Elastic constants identification of anisotropic composite rectangular parallelepipeds

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The macroscopic elastic properties of two composites (Duraluminium/air and Duraluminium/tungsten carbide (WC)) have been calculated using periodical homogenisation method from the elastic properties of each phase (measured by high frequency acoustic microscopy). In order to check the validity of such a modelisation, acoustical resonant spectroscopy has been applied. Here the free resonance of a parallelepiped sample allowed to measure the frequency of mechanical vibration by means of laser interferometer. Thanks to an inverse computation, this paper presents the full elastic characterization of the two composite samples and comparisons with those predicted. Here, experimental results are in good agreement for the Duraluminium/air sample but differences appear in transversal properties for the Duraluminium/WC one.

1 Introduction

In order to prepare the fourth generation of nuclear civil power plants, many nuclear fuel are developed in nuclear centers. Many of these new fuels are ceramic/metal or ceramic/ceramic composites as listed in [1]. In such structures, thermo-mechanical effects due to complex phenomena (source of heat, thermal expansion, swelling, densification,...) has to be simulated to study the fuel behavior in reactor.
For mechanical aspect, finite element simulation are generally used but they are very time and memory consuming. Indeed, one needs to use very sharp meshes to obtain accurate results. Consequently, with complex structures such as porous materials or inhomogeneous composites, parametric studies or fuel simulations on long time of irradiation (many years) become difficult and sometimes impossible. So, effective properties of these materials are useful to assess the structure behavior under irradiation.
The attention is focused one the elastic properties of test samples which are closed to the new future composite fuels. Here, samples are rectangular parallelepipeds (Duraluminium) with cylindrical ceramic parallel fibers (WC). Previous works, based on homogenisation method [2], allowed to deduce effectives properties from each phase of the composite and it is here presented in the first section. In a second section, authors developed an experimental setup based on free resonant ultrasound spectroscopy in order to characterize the full elastic properties of samples and compare it to those predicted by homogenisation.

2 Sample properties

The major characteristics of the sample, Fig.1, manufactured by the Society Rolland Bailly (Besançon, France) are listed bellow:

- Duraluminium/Air sample (#1):
  Size : 1cm×1cm×1cm
  Number of holes : 64
  Holes size : 1mm
  Volume fraction of air : 50.26%
  Matrix : Duraluminium
  Durlauminium Density : 2740kg/m^3
  Sample density : 1377kg/m^3

- Duraluminium/WC sample (#2):
  Size : 1cm×1cm×1cm
  Number of holes : 64
  Holes size : 1mm
  Volume fraction of air : 50.26%
  Matrix : Duraluminium
  Durlauminium Density : 2740kg/m^3
  Holes filled with tungsten carbide (WC) rods
  WC density : 14.308kg/m^3
  Sample density : 8569kg/m^3

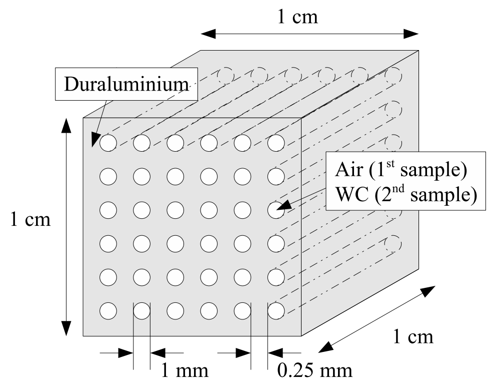

Figure 1: Schematic representation of samples.

2.1 Homogenisation

The determination of elastic effective properties is based on a homogenisation method [2]. The composite is a periodical geometry and it so possible to study it with a Representative Elementary Volume (REV), presented in Fig.2. The mechanical principle, equilibrium and compatibility equations lead to:

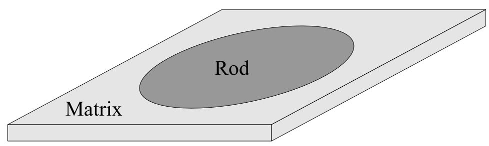

Figure 2: Representative Elementary Volume.

$$\sigma_i = C_i : \varepsilon_i, \tag{1}$$

$$div_z(\sigma) = 0, \tag{2}$$

$$\varepsilon = \frac{1}{2}(grad_z(u) + grad_z^T(u)), \tag{3}$$

where i represents each phase, C the elastic constants and ε the strain. The main idea of the homogenisation method is to decompose the displacement using the periodical displacement on the REV. So, z is substituted by x, the large scale term and y, the periocidal term as below:

$$z = x + \omega y, \tag{4}$$

where ω is a scaling factor. Then the displacement can be written as:

$$\vec{u} = \sum_{i=0}^{+\infty} \omega^i \vec{u_i}. \quad (5)$$

Mean values taken on the REF lead to the effective elastic constants of the composite.

