically. In acoustically active media it is predicted the existence of the stationary solitary pulse.

Then, we consider 1-D relaxing gas dynamics system of equations with simple Landau-Teller model of relaxation. The possible stationary profiles are shown in nonequilibrium degree- stationary wave speed bifurcation diagram. The field of weak shock wave instability is shown in this bifurcation diagram. Unstable shock wave disintegrates into the sequence of solitary pulses described by the general acoustical equation.

2 General acoustical equation

2.1 Equation and low- and high-frequency limits

The general acoustical equation has the form [9]:

$$\begin{aligned} &C_{V\infty}\tau_0\left(\rho_{tt} - c_\infty^2\rho_{xx} - \frac{c_\infty^2\Psi_\infty}{\rho_0}\rho_{xx}^2 - \frac{\mu_\infty}{\rho_0}\rho_{xxt}\right)_t + \\ &+ C_{V0}\left(\rho_{tt} - c_0^2\rho_{xx} - \frac{c_0^2\Psi_0}{\rho_0}\rho_{xx}^2 - \frac{\mu_0}{\rho_0}\rho_{xxt}\right) = 0 \end{aligned} \qquad (2)$$

where $c_\infty = \sqrt{\gamma_\infty T_0/M}$, $c_0 = \sqrt{\gamma_0 T_0/M}$ are the speeds of the high frequency and the low frequency sounds; $\gamma_\infty = C_{P\infty}/C_{V\infty}$, $\gamma_0 = C_{P0}/C_{V0}$; $C_{V0} = C_{V\infty} + C_K + S\tau_T$, $C_{P0} = C_{P\infty} + C_K + S(\tau_T + 1)$ are the low frequency specific heats in vibrationally excited gases at constant volume or pressure; T_0, ρ_0, τ_0 are the stationary values; M is the molecular mass; $S = Q\tau_0/T_0$ is the nonequilibrium degree; $\tau_0 = \tau(T_0, \rho_0)$; $C_K = (dE_e/dT)_{T=T_0}$; $\tau_T = \partial \ln\tau_0/\partial \ln T_0$; $\mu_\infty = 4\eta/3 + \chi m(1/C_{V\infty} - 1/C_{P\infty})$,

$\mu_0 = 4\eta/3 + \chi m(1/C_{V0} - 1/C_{P0})$ are the high frequency and the low frequency shear viscosity – heat-capacity coefficients; $\Psi_\infty = (\gamma_\infty + 1)/2$ is the high frequency nonlinear coefficient; Ψ_0 is the low frequency nonlinear coefficient. It is important that the coefficient Ψ_0 depends on the nonequilibrium degree S and can be even negative. Eq. (2) is valid for the weak dispersion $\widetilde{m} = (c_0^2 - c_\infty^2)/c_\infty^2 \sim \theta << 1$.

For waves travelling in one direction ($\widetilde{\rho} = \rho/\rho_0$, $\varsigma = (x - c_\infty t)/c_\infty \tau_0$, $y = \theta t/\tau_0$), Eq. (2) reduces to:

$$(\widetilde{\rho}_y + \frac{\Psi_\infty}{2}\widetilde{\rho}_\varsigma^2 - \widetilde{\mu}_\infty \widetilde{\rho}_{\varsigma\varsigma})_\varsigma - \\ -\nu(\widetilde{\rho}_y + \frac{m}{2}\widetilde{\rho}_\varsigma + \frac{\widetilde{\Psi}_0}{2}\widetilde{\rho}_\varsigma^2 - \widetilde{\mu}_0 \widetilde{\rho}_{\varsigma\varsigma}) = 0 \tag{3}$$

where $\widetilde{\mu} = \mu/2\tau c_s^2 \rho_0$, $\widetilde{\Psi}_0 = \gamma_0 \Psi_0/\gamma_\infty$, $\nu = C_{V0}/C_{V\infty}$.

