Acoustics'08 Paris
June 29-July 4, 2008
www.acoustics08-paris.org

Analysis of band-pass spectra of phononic defect-mode waveguides based on mode coupling between point-defects

Toyokatsu Miyashita

Dept. Electronics & Informatics, Ryukoku University, Seta Oe-cho Yokotani 1-5, 520-2194 Otsu, Japan
miya@rins.ryukoku.ac.jp

The band-pass structure of the defect-mode waveguides (DMWGs) fabricated in sonic/phononic crystals has desirable characteristics for practical applications to the acoustic band-pass filter. The waveguide mechanism is considered to originate in the mode-coupling of point-defects, a chain of which composes a DMWG, in the crystal. In order to analyze the mechanism which determines the bandwidth of the DMWG, we developed a phenomenological linear coupled-mode equation for an arbitrary number of linearly coupled resonant modes, derived recursive eigenvalue equations, and analytically solved eigenfrequencies up to the seventh order. The eigenfrequencies were quantitatively compared with the peak-frequencies in the transmission spectra obtained by an elastic FDTD simulation of the acoustic wave propagations in the DMWGs. They showed a quite good quantitative agreement. The bandwidth of the DMWGs in a sonic/phononic crystal with an identical coupling coefficient was estimated to be around 1.85 times the resonant-frequency split of the coupled two point-defects. Adjustment of the coupling coefficient is considered to be the easiest way to control the bandwidth of the DMWG filter.

1 Introduction

Sonic/phononic crystals are artificial crystals for sonic or ultrasonic waves, just like photonic crystals for light waves.[1] Their periodic array of scatterers makes a full (omnidirectional, or complete) band gap, in which no plane waves can propagate through the crystal in any direction and the waves are completely reflected.[2] One of most attractive features of these artificial crystals is an ability to fabricate various types of acoustic waveguides. Conventionally, a line of successive scatterers are removed to fabricate a waveguide which is surrounded by two blocks of artificial crystals with a full band gap.[3]

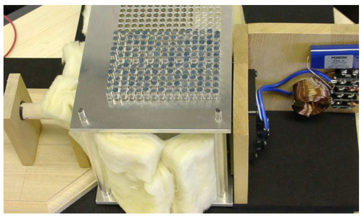
In contrast to the above mentioned conventional linear-defect waveguides, novel ones named defect-mode waveguides (DMWGs) have been studied for acoustic waves.[4–6] A DMWG is composed of a chain of point-defects,[9] where a point-defect or a single-defect is a lack of a scatterer in a sonic/phononic crystal, typically being fabricated by removing every second scatterers of the host crystal along the desired propagation path of the relevant waves. Straight and sharp perpendicularly bending DMWGs have been discussed in comparison to the conventional waveguides.[9] The most distinct feature is a clearly distinguished passband of a nearly 0 dB flat transmission for any DMWGs with and without sharp bends, while the conventional waveguides have a relatively low transmission of −9 dB.

Visualized observation of the numerical simulations of the wave propagation along DMWG revealed a successive transfer of a kind of localized resonant mode between the point-defects and two good characteristics (1) the guided waves travel along the waveguide without noticeable reflection at input and output, and even at sharp bends of the waveguides, and (2) the waves are well confined around the waveguide with a good transmission of about 0 dB.[9] It was found quite easy to fabricate acoustic waveguides of a bandpass characteristic with sharp perpendicular bends, branch or crossroads,[8] and also two coupled waveguides in a sonic/phononic crystal.[9]

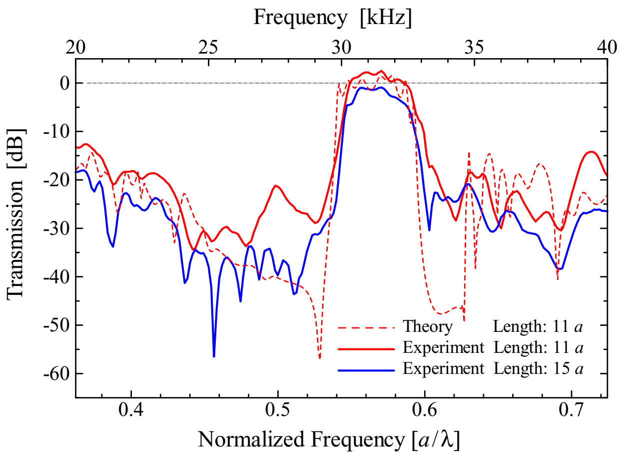
A resonant mode localized in a point-defect, named defect-mode, has no capabilities to travel in the crystal. Acoustic coupling between the defect-modes of the neighboring point-defects through their evanescent fields opens, however, a way for the defect-modes to travel. The most simple case of two coupled defect-modes has been successfully discussed by phenomenologically applying a liner coupled-mode theory, and a coupling coefficient has been estimated precisely both theoretically and experimentally.[7] In this report, the linear coupled-mode theory is developed to an arbitrary number of coupled resonant modes, namely recursive formulae for the eigenfrequencies are derived. Furthermore, analytical solutions are calculated up to the seven coupled-modes, and quite good agreements are obtained with the transmission spectra of the DMWGs calculated by FDTD simulations and with those observed experimentally.