2.2 Acoustic microscopy

Using acoustic microscopy, two modes are possible : imaging or signature [3]. In the latter case, from mechanical waves (generated by a piezoelectric crystal) focused, the sensor is defocused toward the sample and the reflected signal is acquired. Interference phenomena between the specular ray (normal incidence) and three surface waves (longitudinal, transverse and Rayleigh waves) are then created. The reflected signal is pseudo periodic and the velocities of each surface waves can be estimated with the following relation:

$$V_{sw} = \frac{V_{cf}}{\sqrt{1 - \left(1 - \frac{V_{cf}}{2f\Delta z_{sw}}\right)^2}}, \quad (6)$$

where V_{cf} is the ultrasonic velocity in the coupling fluid between the sample ant the sensor, V_{sw} the velocity of the surface wave, f the frequency and Δz_{sw} the pseudo period. An example of acoustic signature for the matrix measurement is presented Fig.3. The Young modulus

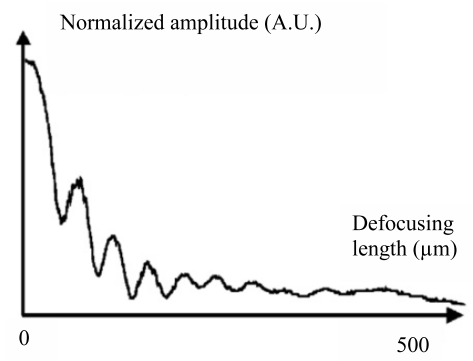

Figure 3: V(z) acquired in the matrix.

(E) and shear modulus (G) can be calculated by:

$$E = \rho V_T^2 \frac{3V_L^2 - 4V_T^2}{V_L^2 - V_T^2}, \quad (7)$$

$$G = \rho V_T^2, \quad (8)$$

where ρ is the density, V_T the transversal wave and V_L the longitudinal one.
Under the assumption that the Poisson coefficient is closed to 0.3, the elastic modulus of Duraluminium is evaluated to 70±5 GPa, which is good agreement with literature [3]. For fibers of WC, the acoustic signature leads to an elastic modulus of 580±5 GPa

2.3 Results

Table 1 shows the results of homogenisation for the Duraluminium/Air sample (# 1) and the Duraluminium/WC one (#2) calculated from each phase properties measured by acoustic microscopy.

Properties	Units	Sample #1	Sample #2
E_x,E_y	GPa	22.7	182
E_z	GPa	36.1	326
G_{xy}	GPa	2.4	51
G_{xz},G_{xz}	GPa	9	62
Nu_{xy},Nu_{xz}		0.122	0.305
Nu_{yz}		0.188	0.153

Table 1: Predicted properties of elastic constants from homogenisation method for the two samples

The aim of the next section is to compare theses results to an experimental characterization. Previous work [2] based on resonant ultrasound spectroscopy shows experimental comparison. In this case samples are excited with glued ceramic piezoelectrics on both faces. All modes of the free vibration of sample are not revealed and the vibration is the result of the system {sample + piezoelectric ceramic}. That's why, authors present a new experimental setup which allows to study the free sample vibration.

3 Free resonant ultrasound spectroscopy

An experimental setup allows here to measure the different frequencies of free vibration of cubes. An inverse resolution based on Simplex methods [4] allows then to deduce experimental elastic constants of sample.

