Critical assessment of Operational Path Analysis: effect of coupling between path inputs

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Classical transfer path analysis (TPA) is a widely used and reliable method for tackling noise and vibration problems. But due to its complexity and time-consuming procedure the industry is constantly seeking for simpler and faster methods. Several have been proposed in the last years, and one of them, most often referred to as operational path analysis (OPA), attracted particular attention as it uses only measured operational input and output signals and calculates the transmissibilities between them to characterize the paths. The claim for its accuracy is based on being able to reproduce the original output signal by summing the calculated partial contributions but it has not yet been compared to other TPA methods. This new method is now critically examined and compared to a reference classical TPA measurement. The results of this examination reveal three significant inherent limitations. This paper deals with the effect of the cross-coupling between the input signals. Due to modal behavior a single force will cause vibrations at all inputs. Thus, there isn't a simple one-to-one relationship between loads and inputs. This coupling then might easily lead to false identification of significant paths in case of the OPA method.

1 Introduction

1.1 A Short Review of Transfer Path Analysis

Over the years, classical transfer path analysis has proved to be a reliable method for assessing the NVH behavior of vehicles. The original idea, to use a source - path - receiver model, dates back to the '80s [1]. Although the method is well known there is often a confusion about the meaning of the different elements of the model. In order to clarify the definitions we start with a review of the basic TPA formulation.

To create a TPA model the global system has to be divided into an active and a passive part, the former containing the sources, the latter the receiver points where the responses are measured. (see Fig. 1) Loads are defined at the interface between the two, and the so-called noise transfer functions (NTF's) - which are also referred to as frequency response functions (FRF's) - characterize the relationship between a load and a receiver. The paths are represented by these NTF's. The individual contribution of each path to the total response can be calculated by multiplying the load with the corresponding NTF. This model presupposes that the load-response relationship is causal and the paths are system characteristic of the global system. The figure also shows the body FRF matrix, denoted by H_{ij}. This matrix describes the relationship between the input forces F and the passive side responses a_{bi} at the input locations.

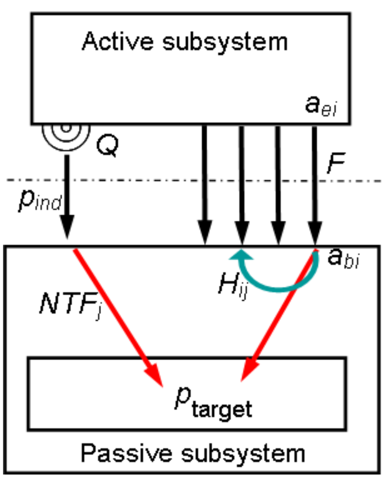

Figure 1: TPA model

Using this model, the target response can be expressed as a sum of the path contributions:

$$p(\omega) = \sum_i NTF_i(\omega)F_i(\omega) + \sum_j NTF_j(\omega)Q_j(\omega) \quad (1)$$

and the body side accelerations can be written as:

$$a_{bi}(\omega) = \sum_j H_{ij}(\omega)F_j(\omega) \quad (2)$$

where F_i represents a force acting as a structural load and Q_j a volume acceleration source acting as an acoustic load. NTF_i and NTF_j are the corresponding noise transfer functions, H_{ij} is the body FRF matrix and a_{ei} and a_{bi} are the measured active and passive side accelerations, respectively. The same equation could be written for an acceleration target response. For the sake of simplicity the following discussions will only deal with a pressure response.

The most common way to analyze the results is to visualize them in a so called partial path contribution (PPC) plot, where each row shows the partial contribution of a single path to the total pressure as a function of rpm for a certain order as shown in Fig. 2. However,

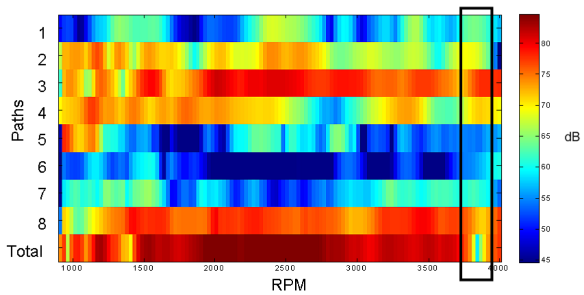

Figure 2: Partial Path Contribution Map

such plot should be handled with care. For example, the black rectangle shows a region where Path 3 seems to have a high contribution but the total contribution remains low. This is due to the inverse phase relationship between the paths, compensating each other's contribution. Phase therefore should always be taken into account during an analysis. For this reason results are also often plotted on Bode plots or vector diagrams.