In the low frequency approximation ($\partial \widetilde{\rho}/\partial y \sim \theta \widetilde{\rho}$), Eq. (3) reduces with an accuracy to $\sim \theta^3$ to the modified Kuramoto-Siwaszynski equation:

$$\widetilde{\rho}_y + \widetilde{\Psi}_0 \widetilde{\rho}\widetilde{\rho}_\varsigma = \mu_\Sigma \widetilde{\rho}_{\varsigma\varsigma} + \widetilde{\beta}\widetilde{\rho}_{\varsigma\varsigma\varsigma} + \widetilde{\kappa}\widetilde{\rho}_{\varsigma\varsigma\varsigma\varsigma} \tag{4}$$

In Eq. (4) $\mu_\Sigma = \widetilde{\mu}_0 + \widetilde{\xi}$, $\widetilde{\xi} = \xi_0/2\rho_0\tau_0 c_0^2$, where

$$\xi_0 = \frac{\rho_0 \tau_0 C_{V\infty}(c_\infty^2 - c_0^2)}{C_{V0}} = \frac{P_0 \tau_0 [(\tau_T - C_{V\infty})S + C_K]}{C_{V0}^2}$$

is the second viscosity coefficient, $\widetilde{\kappa} = C_{V0}\widetilde{\beta}/C_{V\infty} = C_{V0}^2 \widetilde{\xi}/C_{V\infty}^2$ (with neglect of $\sim \widetilde{\mu}_0^2, \widetilde{\mu}_0 \widetilde{\xi}$). For $C_{V0} > 0$, all these coefficients are negative if the second viscosity coefficient is negative, that is, for $(\tau_T - C_{V\infty})S + C_K < 0$.

In the high frequency approximation ($\partial \widetilde{\rho}/\partial y \sim \theta^{-1} \widetilde{\rho}$), Eq. (2) reduces (with an accuracy to $\sim \theta^2$) to the Burgers equation with a source and integral dispersion

$$\widetilde{\rho}_y + \Psi_\infty \widetilde{\rho}\widetilde{\rho}_\varsigma = \widetilde{\mu}_\infty \widetilde{\rho}_{\varsigma\varsigma} - \widetilde{\alpha}_\infty \widetilde{\rho} - \widetilde{\beta}\int \widetilde{\rho} d\varsigma \tag{5}$$

where $\widetilde{\alpha}_\infty = \xi_0 C_{V0}^2 / C_{V\infty}^2 \rho_0 \tau_0 c_\infty^2$ is the dimensionless gain (at $\xi_0 < 0$) of the high frequency sound, $\widetilde{\beta} \approx C_{V0}\alpha_\infty / C_{V\infty}$ is the dispersion coefficient. The solutions of Eq. (4) and (5) are well known. The shortcoming of these equations is their disability to describe a nonstationary evolution of disturbances with a wide spectrum. Moreover, a spectrum of their stationary structures is wider than their application region.

The evolution of a disturbance with an arbitrary spectrum must be investigated on the basis of the complete equation (3), as in this study.

2.2 Stationary structures

For $\Psi_0 > 0$, $C_{V0} > 0$ and the negative second viscosity, Eq. (3) describes three stationary structures that are shown in Fig. 1 [10].

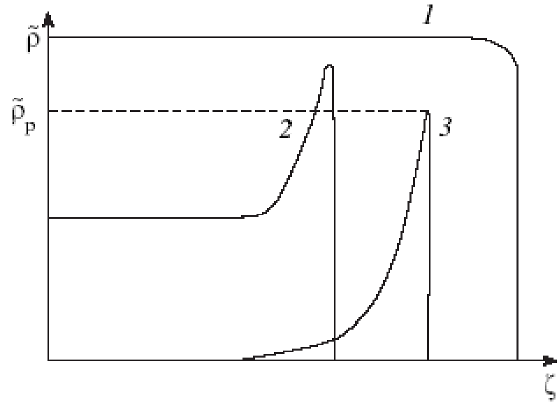

Fig. 1. Stationary structures of Eq. (3).

The most interesting structure is strongly asymmetric solitary pulse (curve3, Fig. 1) with the shock front width $\sim \widetilde{\mu}_\infty / \mu_\Sigma << 1$ and exponential trail

$$\widetilde{\rho} = \widetilde{\rho}_p \exp \nu \widetilde{\Psi}_0 \varsigma / 2\Psi_\infty \tag{6}$$

where $\widetilde{\rho}_p = -4\nu\mu_\Sigma/(2\Psi_\infty - \widetilde{\Psi}_0)$.

