Measurements on Quarterwavelength Tubes and Helmholtz Resonators

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In this work some measurements on quarter-wavelength tubes and acoustic resonators were made. The objective was to see what are factors limiting the prediction of the first frequency resonance in these devices. The prediction of the resonance was made by using the so called "transcendental formula". The conclusion of measurements is that the sound speed, the end corrections and non ideal effects (like visco-thermal, form factor, effect in edges etc.) are the main factors that limit the prediction. Non ideal effects make devices to resonate at lower frequency. The end correction proposed by Levine et Al. seems to underestimate the effective length of the tubes. Results obtained here seem to support the results obtained by Peters et Al.

1 Introduction

The objective of measurements presented in this work was to learn the factors that usually limit the prediction of the first resonance frequency (Helmholtz frequency) in quarter wavelength tubes and cylindrical acoustic resonators.

If the first resonance of a cylindrical acoustic resonator is predicted by the classical theory [1], the result is a value that depends on the sound speed, effective length of the orifice (including end corrections), its section, and the length and section of the cavity. A quarter wavelength tube is a particular case of a resonator but where the section of the orifice equals the section of the cavity. The internal end correction being, therefore, equal to cero.

The, so called, end corrections change the effective length of a tube or orifice of a resonator in such a way that, when the radius increase, the resonance of the device decreases. There are considerable amount of bibliography where different values for end corrections are proposed. Only three of them will be discussed here: The end correction proposed by Levine et Al. [2] for unflanged tubes, the correction proposed by J:W:S Rayleigh [3] for flanged tubes, and the values measured by Peters et Al. [4] in unflanged tubes. The discussion presented is, of course, also consistent with values of end corrections provided by other authors (see for example [5, 6]) although only these three mentioned above will be used here.

It is known that sound speed depends mainly on the temperature but has also a small dependence on the humidity, barometric pressure and the composition of the atmosphere. Serious attempts to provide an equation including all of these variables have been made (see, for example work of O. Cramer [7]). Nevertheless, the author did not find any expression that is widely accepted, tested experimentally and which accounts for all of these variables. In some works, very precise measurements are attempted considering only a correction for the humidity [4], but nevertheless, again the corrections proposed by different authors may differ. Therefore the sound speed is another source of error that introduces uncertainty to the resonance.

Other factors that affect the resonance are those related to non ideal effects, like visco-thermal effects, form factors, effect on edges, impedance related to materials, etc. It is very difficult to discriminate these effects one from another and they often are underestimated in calculations and theoretical models. It is not possible to make a precise measure of them here but it is possible to demonstrate some of the effects they produce.

2 Measurements

A total of 23 tubes and 6 resonators were measured. Each one was marked with a number (1-23, 1-6). All of them were made in aluminium. Dimensions of each tube were measured with the help of a micrometer with a precision of $+/-0.1mm$, except those marked with * (in the tables). Those where measured with a precision of $+/-1mm$.

The resonances were predicted first using the so called "transcendental formula" [8, 9]

$$\frac{l' A}{LS} kL = \cot(kL) \qquad (1)$$

Where $l' = l_0 + l_i + l_e$ and $S = \pi r_0^2$ are the effective length and cross section surface of the orifice of the resonator, L and $A = \pi R^2$ the length and cross section area of the cavity and k the wave number (see Fig.1).

The sound speed was estimated from the formula provided by Pierce [10] for dry air in the temperature range $0-30^oC$:

$$c = 331.4 + 0.6T_c,$$

where T_c is the temperature in centigrade.

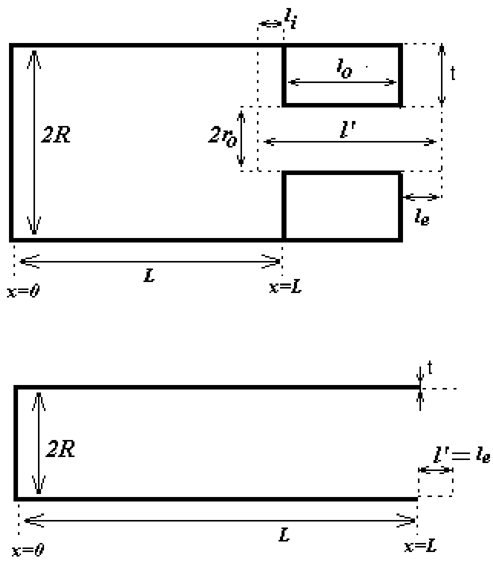

Fig.1. Schema of tubes and resonators measured.

