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Evaluation of the lined duct performances based on a 3D two port scattering matrix

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The two port acoustic dissipation and attenuation interest the designers of lined duct like aircraft engine manufacturers to evaluate the duct performances. These values can be deduced from the two port scattering matrix and from the vector of incoming modal pressures. In this work, a study about the two port acoustic dissipation and attenuation computed from the scattering matrix and for different cases of incoming modal pressures are presented. Scattering matrices used in this study are measured by an experimental procedure developed at the University of Technology of Compiègne based upon the experimental set up realized during the DUCAT project. The experimental acoustic power dissipation and attenuation are computed for different cases of modal structure on the both side of the duct. Then, these results were confronted with ones given by a theorical study of the problem based on the finite element method.

1 Introduction

The increase of airport traffic and the necessity to reduce engine dimensions and noise push engine designers to develop new technologies to reach these objectives. Among these techniques, the use of absorbent liners is mostly common due its simplicity: the used liners are usually composed by a perforated plate, a honey comb structure and a rigid plate. This type of liner is known to have a high heat and mechanical resistances with an acceptable weight and its aptitude to be installed easily in an engine. To evaluate this technology of noise reducing, the acoustic power dissipation and attenuation are used like in the study of Gerhold and al. [1]. In fact, these parameters present well who the liner behaves in front of acoustic waves.

In the Roberval laboratory of the University of Technology of Compiègne a technique of measuring these parameters are developed, this technique is based on the measure of the multimodal scattering matrix of the lined duct. These acoustic power dissipation and attenuation depend on the acoustic waves incoming and out coming the studied duct. We present in this paper a study of theses parameters for different cases of modal structure on the both side of the duct. Then, the experimental results are confronted with ones given by a theorical study of the problem based on the finite element method.

After defining the multimodal scattering matrix and the theorical bases to determine it in the section 2, the technique of measuring this matrix is presented in the third section. The fourth section presents the computation of acoustic power dissipation and attenuation from the scattering matrix. In the fifth section, experimental and theorical results are compared and analyzed.

2 Theorical bases

2.1 Scattering matrix definition

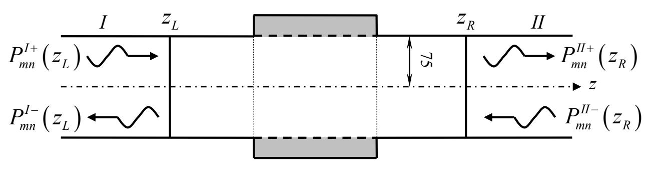


Fig.1. Acoustic waves coming in and out the studied duct.

In no flow conditions, the fluid is assumed ideal and linear acoustic theory to be valid. As the lined duct is connected to two cylindrical rigid ducts with the same radius a at z_L and z_R (Fig. 1), the distribution of the pressure in the duct, in the cylindrical coordinates system (r,θ,z) is written as [2]:

$$p(r,\theta,z,t) = \sum_{m=-\infty}^{+\infty} \sum_{n=0}^{+\infty} P_{mn}(z) \Psi_{mn}^{a}(r,\theta) e^{-i\omega t} \tag{1}$$

where $\Psi_{mn}^{a}(r,\theta) = J_m(\chi_{mn}/a) e^{-i\omega t}$ are the eigenfunctions; the integers m and n are, respectively, the angular wave number and the radial number. J_m is the Bessel function of the first kind of order m, χ_{mn}/a is the n^{th} root satisfying the radial hard wall boundary condition on the wall of the main duct $J_m^{'}(\chi_{mn}/a) = 0$.

Pressure waves travelling in the positive and negative z directions are depicted, respectively, in region I and II by $P_{mn}^{I+}(z_L)$, $P_{mn}^{I-}(z_L)$, $P_{mn}^{II-}(z_R)$ and $P_{mn}^{II+}(z_R)$ respectively the incident, reflected, retrograde and transmitted modal pressures.