3.1 Modeling of the resonant vibration modes of rectangular parallelepipeds

Let's consider a rectangular parallelepiped with characteristic dimensions $L_1 = \frac{A}{2}$, $L_2 = \frac{B}{2}$, $L_3 = \frac{C}{2}$. The material characteristics are the density ρ and elastic constants C_{ijkl}. The Lagrangian of this body of volume v is [5, 6]:

$$L = \frac{1}{2} \iiint_v u_{i,j} C_{ijkl} u_{k,l} - \rho\omega^2 u_i u_i dV, \quad (9)$$

where ω is the angular frequency. A $e^{i\omega t}$ time dependence is assumed for mechanical. In a variational approach, any mechanical displacement $u_i \rightarrow u_i + \delta u_i$ yields a variation of the Lagrangian: $L \rightarrow L + \delta L$. Hamilton's principle leads us to look for the Lagrangian stationary points at which $\delta L = 0$. This determines a motion equation whose solutions correspond to the free vibrations of the body. In the Rayleigh-Ritz method [5], the mechanical displacements are expressed as a linear combination of functions:

$$u_i = \sum_{p=1}^{N} a_p \psi_p. \quad (10)$$

The $\psi_p,\ p = 1, \ldots, N$, functions are chosen to be orthonormal. To determine a_p constants, an eigenvalue problem is expressed by substituting expressions (10) into equation (9), and by the condition of the stationary Lagrangian:

$$\mathbf{\Gamma a} = \rho \boldsymbol{\omega}^2 \mathbf{a}, \quad (11)$$

$\mathbf{a} = (a_1, a_2, \ldots, a_N)^t$ are unknown vectors. $\mathbf{\Gamma}$ is called elastic interaction matrix. It depends on the shape and on the mechanical properties of the body and its description is given in [6]. The determination of the eigenvalues $\rho\omega^2$ and the eigenvectors $\mathbf{a}$ allows the resonant frequency ω and the modal elastic displacements $\mathbf{u}$ to be identified.
As pointed out in the literature [5, 6] a Legendre polynomial basis is well adapted to describe the behavior acoustical fields inside parallelepipeds. As a result, the solutions of the eigenproblem (11) are sought in the form:

$$\psi_{\mathbf{p}} = \frac{1}{\sqrt{L_1 L_2 L_3}} \overline{P}_\lambda \left(\frac{x_1}{L_1}\right) \overline{P}_\mu \left(\frac{x_2}{L_2}\right) \overline{P}_\nu \left(\frac{x_3}{L_3}\right) \mathbf{e_i}, \quad (12)$$

where the p^{th} basic functions ψ_p is defined by the triplets (λ, μ, ν). $\overline{P}_\alpha(x)$ is the Legendre function of order α and $\mathbf{e_i}$ is the unit displacement vector in the direction $\mathbf{x_i}$.
Note that to accelerate computation time, symmetry consideration can be employed to split the matrix $\mathbf{\Gamma}$ and enhance the eigenvalue resolution [7].

3.2 Experimental setup

Several methods already have been used to perform acoustic spectroscopy on parallelepipeds. They generally involve pinducers or ultrasonic transducers positioned at the sample corners [8, 9]. Here, due to the fact that the samples are not too small, they are excited by one piezoelectric ceramic where its own resonance is about 250kHz. Velocity measurements at the surface of the sample are carried out through a laser vibrometer (Polytech OFV-505) so as to detect resonance frequencies as well as the associated mode shapes. The interferometer is positioned at 50 cm from the sample leading to a 20 μm focal area. Fig.4 shows a schematic presentation of the experimental setup. Note that in order to maintain the sample without changing limit conditions a piece of cotton holds the upper corner. The piezoelectric ceramic is directly linked to a frequency generator. Then, a frequency sweep allows (when the amplitude response of the velocity is maximum) to note the different resonance of the cube.

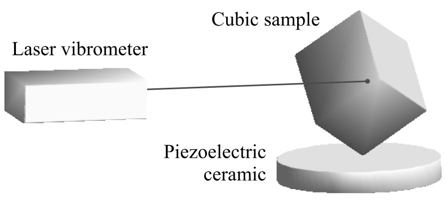

Figure 4: Experimental setup.

Table 2 shows the free resonance measured for the two samples.