2.3 Numerical simulation of equation (3)

The initial step-like disturbance with amplitude $\widetilde{\rho} > \widetilde{\rho}_{cr} = -2\nu\mu_\Sigma/(\Psi_\infty - \widetilde{\Psi}_0)$ transformed to the first stationary structure (curve 1, Fig. 1). The second structure (curve2, Fig. 1) was obtained for $\widetilde{\rho}_{cr} > \widetilde{\rho} > \widetilde{\rho}_{cr1} = -2\nu\mu_\Sigma/(2\Psi_\infty - \widetilde{\Psi}_0)$. The steps with amplitudes $\widetilde{\rho} < \widetilde{\rho}_{cr1}$ were unstable and broke down into a periodic sequence of stationary pulses (Fig.2). Each pulse had previous form and amplitude $\widetilde{\rho}_p$ (curve3, Fig. 1). Thus, such pulse is autowave (self- wave), whose form and amplitude depend on parameters of the nonequilibrium medium only.

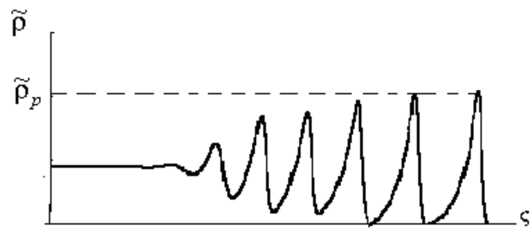

Fig. 2. Nonstationary disintegration of a small-amplitude step into pulse autowaves

The bell-like disturbance (Fig. 3a) transformed into the same pulses and roll waves in trail (Fig. 3b). These roll waves were autowave too. Their amplitude and period did not depend on amplitude and square of the initial bell-like disturbance.

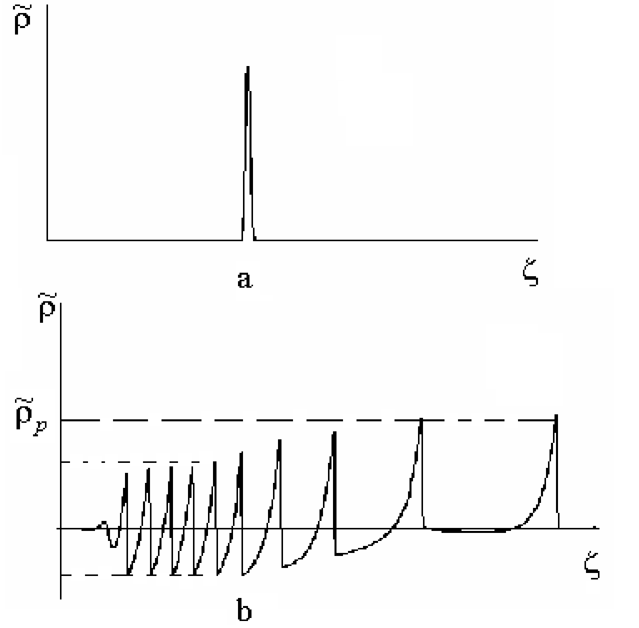

Fig. 3. Nonstationary disintegration of a bell (a) into pulse and roll autowaves (b)

3 Gas dynamic relaxation system

3.1 Shock adiabats in Nonequilibrium Medium

The initial system of gas dynamics contains Eq. (1) and the following equations:

$$P=\frac{\rho T}{M},\quad \frac{d\rho}{dt}+\rho\frac{\partial \mathrm{v}}{\partial x}=0$$

$$\rho\frac{d\mathrm{v}}{dt}=-\frac{\partial P}{\partial x} \tag{7}$$

$$C_{V\infty}\frac{dT}{dt}+\frac{dE}{dt}-\frac{T}{\rho}\cdot\frac{d\rho}{dt}=Q-I$$

where v, T, ρ, P are, respectively, the velocity, temperature, density, and pressure, $Q=I$ is the heat removal and $d/dt=\partial/\partial t+v\partial/\partial x$.

The gas with stationary nonequilibrium has the five fields of the nonequilibrium degree S with qualitatively different properties [8,12,13]:

Field 1: $S<S_{th}=C_K/(C_{V\infty}-\tau_T)$. Here, we have the positive second (bulk) viscosity $\xi_0>0$, the positive dispersion $c_0<c_\infty$, and the positive nonlinearity coefficient $\Psi_0\approx(\gamma_0+1)/2$ similar to equilibrium media.