The multimodal reflection matrix, $\left[R^I\right]_{N\times N}$ relates the vector of modal reflected pressures $\left\{P^{I-}\right\}_N = \left\langle P_{00}^{I-}(z_L),...,P_{PQ}^{I-}(z_L)\right\rangle^T$ to the vector of modal incident pressures $\left\{P^{I+}\right\}_N = \left\langle P_{00}^{I+}(z_L),...,P_{PQ}^{I+}(z_L)\right\rangle^T$:

$$\left\{P^{I-}\right\} = \left[R^I\right] \cdot \left\{P^{I+}\right\} \tag{2}$$

N is the number of modes in both cross sections. P and Q are respectively the angular wave number and the radial number associated to the N^{th} propagatif mode.

The multimodal reflection matrix, $\left[R^{II}\right]_{N\times N}$, relates the vector of modal transmitted pressures $\left\{P^{II+}\right\}_N = \left\langle P_{00}^{II+}(z_R),...,P_{PQ}^{II+}(z_R)\right\rangle^T$ to the vector of modal retrograde pressures $\left\{P^{II-}\right\}_N = \left\langle P_{00}^{II-}(z_R),...,P_{PQ}^{II-}(z_R)\right\rangle^T$:

$$\left\{P^{II-}\right\} = \left[R^{II}\right] \cdot \left\{P^{II+}\right\} \tag{3}$$

The multimodal transmission matrix of the duct, $\left[T\right]_{N\times N}$, relates the vector of modal transmitted pressures $\left\{P^{II+}\right\}_N = \left\langle P_{00}^{II+}(z_R),...,P_{PQ}^{II+}(z_R)\right\rangle^T$ to the vector of modal incident pressures $\left\{P^{I+}\right\}_N = \left\langle P_{00}^{I+}(z_L),...,P_{PQ}^{I+}(z_L)\right\rangle^T$:

$$\left\{P^{II+}\right\} = [T] \cdot \left\{P^{I+}\right\} \quad (4)$$

The multimodal scattering matrix $[S]$ of the duct located between the axial coordinates z_L and z_R (Fig.1.) is a linear relationship between the incoming pressures vector $\left\{P^{in}\right\}_{2N} = \left\langle P_{00}^{I+}(z_L), ..., P_{PQ}^{I+}(z_L), P_{00}^{II-}(z_R), ..., P_{PQ}^{II-}(z_R)\right\rangle^T$ and the out coming pressures vector $\left\{P^{out}\right\}_{2N} = \left\langle P_{00}^{I-}(z_L), ..., P_{PQ}^{I-}(z_L), P_{00}^{II+}(z_R), ..., P_{PQ}^{II+}(z_R)\right\rangle^T$:

$$\left\{P^{out}\right\} = [S] = \begin{bmatrix} \left[S^{11}\right]_{N\times N} & \left[S^{12}\right]_{N\times N} \\ \left[S^{21}\right]_{N\times N} & \left[S^{21}\right]_{N\times N} \end{bmatrix} \times \left\{P^{in}\right\} \quad (5)$$

This matrix represents an intersect characterisation of the duct and depends only on its acoustic and geometric characteristics. $\left[S^{11}\right]$ is the reflection of the wave coming in the element from the left side; $\left[S^{21}\right]$ is the transmission of the wave coming in the element from the left side; $\left[S^{22}\right]$ is the reflection of the wave coming in the element from the right side and $\left[S^{12}\right]$ is the transmission of the wave coming in the element from the right side.

2.2 Scattering matrix measurement

The scattering matrix is obtained by the use of the anechoic termination. This choice of is justified by the gain in time. In fact, the four matrices defined before, are related. In the case of anechoic termination $\left(\left[R^{II}\right] = [0]\right)$, these relations are :

$$\begin{aligned} \left[R^{I}_{anechoic}\right] &= \left[S^{11}\right] \\ \left[T_{anechoic}\right] &= \left[S^{21}\right] \end{aligned} \quad (6)$$

