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Critical assessment of Operational Path Analysis: mathematical problems of transmissibility estimation

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Classical transfer path analysis (TPA) is a widely used and reliable method for tackling noise and vibration problems. But due to its complexity and time-consuming procedure the industry is constantly seeking for simpler and faster methods. Several have been proposed in the last years, and one of them, most often referred to as operational path analysis (OPA), attracted particular attention as it uses only measured operational input and output signals and calculates the transmissibilities between them to characterize the paths. The claim for its accuracy is based on being able to reproduce the original output signal by summing the calculated partial contributions but it has not yet been compared to other TPA methods. This new method is now critically examined and compared to a reference classical TPA measurement. The results of this examination reveal three significant weaknesses. This paper focuses on the problems related to the estimation of transmissibilities which mostly arise from the limited amount of orders present in the signal and the coherence between inputs. It is shown that despite the advantages of the method, it is not applicable in many situations and has to be used with care for it can easily give misleading results.

1 Introduction

1.1 A Short Review of Transfer Path Analysis

Over the years, classical transfer path analysis has proved to be a reliable method for assessing the NVH behavior of vehicles. The original idea, to use a source - path - receiver model, dates back to the '80s [1]. Although the method is well known there is often a confusion about the meaning of the different elements of the model. In order to clarify the definitions we start with a review of the basic TPA formulation.

To create a TPA model the global system has to be divided into an active and a passive part, the former containing the sources, the latter the receiver points where the responses are measured. (see Fig. 1) Loads are defined at the interface between the two, and the so-called noise transfer functions (NTF's) - which are also referred to as frequency response functions (FRF's) - characterize the relationship between a load and a receiver. The paths are represented by these NTF's. The individual contribution of each path to the total response can be calculated by multiplying the load with the corresponding NTF. This model presupposes that the load-response relationship is causal and the paths are system characteristic of the global system. The figure also shows the body FRF matrix, denoted by H_{ij}. This matrix describes the relationship between the input forces F and the passive side responses a_{bi} at the input locations.

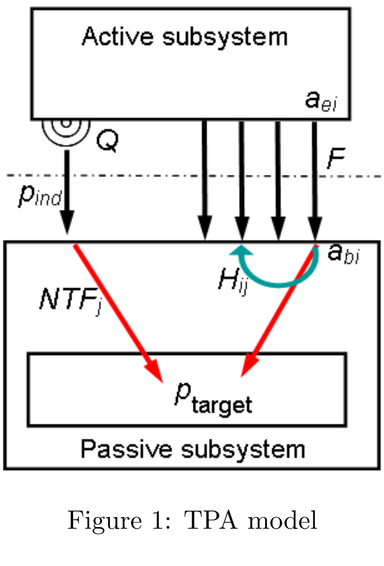

Figure 1: TPA model

Using this model, the target response can be expressed as a sum of the path contributions:

$$p(\omega) = \sum_i NTF_i(\omega)F_i(\omega) + \sum_j NTF_j(\omega)Q_j(\omega) \quad (1)$$

and the body side accelerations can be written as:

$$a_{bi}(\omega) = \sum_j H_{ij}(\omega)F_j(\omega) \quad (2)$$

where F_i represents a force acting as a structural load and Q_j a volume acceleration source acting as an acoustic load. NTF_i and NTF_j are the corresponding noise transfer functions, H_{ij} is the body FRF matrix and a_{ei} and a_{bi} are the measured active and passive side accelerations, respectively. The same equation could be written for an acceleration target response. For the sake of simplicity the following discussions will only deal with a pressure response.

The most common way to analyze the results is to visualize them in a so called partial path contribution (PPC) plot, where each row shows the partial contribution of a single path to the total pressure as a function of rpm for a certain order as shown in Fig. 2. However,

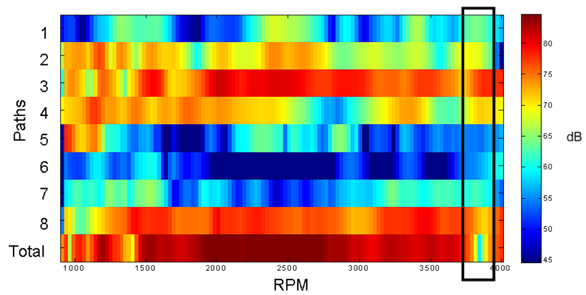

Figure 2: Partial Path Contribution Map

such plot should be handled with care. For example, the black rectangle shows a region where Path 3 seems to have a high contribution but the total contribution remains low. This is due to the inverse phase relationship between the paths, compensating each other's contribution. Phase therefore should always be taken into account during an analysis. For this reason results are also often plotted on Bode plots or vector diagrams.