Field 2: $S_{th}<S<S_n$. The dispersion and the second viscosity are negative ($\xi_0<0\,; c_0>c_\infty$). In fields 2-5 media are acoustically active. The low frequency nonlinear coefficient $\Psi_0>0$. Here S_n is defined from the equation $\Psi_0(S_n)=0$, where

$$\Psi_0=\left[\frac{S\tau_T(1+S)}{C_{P0}C_{V0}}+\frac{1+2C_{V0}}{2C_{V0}}-\frac{S(1+S)^2}{2C_{P0}C_{V0}^2}\tau_{TT}\right]$$

$$\tau_{TT}=\frac{T_0^2}{\tau_0}\frac{\partial^2\tau_0}{\partial T_0{}^2}$$

Field 3: $S_n<S<S_V=\dfrac{C_{V\infty}+C_K}{-\tau_T}$. Here, $\xi<0$, $c_0>c_\infty$, $\tilde{\Psi}_0=\gamma_0\Psi_0<0$.

Field 4: $S_V<S<S_P$; $S_P=\dfrac{C_{P\infty}+C_K}{-(\tau_T+1)}$. ; Here, $\xi<0$, $\tilde{\Psi}_0>0$, $C_{V0}<0$, $C_{P0}>0$.

Field 5: $S>S_P$. Here, $\xi<0, c_0<c_\infty$, $\tilde{\Psi}_0>0$, $C_{V0}<0$, $C_{P0}<0$.

In relaxation gas dynamics, two shock adiabats drawn through a given initial point (P_0, V_0) are considered. One corresponds to total equilibrium of the final states of the gas and, therefore, is called the equilibrium adiabat. The other, referred to as "frozen" assumes that the relaxation processes do not proceed at all. These adiabats can be obtained from the general Rankine-Hugoniot expression

$$\varepsilon_0-\varepsilon_1+\frac{1}{2}(V_0-V_1)(P_0+P_1)=0$$

where subscripts 0 and 1 correspond to stationary states before and after the shock front, $V=1/\rho$ is the specific volume, ε is the specific inner energy.

The frozen adiabat corresponds to $\varepsilon_0=C_{V\infty}T_0+E_0$, $\varepsilon_1=C_{V\infty}T_1+E_1$, where $T=MPV$, from which it follows

$$\frac{P_1}{P_0}=\frac{(\gamma_\infty+1)V_0-(\gamma_\infty-1)V_1}{(\gamma_\infty+1)V_1-(\gamma_\infty-1)V_0}$$

The equilibrium adiabat corresponds to

$$\varepsilon_0=C_{V\infty}T_0+E_0=(C_{V\infty}+S)T_0+E_e(T_0)$$

$$\varepsilon_1=C_{V\infty}T_1+E_1=(C_{V\infty}+S\frac{\tau_1}{T_0})T_1+E_e(T_1)$$

For Landau-Teller dependence

$$\tau(T,\rho)=B\frac{\exp(b/\sqrt[3]{T})}{\rho\sqrt{T}}$$

and an equilibrium vibrational energy in harmonic - oscillator form

$$E_e=\frac{\theta}{\exp(\frac{\theta}{T})-1},$$

where B, b, and θ are constants, we obtain [12]:

$$C_{V\infty}(P_0V_0-P_1V_1)+\frac{1}{2}(V_0-V_1)(P_0+P_1)+$$
$$SP_0V_0\left[1-\sqrt{\frac{V_1P_0}{V_0P_1}}\exp\left(\frac{b}{\sqrt[3]{MP_1V_1}}-\frac{b}{\sqrt[3]{MP_0V_0}}\right)\right]+$$
$$\frac{\theta}{M}\left\{\left[\exp\left(\frac{\theta}{MP_0V_0}\right)-1\right]^{-1}-\left[\exp\left(\frac{\theta}{MP_1V_1}\right)-1\right]^{-1}\right\}=0.$$

For $S \neq 0$, the equilibrium adiabat has two branches with two asymptotes $P \to \infty$ (Figure 4).

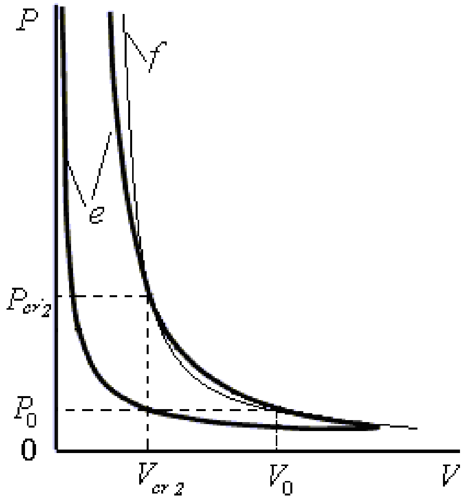

Fig. 4. An equilibrium adiabat. Initial states 1-5 corresponds to five different fields of nonequilibrium

There is the point (P_{cr2}, V_{cr2}) where the frozen and equilibrium adiabats meet. With increase of the nonequilibrium degree, the initial point (P_0, V_0) on the equilibrium adiabat moves from the upper branch to the lower branch.

