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Sound propagation in viscoelastic pipe with liquid-bubble mixture

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Propagation of acoustic waves in thin-walled polymeric tube with viscous liquid is investigated. Dynamics of the tube – liquid interaction is described in conjugated quasi-one-dimensional approximation; the tube material is supposed to follow linear viscoelastic model with appropriate choice of compliance function. It is assumed that the liquid contains fine air bubbles; the concentration of free gas is supposed to be small. Compressibility of liquid in the wave in the presence of bubbles can be almost entirely attributed to compressibility of the gas phase; it is accounted for within dispersion equation for bubbly liquid. Both heat and viscous losses are included in the phase interaction description at the liquid-gas interface. The resulting dispersion equation for the waveguide with liquid-gas mixture is studied in a long-wave range, where sound length is larger from the waveguide diameter. Results of simulations illustrate strong influence of the pipe mechanical properties and parameters of the gas phase on sound dispersion and attenuation.

1 Introduction

Non-stationary processes in pipes with flowing liquid are of great importance for modern power engineering, chemical technology, biotechnology etc. One of the most studied problems, connected with such flows, is sound propagation in the system. Dispersion and attenuation of acoustic waves in tubes with viscous liquid were investigated in a large number of papers; note here [1,2]. The features of liquid viscoelasticity in acoustics of different waveguides were studied in [3-5]. Both tube and liquid properties influence the wave propagation, and relaxational processes in the liquid phase can lead to essential changes in sound dispersion. However, relaxation features in the wave can be originated not only by fluid rheology; another possible reason for frequency dependent response of the liquid phase to periodic pressure changes can be connected with fine bubbles, which very often are trapped by flowing liquid or appear as a result of chemical reaction, technological process or pressure drops. It was shown in [6] that microbubbles even in small amount can influence essentially both low and high-frequency dispersion in a thin-walled elastic tube with polymeric liquid inside.

In the present paper combined effect of small gas bubbles and tube's wall viscoelasticity is investigated within conjugated quasi-one-dimensional formulation. The pipe is treated as a thin cylindrical shell, made from polymeric material, following general linear viscoelastic model. Dynamics of the shell is described within Kirchhoff-Love approximation. The internal problem of momentum transfer is solved in approximation based upon long-wave approach [3]. Acoustic wave propagation in viscoelastic tubes with pure viscous liquid was studied in a number of papers; note, in particular, [7,8].

2 Rheological characterization of the tube wall

The basic assumption of the model, formulated below, is that the pipe diameter is much smaller than the wavelength *l*. The tube is considered as a circular cylindrical shell with the radius of the middle surface R and a constant small thickness $2h$ ($\varepsilon = h/R << 1$). In the case of acoustically induced oscillations in the system all mechanical characteristics (flow velocity, deformations, pressure, etc.) are proportional to $exp(i\omega t)$ and viscoelastic properties of the shell can be accounted for by using of the complex creep compliance $I^* = I_0 + I'(\omega) - iI''(\omega)$ instead of pure elastic parameter I [9]. The Poisson module ν is supposed to be constant [8]. The storage and loss compliances of the shell material $I'(\omega), I''(\omega)$ can be calculated through the creep function $\psi(t)$. Usually the concept of discrete spectrum of retardation times is used (generalized Kelvin-Voight model [9]) that leads to the following equations for I^*:

$$I^* = I_0 + \int_0^\infty \frac{\partial \psi}{\partial t'} e^{-i\omega t'} dt' \tag{1}$$

$$\psi(t) = \int_0^\infty \Phi(\lambda)(1 - e^{-t/\lambda}) d\lambda, \quad \Phi(\lambda) = \sum_{k=1}^n I_k \delta(\lambda - \lambda_k)$$

$$I'(\omega) = \sum_{k=1}^n \frac{I_k}{1 + (\omega\lambda_k)^2}, \quad I''(\omega) = \sum_{k=1}^n \frac{I_k(\omega\lambda_k)}{1 + (\omega\lambda_k)^2}$$

As it follows from Eq.(1), at high frequencies ($\omega\lambda >> 1$) mechanical behaviour of the material is characterized by instantaneous compliance I_0 ($I^* \approx I_0$), while at $\omega\lambda << 1$ the same role plays equilibrium compliance I_∞ ($I^* \approx I_\infty + I_0 + \sum_{k=1}^n I_k$). Dynamic Young module of the material E^* and its limiting values at small and high frequencies are connected with similar creep characteristics by the relations $E^* I^* = 1$, $E_0 = 1/I_0$, $E_\infty = 1/I_\infty$.