1.2 Practical limitations of Transfer Path Analysis

The mathematical formulation is simple enough but unfortunately the same can't be said about the practical measurements. To build a complete TPA model both the operational loads and the *NTF's* must be known. With the help of the recently developed calibrated volume velocity sources and the available reciprocal techniques [2] the *NTF's* can be measured quite easily compared to the earlier direct methods (e.g. impact testing). But the same improvement has not yet been achieved for the measurement of the operational loads. The presently available techniques only allow an estimation of these quantities, for example requiring a priori knowledge of mount stiffnesses and/or removal of the active part, and additional measurements have to be carried out besides the operational measurements.

1.3 Operational Path Analysis

In order to overcome the above mentioned limitations and to speed up the TPA process many solutions and improvements have been proposed[4],[3] making different trade-offs regarding speed, detail of analysis, accuracy and causality. One of them is the so called operational path analysis (OPA). It is based on the idea of the MIMO transmissibility calculation principle that has been around since the late '90s [5], [6] but it is only in the last few years that it has become well known. The goal of the method is to use only operational data to derive "TPA-like" results without the need for all the additional experimental measurements specific to most TPA approaches. This is achieved by using a different model in which the target response is formulated using responses measured at the load locations instead of the loads themselves (Eq (3)).

$$p(\omega) = \sum_i T_i(\omega)a_i(\omega) + \sum_j T_j(\omega)p_j(\omega) \qquad (3)$$

In the following chapters the OPA method will be compared with TPA revealing three potential dangers inherent in using the method without proper consideration. The topic of the paper will be be discussed in detail but the other limitations will be also shortly explained.

2 The limitations of Operational Path Analysis

Since the mathematical formulation of OPA is similar to that of the TPA there is a confusion in the terminology, which might unfortunately lead to an incorrect interpretation on the meaning of the OPA results. So, before we begin the discussion on the weaknesses of the method themselves, it must be pointed out that the TPA and the OPA models are fundamentally different despite their apparent similarity. First of all, as opposed to the TPA model the OPA model is not causal. Instead of a load-response relationship OPA is based on a response-response relationship. This means that while in TPA one can draw a conclusion as to what effect a certain load has on the total response, in OPA one can only talk - with a few exceptions - about a similarity, a "co-existence" between the target and the input responses. It is well known in engineering that co-existence does not necessarily imply causality. Moreover, OPA calculations use transmissibilities instead of NTF's which are not system characteristics but depend on the loading conditions.

Apart from this basic difference to TPA, three significant critical elements can be found in the OPA method:

- effect of neglected paths
- cross-coupling between the input accelerations
- errors in the estimation of transmissibilities

The first two shall be discussed in subsections 2.1 and 2.2 and the third will be analized in detail in chapter 3.

2.1 Effect of neglected paths

First of all, neglecting a path can introduce errors. In TPA, since each path is independent in that model, this error can be recognized because the sum of the path contributions will differ from the measured target response. Furthermore, the individual contributions will remain valid.

In OPA, however, a twofold effect can be observed. In case the neglected path is correlated with the rest, its energy will be spread over the other paths during the crosspower calculation (see Eq (4)). Consequently the individual path contributions will be changed and the mistake can't be recognized by comparing the synthesized and measured target since those will be equal. On the other hand, if the neglected path is uncorrelated the behavior will be similar to the TPA model: the synthesized and the measured target will be different revealing the mistake.

2.2 Effect of cross-coupling

Secondly, as can be seen from Eq. 2, the body side acceleration doesn't only depend on the force acting at the point but also on the other forces. This is called cross-coupling.