3.2 Shock Wave Structures. Bifurcation Diagram

The system of equations (7) for stationary waves propagating with the speed D reduces to one equation [12]

$$\frac{d\rho}{dz} = -\frac{\rho\{[E_e(\rho) - E(\rho)]/\tau(\rho) + Q\}}{\rho_0 D(dE/d\rho)} \equiv \frac{A(\rho)}{B(\rho)} \qquad (8)$$

$$E(\rho) = E_0 + M[C_{P\infty}\frac{P_0}{\rho_0} + \frac{D^2}{2} - \\ -\frac{C_{P\infty}}{\rho}(P_0 + \rho_0 D^2(1 - \frac{\rho_0}{\rho})) - \frac{1}{2}(\frac{\rho_0 D}{\rho})^2]$$

where $z = x - Dt$.

The shock wave structure after the sharp front ρ_d, which is equal to

$$\frac{\rho_d}{\rho_0} = \frac{(\gamma_\infty + 1)D^2}{(\gamma_\infty - 1)D^2 + 2c_\infty^2}$$

was obtained using the numerical solution of an equation (8).

Eq. (8) can be investigated on the phase plane. All results can be presented in the bifurcation diagram (Figure 5 d). Here, the implicit forms of boundaries D_{cr1}, D_{cr2} are

$$S = \frac{\dfrac{\theta/T_0}{\exp\{\frac{\theta}{T_1}\} - 1} - \dfrac{\theta/T_0}{\exp\{\frac{\theta}{T_0}\} - 1} + \dfrac{\gamma_\infty}{2(\gamma_\infty{}^2 - 1)}\dfrac{(D_{cr1}^2 - c_\infty{}^2)^2}{c_\infty{}^2 D_{cr1}^2}}{1 - \dfrac{c_\infty}{D_{cr1}}\exp\{\frac{b}{\sqrt[3]{T_1}} - \frac{b}{\sqrt[3]{T_0}}\}}$$

$$T_1 = M\frac{(c_\infty{}^2 + \gamma_\infty D_{cr1}^2)^2}{\gamma_\infty(\gamma_\infty + 1)^2 D_{cr1}^2}$$

$$S = \frac{\dfrac{\theta/T_0}{\exp\{\frac{\theta}{T_2}\} - 1} - \dfrac{\theta/T_0}{\exp\{\frac{\theta}{T_0}\} - 1}}{1 - \sqrt{\dfrac{T_0}{T_2}}\dfrac{((\gamma_\infty - 1)D_{cr2}^2 + 2c_\infty{}^2]}{(\gamma_\infty + 1)D_{cr2}^2}\exp\{\frac{b}{\sqrt[3]{T_2}} - \frac{b}{\sqrt[3]{T_0}}\}}$$

$$T_2 = T_0\frac{[(\gamma_\infty - 1)D_{cr2}^2 + 2c_\infty{}^2][2\gamma_\infty D_{cr2}^2 - (\gamma_\infty - 1)c_\infty{}^2]}{(\gamma_\infty + 1)^2 c_\infty{}^2 D_{cr2}^2}$$

On this diagram D_{cr2} is the speed, which corresponds to case when $A(\rho_d) = 0$. On the shock adiabats it means that chord meets the point P_{cr2}, V_{cr2} and shock wave has the step-wise structure. The speed D_{cr1} corresponds to case when high-frequency sound speed equals flow speed behind the front of the shock wave. D_t is speed when low-frequency sound speed equals flow speed behind the shock wave front. In field III of the bifurcation diagram the solutions of (7) in form of shock wave propagating with constant velocity is not exist.

3.3 Weak shock wave evolution

By the numerical solution of initial system of gas dynamic equations (1), (7) we obtained the following results.

The strong shock waves corresponding Zones I and II on the bifurcation diagram are evolutionary stable. In the Zone III the condition mechanically stability of shock waves (the flow behind the shock wave front must be subsonic) is broken.