To determinate reflection and transmission matrices, an over dimensioned technique (Sitel and al. [3]) is used which consists on the use of M independent sources configurations $(M > N)$ to produce M independent measures in both cross sections located at z_L and z_R and incident and reflected modal pressure waves separated. The two matrices are given as follow:

$$\begin{aligned} [T]^T_{N\times N} &= \left[\left[\left[P^{I+}\right]^*_{N\times M} \cdot \left[P^{I+}\right]^T_{N\times M}\right]^{-1} \left[P^{I+}\right]_{N\times M}\right] \left[P^{II+}\right]^T_{N\times M} \\ \left[R^{I}\right]^T_{N\times N} &= \left[\left[\left[P^{I+}\right]^*_{N\times M} \cdot \left[P^{I+}\right]^T_{N\times M}\right]^{-1} \left[P^{I+}\right]_{N\times M}\right] \left[P^{I-}\right]^T_{N\times M} \end{aligned} \quad (7)$$

Moreover, because of the symmetry of the studied duct according to its median plan, we have:

$$\left[S^{11}\right] = \left[S^{22}\right]; \left[S^{12}\right] = \left[S^{21}\right] \quad (8)$$

The scattering matrix is then obtained by:

$$[S] = \begin{bmatrix} \left[S^{11}\right] & \left[S^{12}\right] \\ \left[S^{21}\right] & \left[S^{22}\right] \end{bmatrix} = \begin{bmatrix} \left[R^{I}_{anechoic}\right] & \left[T_{anechoic}\right] \\ \left[T_{anechoic}\right] & \left[R^{I}_{anechoic}\right] \end{bmatrix} \quad (9)$$

3 The experimental set up and data processing

3.1 Hardware

The facility of scattering matrix measurement is installed in the anechoic chamber of the University of Technology of Compiègne (Fig. 2). The equipment is made of 0.01 m thick steel wall all have a 0.148 m internal diameter.

From the left to the right of the schema (Fig.3) are shown the following elements:

- The source section with three acoustic drivers flushed mounted in a z line. The axial distance between two drivers is 0.15 m. All this section can rotate over 360°.
- The measurement duct element I which can rotate over 360° is supporting a boom traversing on the radius with a B&K sound intensity probe attached and directed toward the z axis. The distance between the ¼ in. microphones is 0.05 m according to well-known conditions (Akoum and Ville [3]).
- The 0.5 m long duct element will be part of the configuration.
- The measurement duct element II identical to the duct element I.
- The anechoic termination.

Fig. 2. The acoustic duct facility at UTC

Angular and radial displacements are provided by step by step motors. A working station automatically operates the rotation of the source section, the choice of the axial source, and the displacement of the probe in the r and θ directions. It also supplies the noise generation to sources and the signals issued from the four microphones.