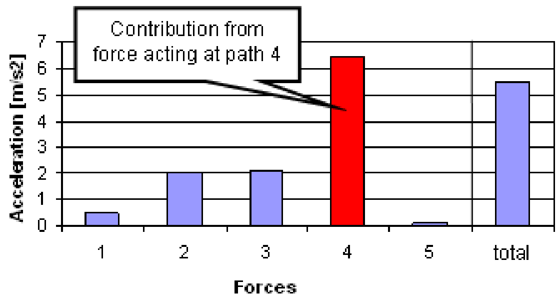

Figure 3: Components of the total input acceleration at Path 1

A strong cross-coupling between two paths can lead to a misjudgment on the importance of the paths. For example Figure 3 shows the individual contribution of the different forces to the total acceleration at path 1 at a given RPM for order 2. It is clear to see that the contribution from path 4 (marked with red) is much higher than that caused by the force actually acting at path 1. Again, this weakness might also lead to incorrect engineering decisions since in OPA the body side accelerations are treated as inputs and there is no insight as to what is the cause of a high acceleration level at a certain point. In short, OPA might indicate a high contribution whereas in reality that path is unimportant.

3 Difficulties in the estimation of transmissibilities

The estimation of transmissibilities has to be done based on the operational data. The simplest approach is to use an H_1 estimation, well-known from classical least-squares estimation (e.g. for FRF's in modal analysis). The ω denoting the frequency dependence of the terms was omitted for the sake of clarity.

$$\begin{aligned} \{p\} &= [T].\{a_b\} \qquad /a'_b \\ \langle p\cdot a'_b\rangle &= \{T\}\langle a_b\cdot a'_b\rangle \\ \{T\} &= \langle p\cdot a'_b\rangle\langle a_b\cdot a'_b\rangle^{-1} \end{aligned} \quad (4)$$

The basic condition for performing this operation is the invertibility of the autopower matrix $\langle a_b\cdot a'_b\rangle$, which is equal to having a full rank matrix.

As the autopower matrix is a dyadic product it has rank 1. During the H1 estimation, many instances of this matrix are calculated from samples over the whole measurement data and are averaged to be able to get an average autopower matrix which has full rank.

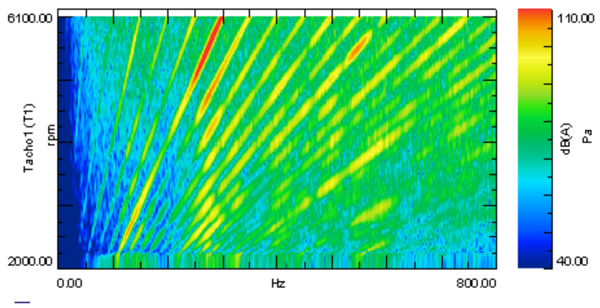

Figure 4: Colormap of engine noise data

By examining typical TPA operational data (Fig. 4), it can be observed that it is dominated by the engine orders and that besides them there are hardly any other phenomena present.

The next figure (Fig. 5) shows a simplified colormap depicting only the meaningful information content.

3.1 Limited number of orders

It can be seen that at each frequency the useful information of the averaged autopower matrix will be limited by the number of orders present at that frequency. In order to examine this effect a simulation was set up using a reference TPA dataset which was further modified to minimize the effects of the other limitations. The input

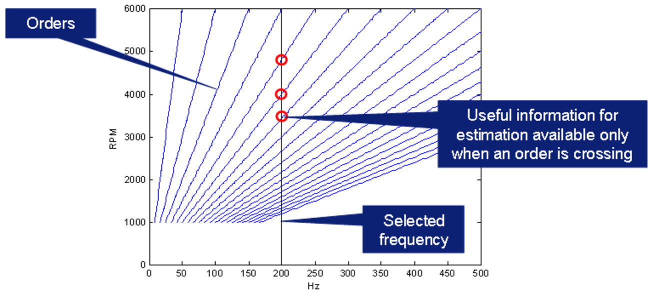

Figure 5: A schematic colormap of engine noise data

forces were randomized to make them incoherent and the cross-coupling between the paths was set to zero. When a low number of paths was used – meaning that there were at least twice as many orders in the signal than the number of paths – the OPA gave similar results to the TPA as shown in Figs. 6 and 7.