1.2 Practical limitations of Transfer Path Analysis

The mathematical formulation is simple enough but unfortunately the same can't be said about the practical measurements. To build a complete TPA model both the operational loads and the *NTF's* must be known. With the help of the recently developed calibrated volume velocity sources and the available reciprocal techniques [2] the *NTF's* can be measured quite easily compared to the earlier direct methods (e.g. impact testing). But the same improvement has not yet been achieved for the measurement of the operational loads. The presently available techniques only allow an estimation of these quantities, for example requiring a priori knowledge of mount stiffnesses and/or removal of the active part, and additional measurements have to be carried out besides the operational measurements.

1.3 Operational Path Analysis

In order to overcome the above mentioned limitations and to speed up the TPA process many solutions and improvements have been proposed[4],[3] making different trade-offs regarding speed, detail of analysis, accuracy and causality. One of them is the so called operational path analysis (OPA). It is based on the idea of the MIMO transmissibility calculation principle that has been around since the late '90s [5], [6] but it is only in the last few years that it has become well known. The goal of the method is to use only operational data to derive "TPA-like" results without the need for all the additional experimental measurements specific to most TPA approaches. This is achieved by using a different model in which the target response is formulated using responses measured at the load locations instead of the loads themselves (Eq (3)).

$$p(\omega) = \sum_i T_i(\omega) a_i(\omega) + \sum_j T_j(\omega) p_j(\omega) \qquad (3)$$

In the following chapters the OPA method will be compared with TPA revealing three potential dangers inherent in using the method without proper consideration. The topic of the paper will be be discussed in detail but the other limitations will be also shortly explained.

2 The limitations of Operational Path Analysis

Since the mathematical formulation of OPA is similar to that of the TPA there is a confusion in the terminology, which might unfortunately lead to an incorrect interpretation on the meaning of the OPA results. So, before we begin the discussion on the weaknesses of the method themselves, it must be pointed out that the TPA and the OPA models are fundamentally different despite their apparent similarity. First of all, as opposed to the TPA model the OPA model is not causal. Instead of a load-response relationship OPA is based on a response-response relationship. This means that while in TPA one can draw a conclusion as to what effect a certain load has on the total response, in OPA one can only talk - with a few exceptions - about a similarity, a "co-existence" between the target and the input responses. It is well known in engineering that co-existence does not necessarily imply causality. Moreover, OPA calculations use transmissibilities instead of NTF's which are not system characteristics but depend on the loading conditions.

Apart from this basic difference to TPA, three significant critical elements can be found in the OPA method:

- errors in the estimation of transmissibilities
- effect of neglected paths
- cross-coupling between the input accelerations

The first limitation lies in the estimation of transmissibilities as this has to be done based on the operational data. The simplest approach is to use an H_1 estimation, well-known from classical least-squares estimation (e.g. for FRF's in modal analysis):

$$\begin{aligned} \{p\} &= [T].\{a_b\} \qquad /a_b' \\ \langle p \cdot a_b' \rangle &= \{T\} \langle a_b \cdot a_b' \rangle \\ \{T\} &= \langle p \cdot a_b' \rangle \langle a_b \cdot a_b' \rangle^{-1} \end{aligned} \qquad (4)$$

The basic condition for performing this operation is the invertibility of the autopower matrix $\langle a \cdot a' \rangle$, which is equal to having a full rank matrix. In most practical cases however this is only satisfied in the high frequency range; for most automotive applications this would be somewhere above 1kHz. In lower frequency ranges the input vibrations will be largely coherent because of the the strong modal behavior, making the autopower matrix rank deficient. In such cases principal component analysis (PCA) or singular value decomposition (SVD) can be applied to get an approximate pseudo-inverse solution [7]. This may lead to an incorrect estimation of the transmissibilities, giving rise to errors in the OPA calculation.

Secondly, neglecting a path can also introduce errors. In TPA, since each path is independent in that model, this error can be recognized because the sum of the path contributions will differ from the measured target response. Furthermore, the individual contributions will remain valid.

In OPA, however, a twofold effect can be observed. In case the neglected path is correlated with the rest, its energy will be spread over the other paths during the crosspower calculation (see Eq (4)). Consequently the individual path contributions will be changed and the mistake can't be recognized by comparing the synthesized and measured target since those will be equal. On the other hand, if the neglected path is uncorrelated the behavior will be similar to the TPA model: the synthesized and the measured target will be different revealing the mistake. Unfortunately, the estimation of the transmissibilities imposes a practical limitation on the accuracy of the individual contributions because the number of averages should be very high to completely eliminate the incoherent signal from the calculation. This condition will not be met in most cases and as a result the individual path contributions will become unreliable.