For $S_{thr} < S < S_e$ gas behind the shock wave front is acoustically active and its dispersion is negative. Therefore, a small step-wise disturbance is transformed into the sequence of autowave pulses (Fig. 5c) with amplitude ρ_d or autowaves with non-zero asymptote (Fig. 5b), propagating with $D = D_{cr1}(S)$. For weak nonequilibrium degree, these autopulses have the shock front and exponential "trail" (6).

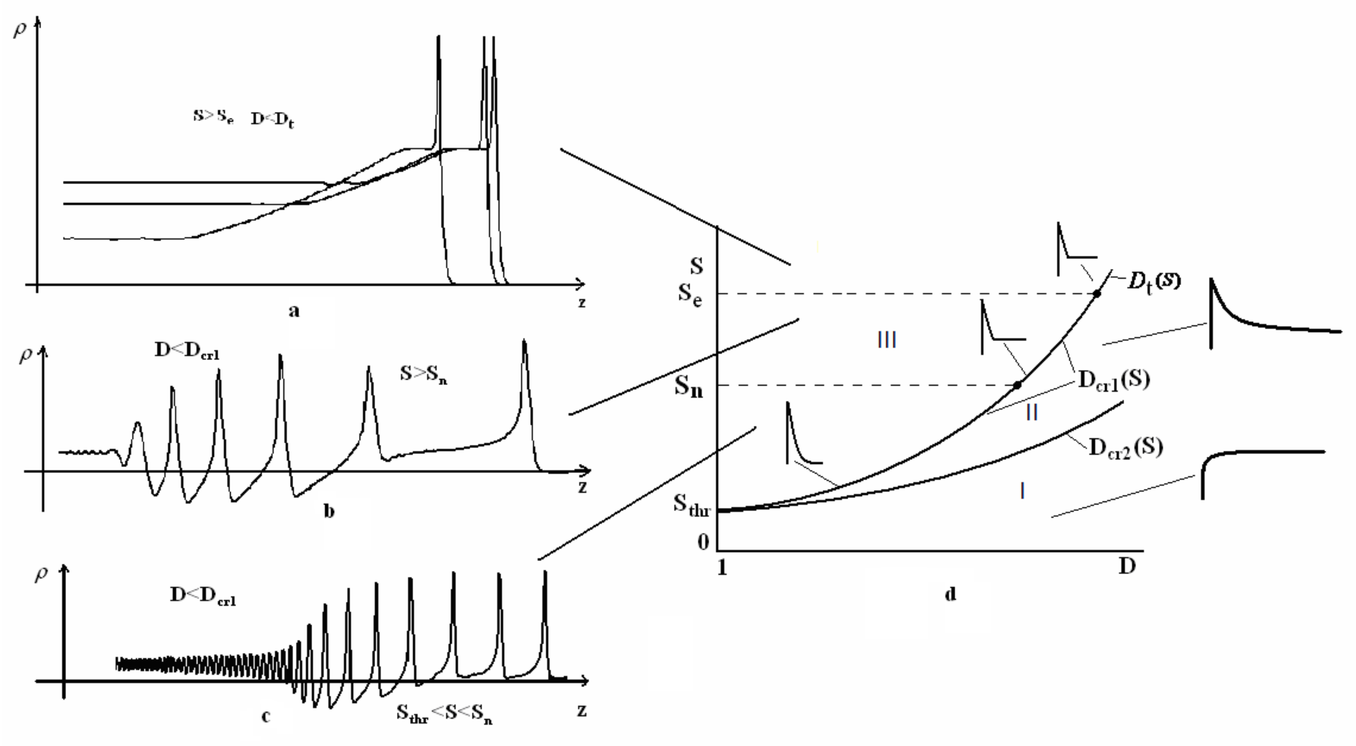

Fig.5 Weak shock wave evolution (a,b,c) and bifurcation diagram (d)

For $S > S_e$ gas behind the shock wave front is acoustically passive and its dispersion is positive. Therefore, a small disturbance is transformed into one autowave with non-zero asymptote (Fig. 5a), propagating with $D = D_t(S)$.

Conclusion

Action of vibrational nonequilibrium sustained by heat source is more significant for weak shock waves. Weak shock waves are unstable in acoustical active nonequilibrium media. In dependence on nonequilibrium degree, the unstable wave accelerates and disintegrates into sequence of self-sustained structures: autopulses or autowaves with non zero asymptote.

Acknowledgments

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