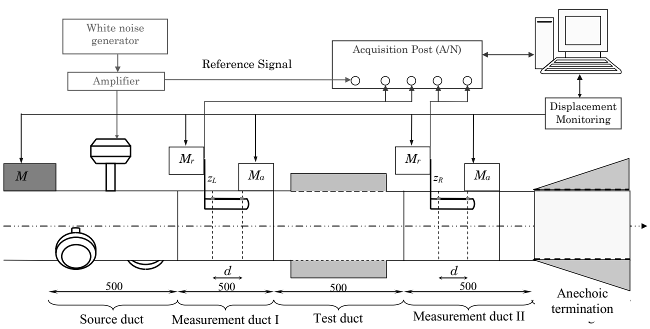


Fig.3. Experimental set up

3.2 Scattering matrix measurement procedure

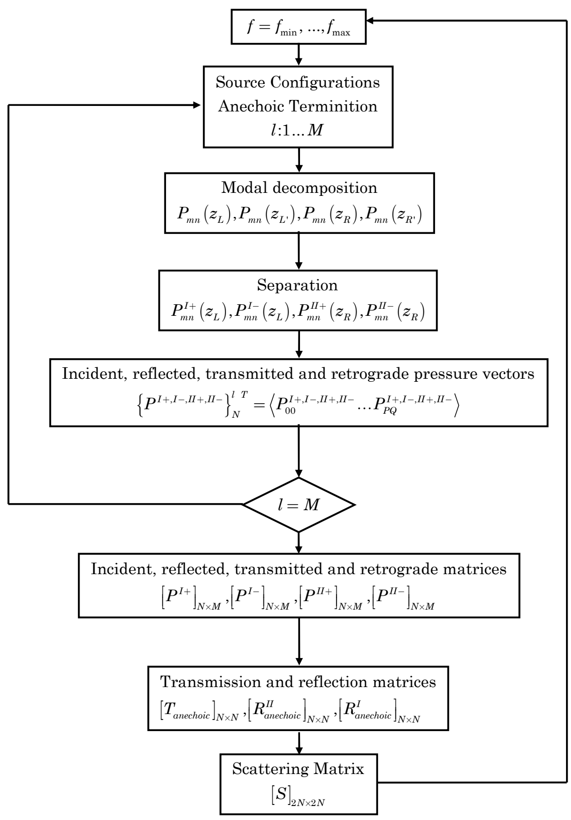


Fig.4. Flow chart of the scattering matrix measurement

The experimental procedure to measure the scattering matrix is presented in Fig. 4. The acoustic driver is driven with a white noise signal that is band limited to 0 – 2700 Hz. For this duct radius and frequency range the total adimensional wave number *ka* is lower than 3.8. Therefore only $N \le 5$ modes (0,0); (±1,0); (±2,0) are cut-on. The transfer functions between the amplifier and microphone signals provide, after calibration, the amplitude and phase of the local total acoustic pressure normalized by the level of the amplifier. The origin of the *z* axis is then given by the reference phase that is in the source *z* axis position. A 240 point discretization of the acoustic pressure field measured in each of the two pairs of cross-section areas is achieved by rotating the measurement duct section at 16 positions equally spaced by 22.5° and displacing the probe to 15 radial positions. Then modal decomposition by a Fourier-Lommel's transform [5] for $m = -7$ to +7 and $n = 0$ to 3 is achieved leading to the cut-on or cut-off complex modal pressure coefficients in both pairs of cross section areas. The incident and reflected modal vectors are then separated and computed at z_L and z_R. All this procedure is repeated for all source configurations and using the anechoic termination. To compute 100 coefficients of the matrix $[S]$, 12 source configurations are used; they have been chosen to ensure generation of linearly independent input modal vectors. The coordinates of the source configurations are given in Table 1.

Configuration n°	$\theta^i_s(°)$	$z^i_s(cm)$
1	0	0
2	120	15
3	240	30
4	8	0
5	128	15
6	248	30
7	16	0
8	136	15
9	256	30
10	24	0
11	144	15
12	264	30

Table 1 Source configurations coordinates

4 Two port acoustical power dissipation/attenuation

The total acoustic power $W(z)$ at measurement sections is:

$$W(z) = \sum_{m=-\infty}^{+\infty} \sum_{n=0}^{\infty} I_{z,mn}(z) N_{mn} \tag{10}$$

$N_{mn} = S J_m^2(\chi_{mn}) \left[1 - \frac{m^2}{\chi_{mn}^2} \right]$ is the normalization factor, $S = \pi a^2$ is the duct section and $I_{z,mn}$ is the acoustic intensity associated to mode (m,n):

$$I_{z,mn}(z) = \frac{1}{2} \mathrm{Re}\left(P_{mn}(z) \cdot V^*_{z,mn}(z) \right) \tag{11}$$

Incident, reflected, transmitted and retrograde intensities are expressed as follow:

$$I^{I\pm}_{z,mn}(z_L) = \frac{N_{mn} \cdot k_{mn}}{2 c_0 \rho_0 k} \left| P^{I\pm}_{mn}(z_L) \right|^2 ;\; I^{II\pm}_{z,mn}(z_R) = \frac{N_{mn} \cdot k_{mn}}{2 c_0 \rho_0 k} \left| P^{II\pm}_{mn}(z_R) \right|^2 \tag{12}$$