The measurement setup is shown if Fig.2. Each resonator or tube was located in an anechoic chamber with the open end facing to a loudspeaker positioned at $1m$ distance. A first microphone was placed at $40cm$ away from the driver's diaphragm in order to regulate the amplitude of the sound pressure radiated from the source. A second microphone was mounted through the wall of the closed end of the device.

A sweep sine around the resonance of the device was performed, the loudspeaker being driven by the source of an Audio Precision System Two analyzer. For each frequency, when the first microphone measures a pressure of $1 Pa$, the pressure measured by the second microphone is recorded. After the sweep, the values of frequency and pressure measured by the second microphone are stored in a file and used later on for detecting the frequency at which the absorption of the device is maximal. This frequency was determined for each device with an error of $+/-1 Hz$.

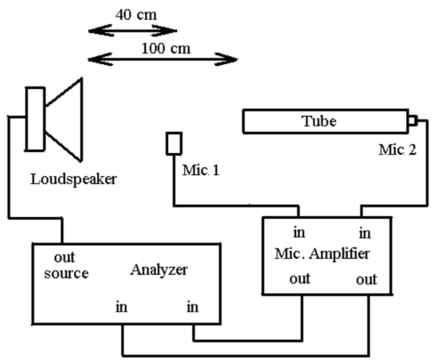

Fig. 2. The measurement setup.

By means of a couple of sensors located in two different corners of the chamber, measurement conditions of barometric pressure, humidity and temperature were recorded before and after each sweep. If a variation of more than one unit was detected in one of the sensors, the measurement was repeated. When the tube is mounted in its position, 15-20 minutes are needed until the temperature of the tube gets equal to the chamber temperature.

Some of the tubes were resonating at a frequency below the cut down frequency of the chamber. In way to verify the results obtained with these tubes, some of these were later measured also in open air locating each tube facing to the ground at 4 meters height and with the loudspeaker radiating from down to top.

Results are presented in graphs but complete results and measurement conditions are given in tables at the end of the paper.

3 Discussion of results

3.1 Measurements in tubes

The first measurements were made to investigate the dependency of the resonance prediction with the radius of the tubes. It is known from theory that the end correction depends on the radius and thickness of the tubes. However in those devices where this correction is negligible, the theory predicts the frequency resonance independently on the radius.

The first results were obtained in long tubes, where the length of each device is so that the end correction is negligible, not affecting its resonance value. The resonance was predicted with the Eq.(1) for each device. After measurement, the error (in %) made in the prediction was estimated. The results are shown in Fig.3. The first five tubes have similar length (that may be considered equal) and the theory predicts the same value for the resonance ($f_{0,61} = 86Hz$, See Table I) for all of them. However the error obtained in the prediction of the resonance increases when the radius of the tube decreases. The last tube has been chosen with a radius very large and, therefore it has been made longer in way to still neglect the end correction. For this last tube, the predicted resonance frequency was obtained with error zero inside the experimental and theoretical tolerance of the measurements and calculations made here.

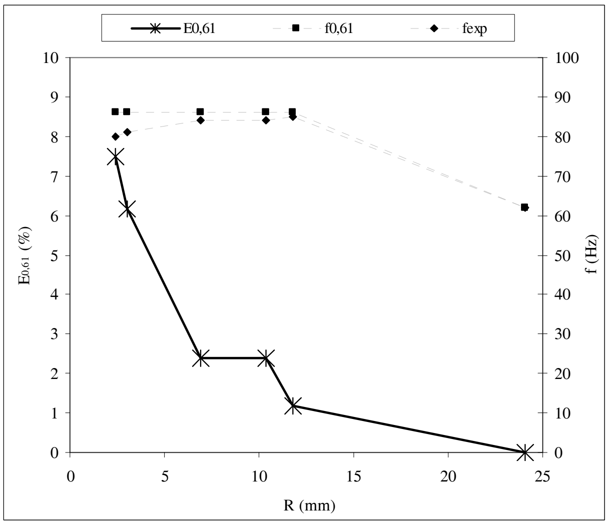

Fig.3. Experimental ($f_{\exp}$) and theoretical ($f_{0,61}$) resonance frequencies are shown against the radius of long tubes. The error $E_{0.61}$ made in the prediction of the resonance decreases when the radius increases.