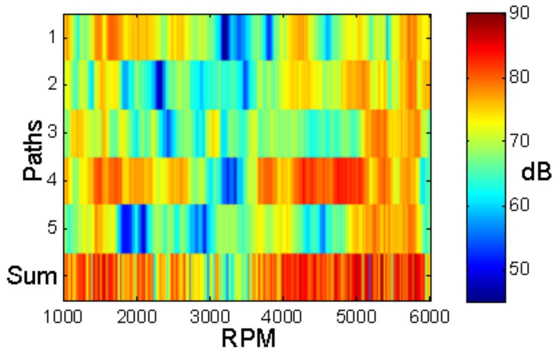

Figure 6: TPA partial path contributions, order 2

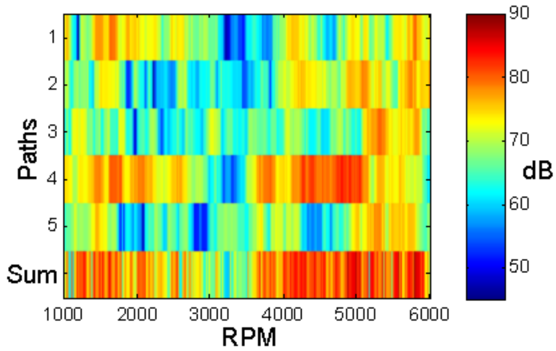

Figure 7: OPA partial path contributions, order 2

By a close examination of the two figures, small differences can still be discovered despite all the efforts to minimize the effects of the weaknesses. The reason for this lies in the process of averaging. The idea incoherence only makes sense if there is a very large number of samples available. Since Eq. (4) contains the cross-power matrix, taking a smaller number of averages will create an effect similar to cross-coupling between the paths and will introduce small errors in the estimated transmissibilities.

In case the number of paths is much higher than the

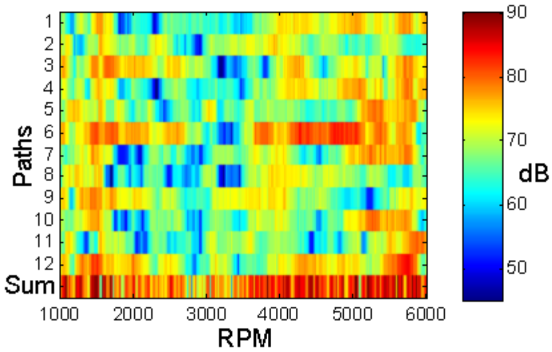

Figure 8: TPA partial path contributions, order 2

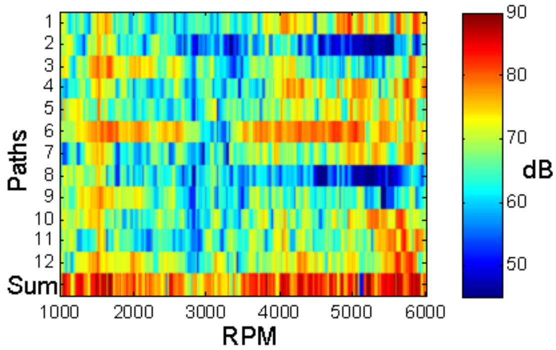

Figure 9: OPA partial path contributions, order 2

number of orders or the measurement data doesn't contain enough variation, the averaged autopower matrix will become rank deficient and only an approximate solution can be found using some kind of a pseudo inverse [7]. But as it will be an approximation, the OPA results will diverge more and more from the TPA results. Figs. 8 and 9 show the resulting PPC plots, Fig. 10 the difference between TPA and OPA and Fig. 11 shows that, for a given number of orders the error of the OPA results increases with the number of paths present in the system.

It has been suggested that including different measurement conditions (e.g. using run-up and run-down data from different gear positions) might alleviate this problem. Earlier studies [5, 8] have shown that the

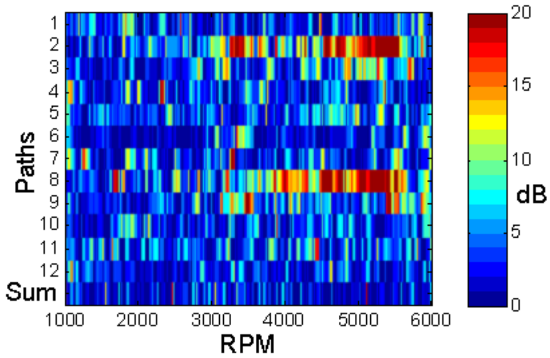

Figure 10: The difference of the TPA and the OPA partial path contributions for order 2