3 Internal problem of momentum transfer

3.1 Dynamics of fluid flow in the wave

The model assumptions about liquid dynamics in tube at acoustic excitation imply:

$$\frac{\partial^2 v_x}{\partial x^2} << \frac{1}{r}\frac{\partial v_x}{\partial r}, \quad \frac{\partial^2 v_x}{\partial x^2} << \frac{\partial^2 v_x}{\partial r^2}, \quad \frac{\partial v_r}{\partial x} << \frac{\partial v_x}{\partial r}$$

$$v_r << v_x, \quad R/L << 1, \quad \dot{u}_x << V$$

Here $\vec{v} = \{v_x, v_r\}$ - vector of liquid velocity, L, R - tube length and radius of the shell middle surface, $\dot{u}_x$ - axial rate of shell displacement in the wave, V - the mean relative flow rate, defined by the relation:

$$V = \frac{2}{R^2}\int_0^R V_x r dr, \quad V_x = v_x - \dot{u}_x$$

Non-dimensional equations of liquid dynamics in the tube, averaged along cross section, can be written now in the form [3]:

$$i\Omega\hat{V} = i\kappa k\hat{P} + 2\kappa\hat{\tau}_R + \Omega^2\hat{u}_2 \tag{2}$$

$$i\Omega\,\hat{\rho} + 2i\Omega\hat{u}_1 - ik\hat{V} = 0 \tag{3}$$

$$\kappa = \rho_s / \rho_{f0}, \quad \Omega = \omega t_0, \quad t_0 = R(\rho_s / p_0)^{1/2}, \quad \tau = t / t_0$$

Here $\hat{P}$, $\hat{V}$, $\hat{\rho}$, $\hat{\tau}_R$, $\hat{u}_1$, $\hat{u}_2$ are complex amplitudes of dimensionless disturbances of mean pressure, flow velocity, liquid density, shear stress in liquid at the tube wall and non-dimensional displacements in shell u_1, u_2 in the radial and axial directions, respectively; k is the non-dimensional wave number; Ω - non-dimensional frequency; p_0 - equilibrium pressure in the waveguide; ρ_s, ρ_{f0} - densities of the shell material and liquid. In Eq.(3) the boundary condition $(v_r)_{|r=R} = \dot{u}_r$ for the radial liquid velocity at the wall is used. The amplitude of transient friction in liquid at the wall $\hat{\tau}_R$ is found from solution of the momentum balance equation for incompressible liquid flow in cylindrical tube (quasi-hydraulic approach [3]):

$$i\Omega\hat{v}_x = \hat{K} + \kappa\overline{\eta}_s\left(\frac{d^2\hat{v}_x}{d\xi^2} + \frac{1}{\xi}\frac{d\hat{v}_x}{d\xi}\right) \tag{4}$$

$$\overline{K} = -\kappa\frac{\partial\left(\Delta\overline{p}_f\right)}{\partial\zeta}, \quad \hat{K} = \overline{K}e^{-i\Omega\tau}$$

$$\Delta\overline{p}_f = (p_f - p_0)/p_0, \quad \overline{\eta}_s = \eta_s/(p_0 t_0)$$

Here $\{\xi, \zeta\} = R^{-1}\{r, x\}$ are non-dimensional radial and axial coordinates of the cylindrical coordinate system, p_f is the pressure in liquid in the wave, and η_s is the liquid viscosity. In Eq.(4) the difference between mean flow rate in the tube and mean relative flow rate is neglected in view of the obvious relation $\dot{u}_x << v_x$ [4]. The result has the form:

$$\hat{\tau}_R = -4\overline{\eta}_s D\hat{V} \tag{5}$$

$$D = -\frac{1}{4}\frac{\mu T(\mu)}{1 - 2\mu^{-1}T(\mu)}, \quad T(\mu) = \frac{J_1(\mu)}{J_0(\mu)}$$