2 Experimental transmission spectra of DMWGs

First we show a most fundamental example of DMWGs, namely, a short straight 2-D DMWG consisting of a chain of seven point-defects, in Fig. 1(a), among already reported[8] short or long DMWGs with a bend, branch or crossroads. They were fabricated in a sonic crystal consisting of an array of 15 × 11 acrylic-resin cylinders in



(a) 2-D defect-mode waveguide



(b) Transmission spectra

Figure 1: Experimental transmission of defect-mode waveguides compared with theoretical one obtained by a finite-difference time-domain numerical simulation.

air with a lattice constant a of 6.25 mm and a diameter of the scatterers of 5.0 mm. The measured frequency characteristics of the normalized transmission, shown in Fig. 1(b), have a clearly distinguishable and relatively flat passband between 0.545 to 0.594 in the normalized frequency (a/λ) with a nearly 0 dB transmission. They agree closely with the theoretical results[9] as shown in the same figure. The purpose of this paper is to show quantitatively that those good characteristics of DMWG result from a linear mode coupling between a chain of point-defects.

3 Theoretical analysis

Mode coupling phenomena between adjacent two point-defects and split resonant frequencies making ripples in the frequency characteristics of the wave transmission through a chain of point-defects are considered fundamental features of the DMWG mechanism. The finite-difference time-domain (FDTD) method is the most powerful one to simulate precisely waves in an arbitrary structure of materials. We have simulated the behaviors of acoustic waves in DMWGs by the elastic FDTD method.[8] The lattice constant is $a = 50\Delta x$ and the radius of the acrylic resin cylinders is $r = 20\Delta x$, consequently the filling ratio is 0.503, where Δx is the spatial sampling interval for the FDTD calculation.

3.1 Internal excitation

We have investigated the magnitude of the mode coupling not only between adjacent two point-defects but also between two point-defects separated along a lattice axis by three or four times the lattice constant.[10] The temporal evolution of the sound pressure at the center of the two point-defects are shown in Fig. 2. Excitation of

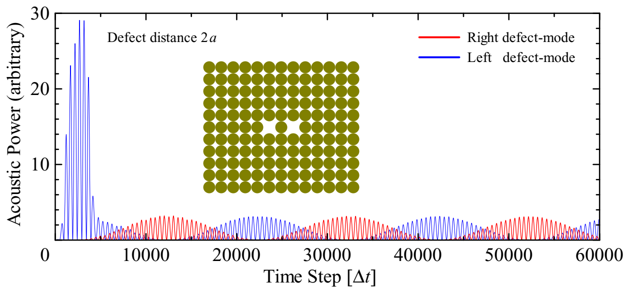
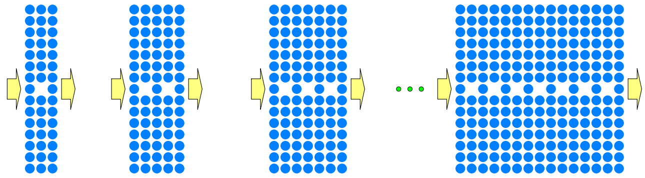


Figure 2: Internal excitation of the coupled two point-defects simulated by FDTD method.

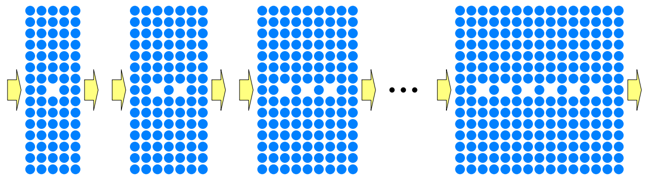
sound pressure at the center of the left point-defect with a five-period tone-burst makes, after a short transient, a steady oscillation in the left point-defect, as shown by a blue curve. A defect-mode is excited also in the right point-defect, as shown by a red curve, through an energy transfer by the mode coupling. An alternative excitation of the acoustic fields in the two point-defects occurs periodically. The period of the energy transfer has a direct relation to the magnitude of the mode-coupling between the point-defects, as discussed in the later sections.