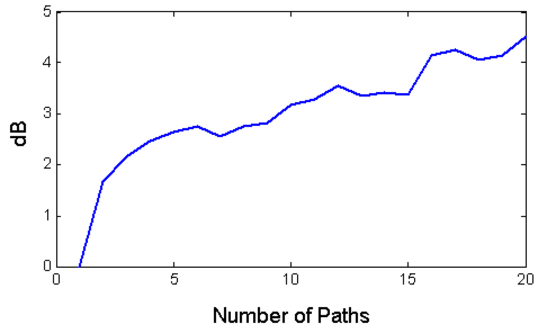

Figure 11: Average error in the OPA path contributions

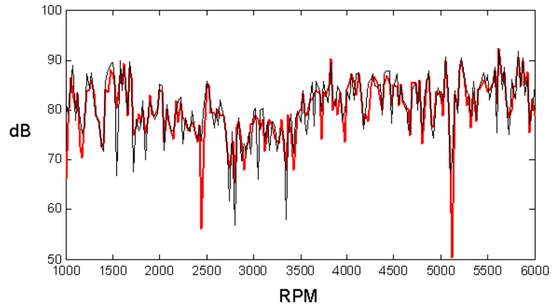

Figure 12: Comparison of the OPA and the reference TPA synthesized targets for order 2

transmissibilities – as opposed to the NTF's – depend on the loading conditions. Changing the amplitude or the phase of the loads will have no effect on the transmissibilities but if a new load is introduced (e.g. a different airborne source, extra impact tests on the passive side) to the system then the transmissibilities will be changed, so averaging them will no longer give valid results. Therefore, one has to be careful when using multiple operating conditions together.

3.2 Coherence between signals

So far we have dealt with an ideal situation, where all the signals in the analysis were randomized to make them incoherent. However, in a real test situation, the orders will be partly correlated due to the modal behavior of the system. Practically this means that the correlated components will carry the same information and this way will reduce the rank of the autopower matrix and increase the error of the estimation.

3.3 Good synthesized target = Reliable analysis?

Last but not least, it must be emphasized that since the OPA method is based on a backward-forward calculation (the target data is included in the calculations from the beginning), in general these limitations will not show up in the measured vs. synthesized target comparisons. It is quite obvious that one gets the same data back as one started from. This can be well observed on Fig. 12, in which the synthesized target pressure is compared against the reference TPA result. The two curves show the same data as the bottom lines on the PPC plots in Figs. 8 and 9. Looking only at the comparison of the targets, one might conclude that the agreement is fairly good and the synthesis is reliable. But the comparison of the PPC's reveal that in this case OPA gives unreliable results. Then, since in a real situation there is often no opportunity for such a comparison, one is left in doubt about the quality of the partial contributions even when a good synthesized target is achieved.

4 Summary

In this paper, a new operational transfer path analysis method was examined and compared with the classical TPA method. Besides revealing that the results can't be interpreted the same way as for classical TPA, three significant limitations were also found that might introduce errors and thus lead to an incorrect engineering decision and hinder the solution of the problem. These limitations are the following: (i) the effect of neglected paths (ii) cross-coupling between the input accelerations and (iii) difficulties in the estimation of transmissibilities. The latter was discussed in detail. The analysis revealed that number of different conditions at least equal to the number of references must be found at each frequency in order to get a good estimate. In practice the accuracy is influenced by the number of orders present in the signal, the number of paths in the analyzed system, the correlation between the inputs and the length of the measurement. In case the number of paths is larger then the number of orders, some error will inevitably be introduced in the partial contributions. But even if the number of orders is high enough, in a real situation, the orders will be correlated to some degree and this way will not produce different conditions. The length of the measurement plays a role in the averaging process where an insufficient number of averages will also introduce errors. Still, it might be possible to achieve usable results by averaging over different measurements, however great care must be taken, since the transmissibilities can vary between the measurements since they depend on the loading conditions. Even then, there isn't a simple indicator which would confirm the reliability of the results since it was also pointed out that a good agreement between the synthesized and measured total pressure in itself doesn't necessarily indicate the quality of the synthesis. All in all, although OPA can be useful as a first troubleshooting tool the results have to be treated with proper caution, keeping the limitations in mind. In the meantime, alternative methods, faster than classical TPA yet having the same precision and avoiding these limitations, are being developed as part of an ongoing research.

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