$$\mu = i\left(\frac{i\Omega}{\kappa\overline{\eta}_s}\right)^{1/2}$$

where J_0, J_1 are Bessel functions of the first kind of the zero and first order, respectively. The physical meaning of the approximation used above is that we neglect both by small cross effect of the liquid viscosity and compressibility and by the influence of radial shell displacements on the losses at the liquid motion in the waveguide. Another assumption, involved here, is that the influence of bubbles on the friction losses at the fluid motion in the waveguide can be neglected also. It is valid for sufficiently small volume concentration of free gas. Note that at $\Omega \to 0$ $D \to 1$ and Eq.(5) coincides with Poiseuille relation between shear stress at the pipe wall and the average flow velocity in steady laminar flow of viscous liquid.

3.2 Free gas effect

Free gas, when it is present in liquid in the form of bubbles, strongly changes its acoustic properties [10]. The key features of sound propagation in bubbly liquid are high dispersion and low sound speed in a wide range of frequencies. Low velocity of acoustic signal is explained by high compressibility of gas coupled with large density of the liquid, while dispersion of sound waves results from relaxational behaviour of the mixture specific volume at pressure variations. Both sound speed and attenuation changes can be attributed to the mixture dynamic bulk module K_m, which in the presence of bubbles is frequency-dependent [6] and is linked to the complex sound speed in the mixture c_m by equation $K_m = \rho_{m0}c_m^2$, where ρ_{m0} is the mixture equilibrium density. Our analysis will be limited to small volume concentrations of free gas α_0 ($\alpha_0 \le 0.01$), when the bubbles influence on the mixture density can be neglected ($\rho_{m0} \approx \rho_{f0}$). As a result, the complex amplitudes of pressure and density variations in the wave are supposed to follow the equation:

$$\hat{P} = \kappa^{-1}\overline{c}_m^2\hat{\rho}, \quad \overline{c}_m = c_m t_0 / R \tag{6}$$

Sound speed in viscous liquid with fine bubbles can be calculated from dispersion equation, which is given below in the form [11]. Microbubbles are supposed to be uniformly distributed within the liquid; thermal dissipation is connected with rectified heat diffusion at liquid-bubbles interface; the thermal boundary layer thickness in liquid around pulsating bubble is much smaller from its radius; the pressure in bubbles is spatially homogeneous ($p_g = p_g(t)$); the input of liquid's compressibility in effective elasticity of the bubble can be neglected as compared with elasticity of gas [11]. It is implied also that the equilibrium bubble radius $a_0 << R$, which allows neglecting by interaction of bubbles with the tube wall. It was shown [12] that this assumption is acceptable for $\overline{a}_0 = a_0 / R << 0.1$.The resulting equation for non-dimensional complex sound speed in the mixture has the form:

$$\frac{1}{\overline{c}_m^2} = \frac{1}{\overline{c}_f^2} + \frac{3\alpha_0(1-\alpha_0)\kappa^{-1}}{2i\Omega\overline{\nu}_b + (\overline{\chi}_b - (\overline{a}_0^2/\kappa)\Omega^2)} \tag{7}$$

$$\overline{\nu}_b = \frac{\overline{a}_0^3}{\kappa}\frac{\Omega^2}{2\overline{c}_f} + 2\overline{\eta}_s + \frac{\operatorname{Im}\{D_T\}}{2\Omega}, \quad \overline{\chi}_b = \operatorname{Re}\{D_T\} - 2\overline{\sigma}/\overline{a}_0$$

$$D_T = \frac{3\overline{p}_{g0}\beta_T^2\gamma}{\beta_T^2 - 3(1-\gamma)(\beta_T \operatorname{cth}\beta_T - 1)}, \quad \beta_T^2 = \left(\frac{\overline{a}_0}{\sqrt{\kappa}}\right)i\Omega\,\mathrm{Pe}_g$$

$$Pe_g = \frac{a_0(p_0/\rho_{f0})^{1/2}}{a_g}, \quad a_g = \frac{k_g}{\rho_{g0}c_{pg}},$$

$$\rho_{g0} = \frac{p_{g0}}{T_0 c_{pg}(1-\gamma^{-1})}$$

$$\overline{c}_f = (t_0/R)c_f, \quad \overline{\sigma} = \sigma/(p_0 R)$$

Here c_f is the sound speed in pure liquid; p_{g0}, ρ_{g0}, T_0 - equilibrium pressure, density and temperature of the gas phase; k_g, c_{pg}, γ - the gas thermal conductivity, specific heat capacity at constant pressure and adiabatic exponent, respectively; σ - liquid-gas surface tension coefficient.