Therefore there is here a dependency of the resonance with the radius and that is neither related to the end correction nor to any variation of length of the tube. This shift of the resonance may be consequence of non idealities not been considered in the Eq.(1). These non idealities are, most probably, related to visco-thermal effects, but also a form factor and irregularities on the surface of tubes may have an impact. In addition, other effects like impedance of walls, effects of edges and corners, may be added and cause the shift of the resonance. Because it was not possible to discriminate these effects and demonstrate which one is dominant, it has been decided to call all these “non ideal effects”. The dependency of the shifting with the radius of the tube is clearly demonstrated. However it is important to note here that, the amount of shifting experimented by the resonance may be different if tubes were of different length than these measured here or the end corrections were not negligible.

The second set of measurements was made for shorter tubes, where the end correction is not negligible. All of them have similar length and thickness but the radius increasing (see Table II). The measurements were made as in the case of long tubes and the calculations were made using the Levine end correction ($l_e = 0.61R$). Results were plotted in the Fig.4.

The graph shows that the error in the prediction of the resonance again decreases when the radius increases. However now it was not possible to decrease the error to zero as in the case of long tubes. If the radius of tubes is still increased more, the error starts to increase again.

When the radius of the tube is increased, the same "non-ideal effects" discussed for long tubes become more and more negligible, but the contribution of the end correction is more relevant (the end correction increases with radius). Therefore, the end correction introduces an uncertainty increasing the error in the prediction of the resonance.

Therefore, the resonance frequency in a tube is predicted better if the tube has wide radius for make non ideal effects negligible and its length is long enough for avoid the error introduced by the end correction.

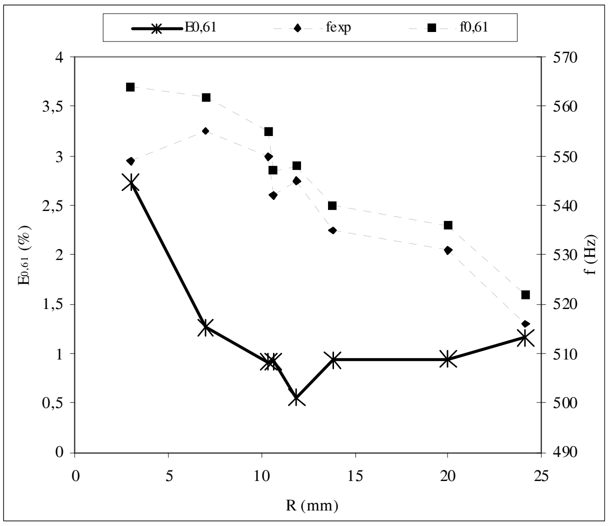

Fig.4. The graph shows the resonance frequency (experimental f_{exp} and theoretical $f_{0,61}$) against of radius of short tubes. The error $E_{0.61}$ in the prediction of the resonance is affected by the uncertainty introduced by the end correction.

If calculations were made using the Rayleight end correction ($0.85R$) it could give the wrong impression that the error decreases when the radius decreases. When a too long end correction is considered, the frequency resonance may be predicted below the experimental. If, after that, the radius of the tube is decreased, the non ideal effects will make the measured resonance to decrease approaching the value predicted (see Table II, $f_{0,85}$ and $E_{0,85}$). Note also that the error in these cases could be even smaller than in the case shown in the Fig.4, appearing as more acceptable when, in fact, errors are compensating between them.