Before we begin the detailed discussion of the third limitation it must be emphasized that since the OPA method is based on a backward-forward calculation (the target data is included in the calculations from the beginning),in general these limitations will not show up in the measured vs. synthesized target comparisons. It is quite obvious that one gets the same data back as one started from.

3 Effect of cross-coupling

And finally we arrive to the main topic of the paper. Cross-coupling means that the body side acceleration at an input point doesn't only depend on the force acting at that certain point but also on the other forces, as can be seen from Eq. (2). Two aspects will be considered to give a better overview on the physical meaning of this limitation, using a reliable engine noise TPA dataset as reference. For the sake of simplicity, the dataset was reduced to contain only five structural paths, the corresponding body FRF matrix and NTF's and one pressure target. It contains order cuts for each input from order 0.5 to 10. Although only acceleration inputs are considered, the described limitations also hold for pressure inputs. As the main goal of this paper is to show the effects of cross-coupling on the method, the dataset was further modified to minimize the effect of other weaknesses.

For the first example one of the forces was set to zero in the TPA dataset. Since the path contributions in the TPA model are expressed as a product of the input force and the NTF ($F_i \cdot NTF_i$) the corresponding path will show zero partial contribution as shown in Fig. 3. As opposed to this, the OPA partial contributions depend on the degree of cross-coupling between the paths. Using Eq. (2) the individual contributions can be expressed as:

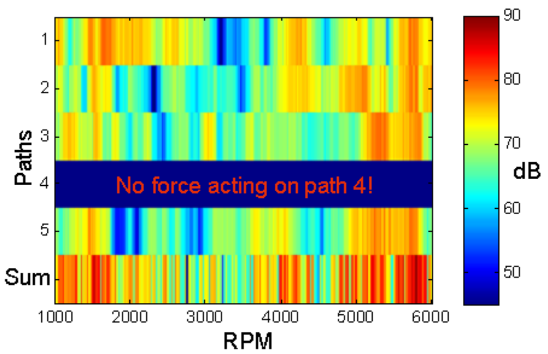

Figure 3: TPA partial path contributions, order 2

$$p_i(\omega) = \left(\sum_j H_{ij}(\omega) F_j(\omega) \right) T_i(\omega) \quad (5)$$

from this it follows that even when there is no force acting at an input point OPA might still show an important partial contribution if there is a strong cross coupling. This effect is clearly visible in the simulation results as shown in Fig. 4 as Path 4, marked with red, shows a quite high contribution in some areas despite having no excitation at that point.

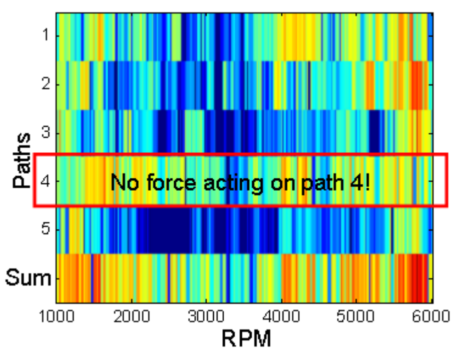

Figure 4: OPA partial path contributions, order 2

As for the second aspect, two examples will be considered. In the first one, the effect of cross-coupling is excluded. Then it can be shown that Eq. (5) simplifies to:

$$p_i(\omega) = H_{ii}(\omega) F_i(\omega) T_i(\omega) \quad (6)$$

Furthermore if the other limitations are also minimized, that is, the forces acting on the system are incoherent and there are enough conditions for the estimation of the transmissibilities, then in this case and only in this case the two methods - TPA and OPA - give the same results as shown on the Figs. 5 and 6.