4 Shell dynamics and boundary conditions

Dynamics of thin-walled elastic shell in the long-wave region is described by the following equations (Kirchhoff - Love approximation [13]):

$$i\nu k\hat{u}_1 + (k^2 - \overline{E}^{-1}\Omega^2(1-\nu^2))\hat{u}_2 = 0 \tag{8}$$

$$Q\hat{u}_1 - ik\nu\hat{u}_2 - [(1-\nu^2)/(2\varepsilon\overline{E})]\hat{\overline{p}}_c = 0 \tag{9}$$

$$Q = 1 - \Omega^2\overline{E}^{-1}(1-\nu^2)$$

$$\overline{p}_c = \Delta p / p_0, \quad \overline{E} = E / p_0$$

Here Δp is the contact pressure equal to the normal stress component in liquid at the pipe wall; E is the Young module of the shell material. The expression for Q in Eq. (9) was simplified with account for smallness of the bending stresses in the shell with respect to the membrane ones in the case of long waves [4].

The contact pressure is coupled with the liquid flow characteristics by dynamic boundary condition $\Delta p = \Delta p_f - \tau_{rr}$ at $r = R - h \approx R$, or, in terms of dimensionless complex amplitudes $\hat{p}_c = \hat{P} - \hat{\sigma}_R$. Here the dimensionless amplitude $\hat{\sigma}_R$ of the normal component of deviatoric stress at the tube wall τ_{rr} can be calculated by a technique, similar to [3]. Using results of the previous section, this relation can be written in the form

$$\hat{p}_c = (1 - \frac{2i\Omega\kappa\overline{\eta}_s}{3\overline{c}_m^2})\hat{P} - 2i\Omega\overline{\eta}_s\hat{u}_1 \tag{10}$$

The dispersion equation for acoustic waves in the waveguide follows from Eqs. (2), (3), (5), (6)-(10) after replacement of the Young module E by dynamic module $E^*(\omega) = 1/I^*(\omega)$ according to Eq. (1). It can be reduced to the following relation for the complex sound speed c:

$$A_1 z^2 - B_1 z + C_1 = 0, \quad z = c^{-2}, \quad c = \Omega / k \tag{11}$$

$$A_1 = i\Omega\varepsilon\overline{E}^* Q\left(\frac{1-\nu^2 Q^{-1}}{1-\nu^2}\right) - \Omega^2\overline{\eta}_s$$

$$B_1 = \frac{\varepsilon\overline{E}^* NQ}{\overline{c}_m^2}\left(\frac{1-\nu^2 Q^{-1}}{1-\nu^2}\right) + \kappa^{-1}N - (\overline{E}^*)^{-1}\Omega^2\overline{\eta}_s(1-\nu^2) +$$

$$+ i\Omega[\varepsilon Q + \frac{\nu}{2\kappa} + \frac{\overline{\eta}_s N}{\overline{c}_m^2}]$$

$$C_1 = \frac{(1-\nu^2)}{\overline{E}^*}[\kappa^{-1}N + \frac{i\Omega\overline{\eta}_s N}{\overline{c}_m^2}] + \frac{\varepsilon QN}{\overline{c}_m^2},$$

$$N = i\Omega + 8\kappa\overline{\eta}_s D$$

The coefficients in Eq. (11) were simplified in view of the relation $2i\Omega\kappa\overline{\eta}_s/(3\overline{c}_m^2) << 1$. For pure elastic shell ($E^* = E = const$), Eq.(11) coincides with the dispersion equation [6] in the limiting case of pure viscous liquid.