One additional observation in the graph of Fig.4 above is that all resonances predicted using the Levine end correction have been predicted by excess ($E_{0,61}$ was positive in all cases). By using Rayleigh end correction ($0,85R$) the resonance would be predicted with negative error (See $E_{0,85}$ in Table II). So the question arising now is what value of the end correction is the correct one. By looking these two values and results achieved above it is somewhat intuitive that the value for a tube of a certain thickness, must be somewhere in between these two. This result was demonstrated by Y. Ando [5], Nomura et Al.[6 and, more recently, by Peters et Al [4]. The first author calculated the end correction of a circular tube and demonstrated that it depends on its thickness. He also provided theoretical values for the correction that were well supported by experimental results and consistent with corrections of Levine and Rayleigh. The last author attempted a precise measurement of the end correction of a circular tube of different thickness in a somewhat more elaborated experiment. In the work of Peters et Al., measurement results in a circular pipe were provided for three values of the ratio $\delta = R/(R+t)$: $\delta = 1$ (sharp edges), $\delta = 0.85$ and $\delta = 0.70$. From tables provided of that work, it was possible to estimate an approximate value for the end correction for each one of these three cases. Values of l_e/R determined are 0.65, 0.68 and 0.72 respectively. These values are consistent with those provided by Y. Ando and have been used here to predict the frequency resonance of 9 additional tubes.

For each tube, the value of δ is calculated and compared to these three values estimated. The closest one determines the value of the correction l_e/R to be used in the calculation. Table III provides values of δ for each tube, as well as the value of the end correction used. The value of the resonance calculated is noted by f_p and E_p is the relative error obtained. Results are given in Fig.6 (and Table III).

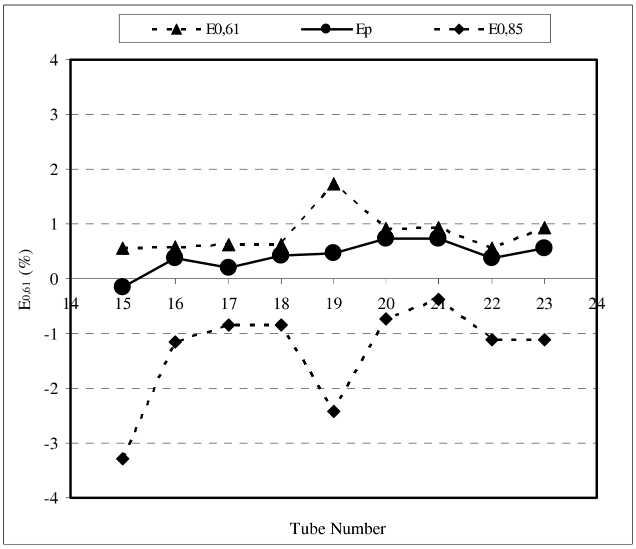

Fig.6 Error in the prediction of the Frequency Resonance when the end correction is considered like Levine (▲), Peters et Al (●) and Rayleigh (♦).

Values provided in the work Peters et Al. are limited and the estimation of the correction made here is somewhat rough. Nevertheless, the Fig.6 shows that, after the calculation, the resonance frequencies were predicted with smaller error in all cases and results are consistent with all discussion developed above.

Results obtained indicate that, when the tube has a thickness (even if it is small), a larger value of the end correction than the ones provided by Levine et Al and smaller that the ones of Rayleigh is needed. Values provided by Peters et Al. enabled to predict the resonance of tubes more accurately here.

The value of Sound Speed estimated here was for dry air. If corrections for humidity, pressure and atmosphere composition were made a larger value would be obtained. Therefore, for each tube, a larger value of the predicted frequency would be obtained and the error using the Levine end correction would be slightly larger. Even if not any definitive conclusion can be made in this work, the results obtained suggest that the end correction of an unflanged tube is larger than the ones proposed by Levine et Al. and supports the results obtained by Peters and Al.. However not precise values for the end corrections may be given in this work.

Because there are no idealities no considered in the theory, an error in the estimation of the sound speed and the uncertainty of measurements and end corrections, it has been no possible to predict more accurately the resonances here.

3.2 Measurements in acoustics resonators

The effective length of the orifice of a resonator is the result of the contribution of the length of the orifice itself, the external and the internal end corrections. Ingard [1] investigated the interior end correction of concentric circular tubes and showed that, when $\xi = r_0/R < 0.4$ the interior end correction can be approximated by the following expression written here again for convenience:

$$l_i = \frac{8r_0}{3\pi}\left(1 - 1.25\frac{r_0}{R}\right). \qquad (2)$$

If $\xi > 0.4$, the value to be used must be deduced directly from the tables provided in Ingard's work. Values for the internal end corrections were calculated from Eq.(2) and these tables.