$k_{mn} = \sqrt{k^2 - (\chi_{mn}/a)^2}$ is the axial wave number of mode (m,n) in the main duct, $k = 2\pi f / c_0$, *f* the frequency, and c_0 the speed of sound in air. The two port acoustical power dissipation W_{dis} of a duct is the difference between the acoustic power of incoming waves on both sides W^{in} and the acoustic power of outgoing waves on both sides W^{out} :

$$W_{dis} = W^{in} - W^{out} \tag{13}$$

$$\begin{aligned} W^{in} &= \sum_{m=-P}^{P} \sum_{n=0}^{Q} \frac{N_{mn} \cdot k_{mn}}{2 c_0 \rho_0 k} \left(\left| P^{I+}_{mn}(z_L) \right|^2 + \left| P^{II-}_{mn}(z_R) \right|^2 \right) \\ W^{out} &= \sum_{m=-P}^{P} \sum_{n=0}^{Q} \frac{N_{mn} \cdot k_{mn}}{2 c_0 \rho_0 k} \left(\left| P^{I-}_{mn}(z_L) \right|^2 + \left| P^{II+}_{mn}(z_R) \right|^2 \right) \end{aligned} \tag{14}$$

These powers can be written as follow:

$$W^{in} = \{\Pi^{in}\}_{2N}^{T*} \cdot \{\Pi^{in}\}_{2N} \ ; \ W^{out} = \{\Pi^{out}\}_{2N}^{T*} \cdot \{\Pi^{out}\}_{2N} \quad (15)$$

$$\{\Pi^{in}\} = [X]_{2N\times 2N} \cdot \{P^{in}\}_{2N} \ ; \ \{\Pi^{out}\} = [X]_{2N\times 2N} \cdot \{P^{out}\}_{2N} \quad (16)$$

$$[X] = \begin{bmatrix} [diag(X_{mn})]_{N\times N} & [0]_{N\times N} \\ [0]_{N\times N} & [diag(X_{mn})]_{N\times N} \end{bmatrix} \quad (17)$$

$X_{mn} = \sqrt{\dfrac{N_{mn}k_{mn}}{2\rho_0 c_0 k}}$. By considering the definition of the multimodal scattering matrix, the power dissipation can be written as:

$$W_{dis} = \{\Pi^{in}\}^{T*}\big[[I]-[H]\big]\{\Pi^{in}\} \quad (18)$$

$$\{\Pi^{out}\} = [S'] \cdot \{\Pi^{en}\} = [X][S][X]^{-1}\{\Pi^{en}\} \quad (19)$$

$$[H] = [S']^{T*} \cdot [S'] \ ; \ [S'] = [X][S][X]^{-1} \quad (20)$$

The determination of eigen values of this matrix, as shown in Aurégan and Starobinski [6], allows to get a physical interpretation of absorption and generation of energy. The two power acoustic dissipation is then given by:

$$W_{dis} = \{d\}_{2N}^{T*}\left[diag(1-\lambda_i)\right]\{d\} \quad (21)$$

λ_i are the eigen values of $[H]$ and $\{d\} = [U]^{T*}\{\Pi^{in}\}$, $[U]$ is the matrix of eigen vectors of $[H]$. Finally, the acoustic power dissipation is written as:

$$W_{dis}(Watts) = \sum_{i=1}^{2N}(1-\lambda_i)|d_i|^2 = \sum_{i=1}^{2N}\left[(1-\lambda_i)\left|\sum_{j=1}^{2N}\left(TR_{ij}P_j^{in}\right)\right|^2\right] \quad (22)$$

$$W_{dis}(dB) = 10\log\left(\frac{W_{dis}(Watts)}{10^{-12}}\right)$$

Eq. (23) points out that the overall acoustic power dissipated depends upon the acoustic and geometrical properties of the duct element depicted by $[TR]_{2N\times 2N} = [U]^{*T} \cdot [X]$ and upon $\{P^{in}\}$ the vector of the modal pressures incoming from the left and the right sides of the test duct element.