5 Results and discussion

Numerical simulations have been performed for waveguide with the following parameters: $R = 10^{-2}$ m, $\varepsilon = 0.05$, $E_0 = 1/I_0 = 0.65\cdot10^9$ N/m^2, $\rho_s = 10^3$ kg/m^3, $I_1 = 0.754\cdot10^{-9}$ m^2/N, $I_2 = 1.046\cdot10^{-9}$ m^2/N, $I_3 = 1.237\cdot10^{-9}$ m^2/N, $\lambda_1 = 0.89\cdot10^{-4}$ s, $\lambda_2 = 0.0222$ s, $\lambda_3 = 1.864$ s (polyethylene at 25^0C, according to experimental data [10]), $p_0 = 10^5$ Pa, $\rho_{f0} = 10^3$ kg/m^3, $c_f = 1500$ m/s, $\eta_s = 0.001$ $Pa\cdot s$, $\sigma = 0.05$ N/m (water). Gas parameters correspond to air at normal conditions.

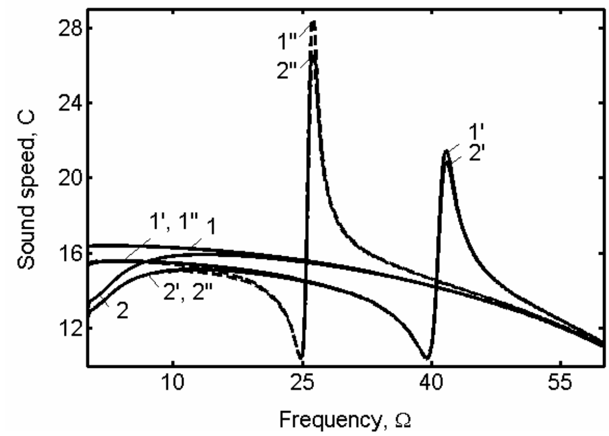

Fig.1 Sound dispersion in PE tube with water-bubble mixture.

For plots 1, 1', 1'' $I = I_0$; for 2, 2', 2'' – $I = I^*(\Omega)$; for curves 1', 2' (solid lines) - $a_0 = 5\cdot10^{-4}$ m; for 1'', 2'' (dashed lines) - $a_0 = 8\cdot10^{-4}$ m. Plots 1, 2 were calculated for pure liquid without bubbles, the rest curves correspond to $\alpha_0 = 5\cdot10^{-4}$.

The range of frequency variation, in accordance with the model assumptions, is bounded from above by relation $l >> R$. For the set of parameters, listed above, it gives $\Omega_{max} \sim 10^2$ and corresponding dimensional frequency value $f_{max} \sim 15$ KHz.

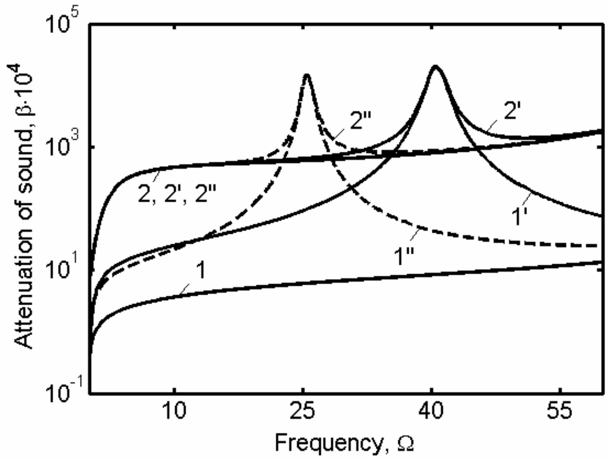

Fig.2 Attenuation of sound in PE tube with water-bubble mixture.

The plots 1-1'', 2-2'' correspond to the same parameters of the system as similar plots in Fig. 1.

From (11) approximate relation for c can be obtained at $\Omega \to 0$ by setting $\nu = 0$, $Q = 1$ (longitudinal deformations and inertia of the pipe wall are neglected, the same as the extra stresses input in the boundary condition (10)). This relation has the form:

$$c \approx \bar{c}_K (1 + 8(i\Omega)^{-1} \kappa \bar{\eta}_s D)^{-1/2} \qquad (12)$$

$$\bar{c}_K^2 = \bar{c}_S^2 \bar{c}_f^2 (\bar{c}_S^2 + \bar{c}_f^2)^{-1}, \quad \bar{c}_S^2 = \varepsilon \bar{E}_\infty \kappa \qquad (13)$$

At $\bar{\eta}_s = 0$ Eq. (12) defines Korteweg speed $\bar{c}_K$ of water hammer in elastic tube with ideal liquid [14].