Six devices were measured. First, resonances were calculated using the Levine external end correction for the first three devices. For the other three, the Rayleigh end correction seems more convenient after the value of the thickness t (Table III). Resonators were measured and the error made in the prediction was calculated. Results are shown in the Fig 7 and Table IV.

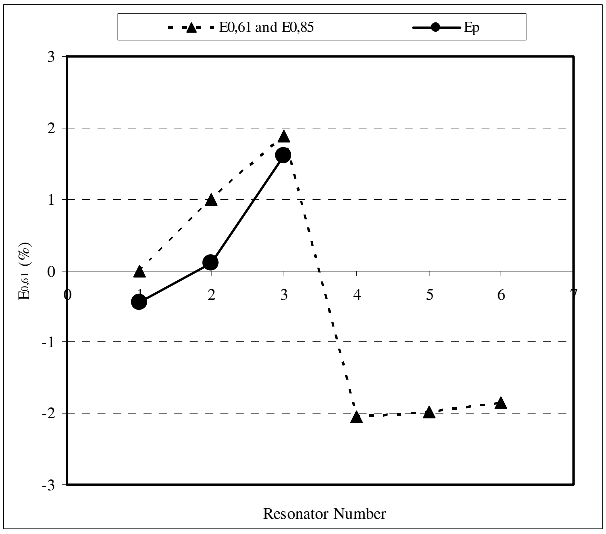

Fig.7 The graph shows the error made in the resonance prediction in six acoustics resonators. In the three first the resonance was calculated using Levine and Peters end correction. In the last three the prediction was made using Rayleigh end correction only.

In general, a good agreement between prediction and measurements is again obtained but also a deeper analysis is needed. In the case of the resonator 1 the resonance using Levine end correction was predicted with no error. Nevertheless, as it was suggested earlier in the case of tubes, this correction is too short. Therefore, the error introduced by it is compensating the error introduced by the interior end correction and no ideal effects. The same problem may happen in the other resonators measured.

In the same graph and table is also shown the error made when considering the external correction as Peters et Al for the first three resonators. Note that, the overall result is better even if in the first resonator the error have increased compared to prediction made with Levine because the error in the internal end correction.

4 Conclusion

Some measurements on unflanged quarter wavelength tubes and resonators have been carried out. There are three factors that limit the prediction of the resonance frequency these devices: sound speed, end corrections (internal and external) and non ideal effects like visco-thermal effects, form factors etc.

An important consequence of the non-ideal effects is that devices resonate at lower frequencies. This is a result that is not completely explained by the theory. The error introduced by all these factors may compensate between them often giving the impression e.g. that different values of end corrections are suitable.

There is considerable literature about external end corrections. However the conclusion that may be made here is that the measured by Peters et Al. are not far from being correct. Therefore all theories that explain these values could be considered as acceptable. Concerning the internal end correction proposed by Ingard is not experimentally proved and they may introduce an error in calculations.

The accuracy achieved here when predicting resonance in tubes and resonators considering the sound speed for dry air was around 2%.

Tables

	Meas. Conditions			Tube Dimensions			Meas.
Tubes	**P**	**H**	**Tc**	**L**	**R**	**t**	**f_{exp}**
number	*hPa*	*%*	*oC*	*mm*	*mm*	*mm*	*Hz*
1	993	37	24	1002*	2.4	0.5	80
2	993	37	23.8	1002*	3	1	81
3	993	37	23.6	1002*	6.9	1	84
4	993	38	23.8	1001*	10.4	2	84
5	993	38	24	1001*	11.8	1	85
6	1014	28	22.8	1349*	24.1	1	62

	Unflanged Tube	
Tubes	**$f_{0.61}$**	**$E_{0.61}$**
number	*Hz*	*%*
1	86	7.5
2	86	6.2
3	86	2.4
4	86	2.4
5	86	1.2
6	62	0.0

TABLE I Dependency with the radius of tubes

	Meas. Conditions			Tube Dimensions			Meas.
Tubes	P	H	Tc	L	R	t	f_{exp}
number	*hPa*	*%*	*oC*	*mm*	*mm*	*mm*	*Hz*
7	1001	22	22.4	151	3	1	549
8	1001	22	22.6	149.2	7	1	555
9	1000	22	22.6	149	10.4	0.4	550
10	1011	19	22.4	150.7	10.6	2	542
11	1001	22	22.4	150	11.9	0.4	545
12	1012	22	22.5	151.2	13.8	1.1	535
13	1011	22	22.4	150.1	20	1	531
14	1013	22	22.3	150.6	24.1	1.1	516