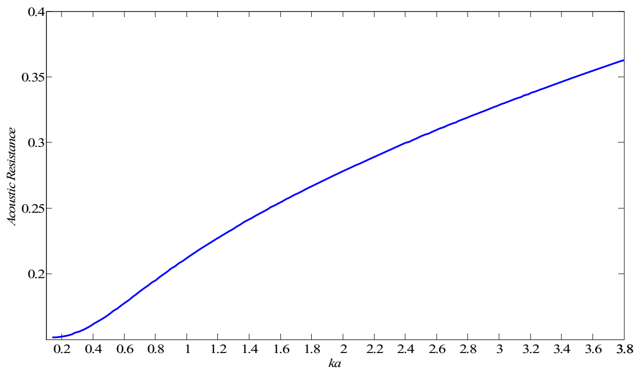
Most often to characterize the efficiency of a muffler the acoustic power attenuation defined by the ratio between the incoming and outgoing acoustic powers is used:

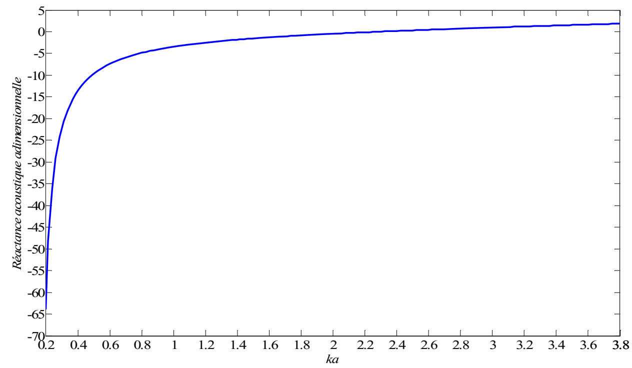
$$W_{attenuation}(dB) = 10\log\left(\frac{W^{in}}{W^{out}}\right) 10\log\left(\frac{\sum_{i=1}^{2N}|d_i|^2}{\sum_{i=1}^{2N}\lambda_i |d_i|^2}\right) \quad (23)$$

$W_{attenuation}(dB)$ also depends on the scattering matrix of the duct element and on the incoming pressure waves $\{P^{in}\}_{2N}$.

5 Results

The axial positions z_L and z_R presented in Fig.1. are respectively, the axial coordinates of the two microphones located on both sides of the test duct (Fig.3.). The scattering matrix is measured for a 1 m duct composed of three elements: 0.35 m wall duct, 0.3 m lined duct and 0.35 m wall duct. The theorical matrix is computed using a finite element modelling presented in an above work Taktak et al. [7]. For this theorical model, the impedance is given both by an impedance model presented in Elnady ey Bodén [8] and the impedance measured by the two microphones method (TMM). The used acoustic resistance and reactance are presented respectively in Fig. 5 and 6.





6 Conclusion

Acknowledgments

References

[1] C.H. Gerhold, M.C. Brown, W.R. Watson, M.G. Jones, " Investigation of liner characteristics in the NASA Langley curved duct test rig",

[2] P.M. Morse, K.U. Ingrad, *Theorical Acoustics* (McGraw-Hill, New York, 1968).

[3] A. Sitel, J.M. Ville, F. Foucart, "An experimental facility for measurement of acoustic transmission

matrix and acoustic power dissipation of duct discontinuity in higher order propagation conditions", *Acustica & Acta Acustica,* 89, 586 – 594 (2003).

[4] M. Akoum, J.M. Ville, "Measurement of reflection matrix of a discontinuity in a duct" , *Journal of Acoustical Society of America,* 103(5), 2463 – 2468 (1998).

[5] J.M. Auger, J.M. Ville, "Measurement of liner impedance based on the determination of duct eigenvalues by Fourier-Lommel's transform", *Journal of Acoustical Society of America,* 88,19 – 22 (1990).

[6] Y. Aurégan, R. Starobinski, "Determination of acoustical energy dissipation/production potentiality from the acoustical transfer functions of a multiport", *Acustica & Acta Acustica,* 85, 788 – 792 (1999).