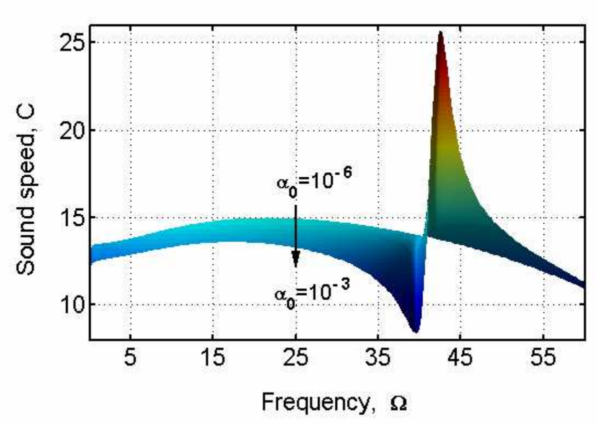

Fig.3. Gas content effect on the dispersion range

(projection of 3*D* plot $C = C(\Omega, \alpha_0)$ on $C - \Omega$ plane)

Bubble radius $a_0 = 5 \cdot 10^{-4}\ m$, viscoelastic PE tube

Plots of non-dimensional sound speed $C = \Omega / \mathrm{Re}\{k\}$ and attenuation $\beta = -\mathrm{Im}\{k\}$ versus frequency Ω, calculated from Eq.(11) for typical parameter values, described above, are presented on the Fig. 1, 2. It follows from Fig.1 that sound speed in a waveguide, made from polymeric material, is essentially less than, for instance, in aluminum one [4], which is explained by great difference in elastic modules. Sound speed, calculated with account for the wall viscoelasticity, is less than that one for pure elastic waveguide with E=E_0 in the low frequency range. However, as it can be expected, this difference vanishes with the frequency growth, when dynamic compliance approaches its instantaneous value.

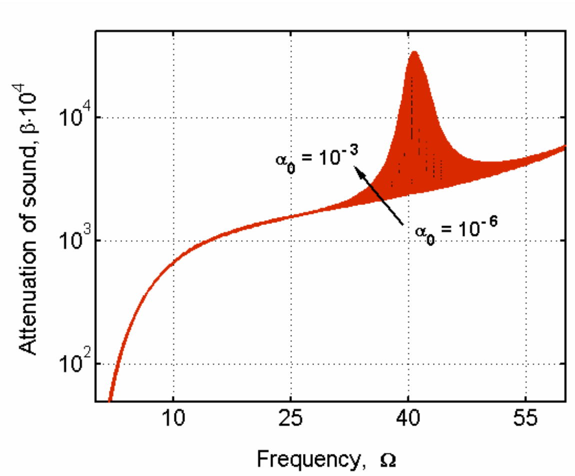

Fig.4. Influence of gas content on attenuation range

(projection of 3*D* plot $\beta = \beta(\Omega, \alpha_0)$ on $\beta - \Omega$ plane).

Parameter values are the same as for Fig.1.

Attenuation of sound wave in polymeric waveguide is much greater from that one for pure elastic tube (Fig.2, curves 1 and 2).

Bubbles, when present in the liquid, lower the sound speed in the low-frequency range and lead to drastic changes of dispersion curve near the bubble resonance frequency. This kind of dependency $C = C(\Omega)$ is quite similar to that takes place in unbounded bubbly liquid. Effect of the wall viscoelasticity in this region is small, as distinct to the frequency range far from the bubble natural frequency.

The Fig. 3, 4 illustrate influence of free gas content on the ranges of sound dispersion and attenuation. It can be seen that this effect is especially large near the resonance frequency. Note that influence of bubbles on sound attenuation far from the resonance is very small.

5 Conclusion

The model of sound propagation in flexible polymeric tube with viscous liquid, containing fine bubbles, has been developed. The resulting dispersion equation accounts for frequency dependence of the tube mechanical properties and volume viscoelasticity of the liquid due to microbubbles presence. Its analysis has showed that sound speed and the overall effect of microbubbles in the studied PE waveguide is less than in similar pure elastic and less compliant tube, while attenuation is larger. Results of simulations allow estimating of the ranges of sound speed and attenuation changes with free gas concentration, which can find application in acoustic diagnostics of bubble content in a flowing liquid, based on attenuation and dispersion measurements.

Acknowledgments

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