3.2 External excitation

Standard DMWGs are considered with a scatterer at both input and output. Even a chain of several point-defects works as a good practical waveguide bandpass filter as shown in Fig. 1. So, from a point-defect to a chain of seven point-defects, we have calculated their transmission spectra by the FDTD method.[7] Their cross-sections are shown in Fig. 3(a). All point-defects



(a) Standard chains of point-defects

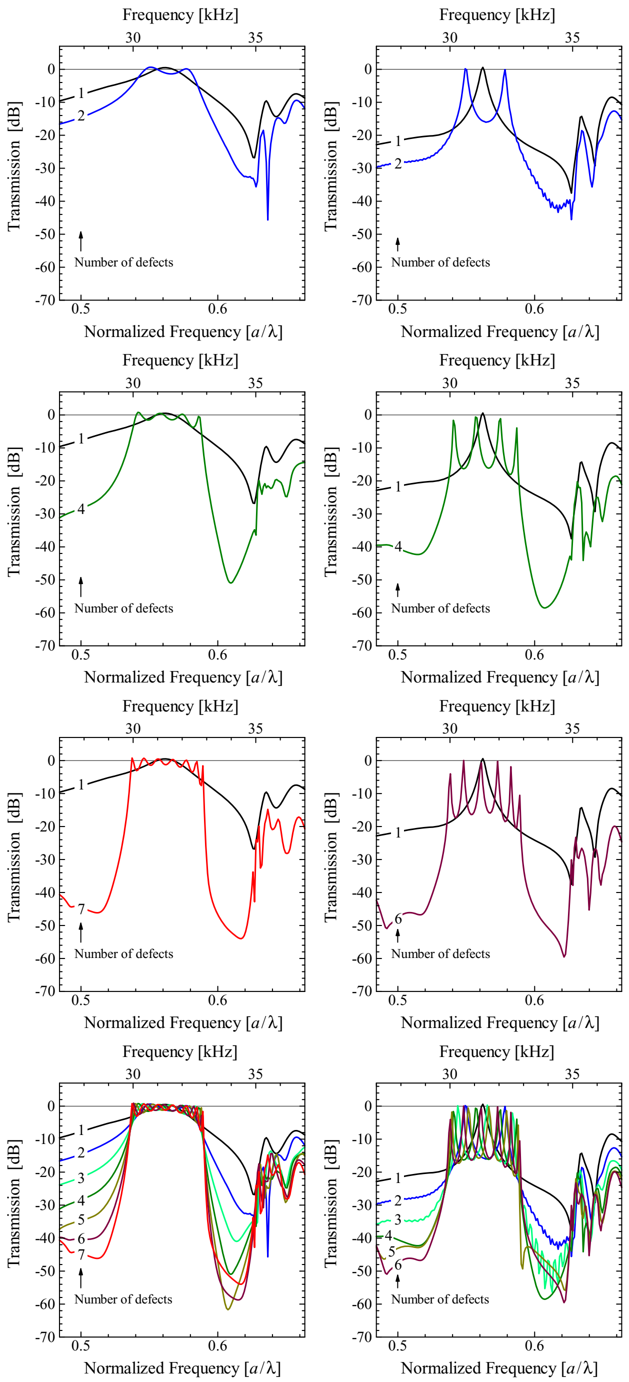


(b) Chains ended with an additional scatterer at both input and output

Figure 3: Two types of chains of point-defects.

are neighbored with a scatterer between them, and also the output and input ones have a scatterer between the outside of the crystal. The calculated transmission spectrum of the shortest one with a point-defect has a broad and smooth resonance as shown by a black curve in Fig. 4(a). Increasing the number of point-defects in a chain as shown in Fig. 3(a), the transmission has as many peak frequencies as the number of point-defects, and makes small ripples in the high transmission range, namely in the passband. The transmission through a chain of seven point-defects is shown by a red curve. It has approximately 2 dB ripples in the center of the passband. Note that the bottoms of the left and right outer-most ripples level just on the broad transmission for a point-defect. Outside the passband are the forbidden bands, whose levels are deeper with the number of point-defects unsymmetrically from −4 dB to −47dB.