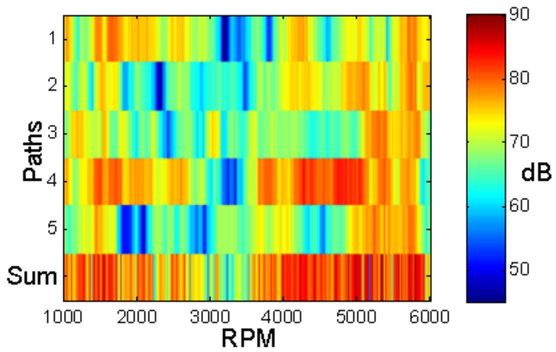

Figure 5: TPA partial path contributions, order 2

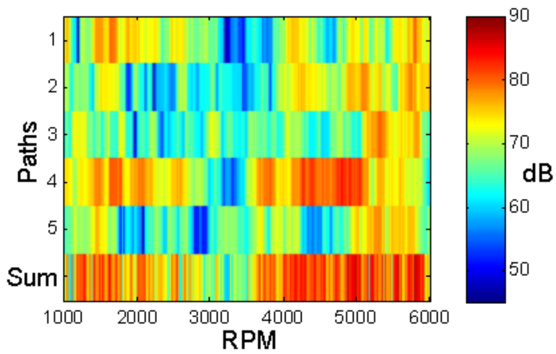

Figure 6: OPA partial path contributions, order 2

In most real-life situations, though, this hardly ever happens. Most structures will exhibit a strong cross coupling, or in other words, a strong modal behavior. For the second example the the cross-coupling is restored to the original measured values. Here the simulation results are not quite the same as before. Comparing TPA (Fig. 7) with OPA (Fig.8), the difference is striking.

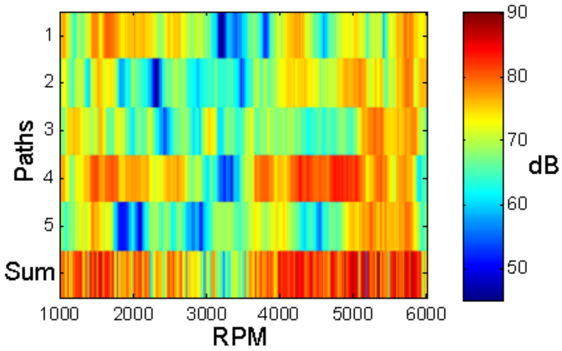

Figure 7: TPA partial path contributions, order 2

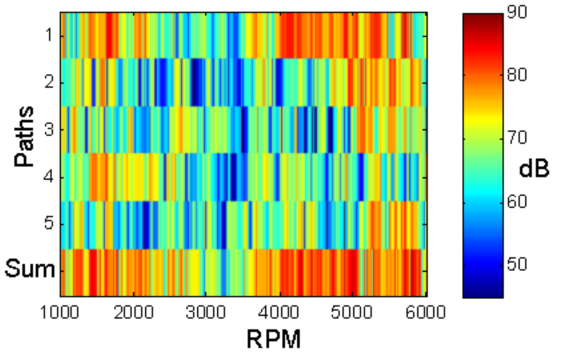

Figure 8: OPA partial path contributions, order 2

The path contributions in OPA are fundamentally different even though the summed contributions show good agreement. For example, whereas Path 4. had an important contribution around 4500 RPM in the reference TPA set, the OPA analysis indicates that Path 1. is the most dominant in that region. Thus, in this particular example, OPA leads to an incorrect engineering conclusion. On the other hand the method should not be completely dismissed. In the region around 1500 RPM the dominant paths are correctly identified.

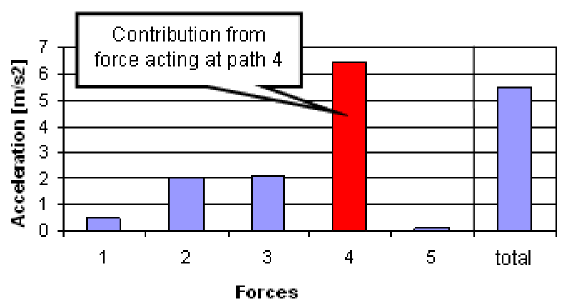

Figure 9: Components of the total input acceleration at Path 1

An analysis of Eq. (5) can help the reader to better understand the mechanism that causes this error. Fig. 9 shows the single members of the summation in the equation - the contribution of each input force to the measured acceleration at Path 1 - at an RPM of 4500. The bar graph reveals that the high vibration level is caused by the force acting at Path 4 and not the force at Path 1. This error should serve as a warning sign against blindly applying OPA without understanding the method.

4 Summary

The goal of this paper was to critically examine a new operational transfer path analysis method. The investigations first of all revealed that the results can't be interpreted the same way as for classical TPA and secondly that this OPA method itself has 3 significant limitations: (i) errors in the estimation of transmissibilities, (ii) the effect of neglected paths and (iii) cross-coupling between the input accelerations; these, depending on the actual application, might prevent the engineer from reaching the right decision and solving the problem. This paper particularly dealt with the last one. One conlusion was that in most real life situations OPA will give different results than TPA owing to the cross-coupling between the input accelerations. At the same time it was also shown that having a good agreement between the synthesized and measured total pressure doesn't necessarily indicate the quality of the synthesis and the reliability of the analysis results, therefore such comparisons should be used with care. Although OPA can be useful as a first troubleshooting tool the results have to be treated with proper caution, taking the limitations into account. Naturally, alternative methods, faster than classical TPA yet having the same precision, are being developed as part of an ongoing research.

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