	Unflanged Tube		Flanged Tube	
Tubes	$f_{0.61}$	$E_{0.61}$	$f_{0.85}$	$E_{0.85}$
number	*Hz*	*%*	*Hz*	*%*
7	564	2.7	562	-0.4
8	562	1.3	556	-1.1
9	555	0.9	546	-1.6
10	547	0.9	540	-1.3
11	548	0.6	539	-1.6
12	540	0.9	529	-2.0
13	536	0.9	516	-3.7
14	522	1.2	505	-3.3

TABLE II Dependency with the end correction

	Meas. Conditions			Tube Dimensions			Meas.
Tubes	P	H	Tc	L	R	t	f_{exp}
number	*hPa*	*%*	*oC*	*mm*	*mm*	*mm*	*Hz*
15	987	46	22.3	61.7	11.8	0.6	1244
16	993	37	24.1	157	11.8	1	523
17	989	45	22.5	173.2	10.4	2	477
18	993	37	24.2	173.5	10.4	2	477
19	989	45	22.4	50.4	10.4	2	1493
20	1000	22	22.6	149	10.4	0.4	550
21	1011	19	22.4	150.7	10.6	2	542
22	1001	22	22.4	150	11.9	0.4	545
23	1012	22	22.5	151.2	13.8	1.1	535

	Unflanged Tube		Flanged Tube		Peters et Al. End Corrections			
Tubes	$f_{0.61}$	$E_{0.61}$	$f_{0.85}$	$E_{0.85}$	δ	l_i/R	f_p	E_p
number	*Hz*	*%*	*Hz*	*%*			*Hz*	*%*
15	1251	0.6	1203	-3.3	0.95	0.65	1242	-0.2
16	526	0.6	517	-1.1	0.92	0.65	525	0.4
17	480	0.6	473	-0.8	0.84	0.68	478	0.2
18	480	0.6	473	-0.8	0.84	0.68	479	0.4
19	1519	1.7	1457	-2.4	0.84	0.68	1500	0.5
20	555	0.9	546	-0.7	0.96	0,65	554	0.7
21	547	0.9	540	-0.4	0.84	0,68	546	0.7
22	548	0.6	539	-1.1	0.97	0,65	547	0.4
23	540	0.9	529	-1.1	0.93	0,65	538	0.6

TABLE III Comparation of resonances predicted using different end Corrections: Levine and Schwinger, Rayleigh and Peters et Al.

	Meas. Conditions			Resonator Dimensions					Meas.
Resonators	P	T_c	H	L	R	t	l_0	r_0	f_{exp}
number	*hPa*	*°C*	*%*	*mm*	*mm*	*mm*	*mm*	*mm*	*Hz*
1	997	23.4	41	157	11.8	2	19.8	10.4	448
2	997	23.3	41	61.7	11.8	2	19.8	10.4	905
3	997	23.4	41	157*	11.8	2	50.4	10.4	373
4	996	22.3	46	61.7	11.8	19.7	0.8	2.4	634
5	996	22.2	46	61.7	11.8	16.5	0.8	5.6	956
6	996	22.2	46	61.7	11.8	18.2	0.8	3.9	810

	Unflanged Resonator		Peters et Al. End Corrections	
	ξ= 0.9; li/R= 0.12			
Resonators	$f_{0.61}$	$E_{0.61}$	f_p	E_p
number	*Hz*	*%*	*Hz*	*%*
1	448	0.0	446	-0.4
2	914	1.0	906	0.1
3	380	1.9	379	1.6

				Flanged Resonator	
Resonators	ξ	l_i/R	δ	$f_{0.85}$	$E_{0.85}$
number				*Hz*	*%*
4	0.20	Eq. (2)	0.11	621	-2.1
5	0.47	0.36 (Ingard)	0.25	937	-2.0
6	0.33	Eq. (2)	0.18	795	-1.9

TABLE IV Measurements and calculations in resonators

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