Sample #1 Frequency (kHz)	Sample #2 Frequency (kHz)
72.2	101.6
88.9	126.4
114.6	138.9
120.2	166.0
127.0	190.6
156.8	207.6
157.9	216.0
161.2	248.8

Table 2: First measured free resonance frequencies of the two tested sample

3.3 Inverse problem and identification of elastic constants

To identify the material characteristics, a fit procedure has been developed. Modifying the C_{ijkl} tensor enables the computed resonant frequencies to be matched to the measured data. The fit (based on the simplex routine [4]) is carried out in order to determine the elastic constants by minimize the distance Δ_{mc} between the experimental resonance frequencies $f^{(i)}_{\text{measured}}$ and the computed vibrations $f^{(i)}_{\text{computed}}$:

$$\Delta_{mc} = \frac{\sum_i \left| f^{(i)}_{\text{mesured}} - f^{(i)}_{\text{computed}} \right|}{\sum_i f^{(i)}_{\text{measured}}} \quad (13)$$

Table 3 shows the result of elastic identification for the two samples. Here, results are given with the C_{ijkl} notation and converted into E, G and Nu modulus to compare with theoretical results.

3.4 Discuss

One can see on Table 3 that for the Duraluminium/Air sample, experimental characterization is in good agreement with those computed by homogenization method (Table 1). This can be checked by Table 4 which shows that difference between measured frequencies and computed frequencies after fitting is less than 0.5%.
For the Duraluminium/WC sample, the fit procedure has been more difficult to realize as one can see on Table 4. First frequencies computed are not all measured due to low signal and the difference reach 4.5% for

Properties		Units	Sample #1	Sample #2
Elastic	C_{11}	GPa	22.3	130.4
constants	C_{33}	GPa	49.1	373.1
	C_{12}	GPa	3.7	40.0
	C_{13}	GPa	5.3	84.2
	C_{44}	GPa	2.7	41.9
	C_{66}	GPa	8.8	66.4
Elastic	E_x	GPa	21.1	106.7
modulus	E_z	GPa	36.9	270.0
	G_{xy}	GPa	2.7	41.9
	G_{xz}	GPa	8.8	66.4
	Nu_{xy}		0.137	0.195
	Nu_{yz}		0.206	0.180

Table 3: Experimental elastic properties of sample #1 and #2

the first mode identified. It corresponds to a torsional mode as it is presented in Fig.5 computed on 106.2kHz while the measured one corresponds to a lower frequency (101.6kHz) which allows to deduce that shear properties are softer than those presented in Table 3. However, results in Table 3 are computed from a global fit and show that the most discrepancy appears for the value of E_x ($= E_y$) where the difference is about 40%. This difference was already showed in [2] and should be explained by a fiber/matrix debonding.

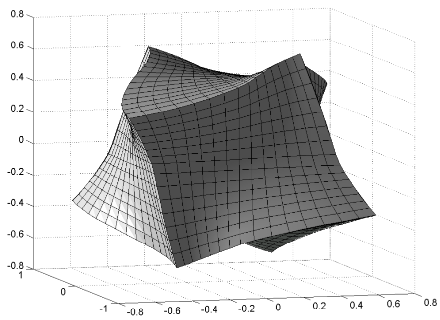

Figure 5: Computed modal shape for the 106.2 kHz mode (Sample #2).

4 Conclusion

In this paper, the ultrasound spectroscopy has shown that homogenisation for composite is a good method to compute effective elastic properties due to the fact that measured constants are in good agreement with those predicted. A discrepancy can appear when there is a partial debonding between phase. In order to study this particularity, future works will focus on a full scan of the vibration. Indeed, the laser interferometer used here has a 20 μm focus area which allows to point one fiber individually and should show if vibration has a particularly aspect. Furthermore, authors have not measured here all resonances and the measurement of modal shapes will bring more precision on computation and fit process.

Sample #1		Sample #2	
Measured	Computed	Measured	Computed
72.2	72.0	101.6	106.2
88.9	88.4	126.4	126.0
114.6	114.7	138.9	137.3
120.2	121.0	166.0	167.8
127.0	127.0	190.6	188.2
156.8	156.7	207.6	204.6
157.9	157.6	216.0	216.8
161.2	161.2	248.8	250.4
$\Delta_{mc} = 0.23\%$		$\Delta = 1.35\%$	

Table 4: Comparison of measured and computed free resonance after fitting (in kHz)

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