One of the important requirements to this desirable characteristics for an acoustic bandpass filter is to design easily the bandwidth. In order to make clear the peak frequencies in the passband and to investigate them quantitatively, we embedded the chains of point-defects deeper in the crystal with an additional scatterer at both input and output, as shown in Fig. 3(b). Resonant frequencies are clearly calculated as shown in Fig. 4(b). The transmission of a point-defect has a sharp narrow resonance as shown by a black curve. The transmission has, in general, as many sharp peaks as the number of point-defects, and makes large ripples in the passband with a wide swing of approximately 18 dB. These peak frequencies are numerically evaluated and compared with the analytical results from a liner coupled-mode theory in the following sections to know factors which determine the bandwidth of the passband.



(a) Standard (b) With additional scatterers

Figure 4: Transmissions along chains of point-defects.

4 Analysis based on coupled-mode theory

The resonant mode in a point-defect, the defect mode, deep in a sonic/phononic crystal is well confined therein and has a high Q value.[10] It is considered reasonable that the defect-modes couple only with their next neighboring ones. We have applied successfully a linear coupled mode equation for two identical resonant modes to a quantitative analysis of the mode coupling between two point-defects, whose coupling coefficients are conditioned so that the whole system fulfills the energy conservation law.[10] We have extended the coupled-mode theory to the coupling among an arbitrary number of resonant modes, derived a recursive formula of the eigenvalue-equations, and found that they are easily solved analytically till the seventh order.

4.1 Linear coupled-mode theory

The coupled-mode equations for two adjacent point-defects embedded deep in a crystal was discussed and proved to have a good numerical coincidence among the results obtained by the FDTD simulation and the experimental results for a sonic crystal of acrylic-resin cylinders in air.[7,10]

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\varphi_1(t) = j2\pi f_0\,\varphi_1(t) + C_a\,\varphi_2(t), \tag{1}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\varphi_2(t) = j2\pi f_0\,\varphi_2(t) - C_a^*\,\varphi_1(t), \tag{2}$$

where $\varphi_1(t)$ and $\varphi_2(t)$ denote scalar potentials of two adjacent defect-modes with an identical resonance frequency f_0 and a complex coupling coefficient C_a. Note that the coupling coefficient $-C_a^*$ in the second equation is so determined as to fulfill the energy conservation law, which is easily confirmed by calculating the temporal variation of $|\varphi_1(t)|^2 + |\varphi_2(t)|^2$ as

$$\frac{\mathrm{d}}{\mathrm{d}t}\{|\varphi_1(t)|^2 + |\varphi_2(t)|^2\} = 0. \tag{3}$$

The eigenfrequencies of Eqs (1) and (2) are given by

$$f = f_0 \pm \frac{|C_a|}{2\pi}. \tag{4}$$

Experimental frequency split was measured as 1.52 kHz for the externally excited two adjacent point-defects in a sonic crystal, as shown in Fig. 3, and the magnitude of their coupling coefficient is evaluated as $4.775 \times 10^3\,\mathrm{s}^{-1}$ according to Eq (4). The temporal solutions of Eqs (1) and (2) show sinusoidally amplitude-modulated sound waves of a frequency f_0 with a complete alternative energy transfer between two defects, whose period is inversely proportional to the magnitude of the coupling coefficient $|C_a|$. This phenomenon was simulated for internally excited two adjacent point-defects in the sonic crystal of acrylic resin cylinders in air by means of the FDTD method. The coupling coefficient $|C_a|$ determined from this simulation is $4.85 \times 10^3\,\mathrm{s}^{-1}$. The fact that these evaluated values of $|C_a|$ are in good agreement indicates a validity of application of the linear coupled-mode theory to a chain of point-defects which fabricates an acoustic DMWG.

The above good results have encouraged us to extend further the linear coupled-mode theory to a general number of identical point-defects.

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\varphi_1(t) = j2\pi f_0\,\varphi_1(t) + C_a\,\varphi_2(t), \tag{5}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\varphi_2(t) = j2\pi f_0\,\varphi_2(t) - C_a^*\,\varphi_1(t) - C_b^*\,\varphi_3(t), \tag{6}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\varphi_3(t) = j2\pi f_0\,\varphi_3(t) + C_b\,\varphi_2(t). \tag{7}$$

In this system, three defect-modes have an identical natural resonance frequency f_0, but they may have different coupling coefficients $|C_a|$ and $|C_b|$. It is easy to confirm the energy conservation. Three eigenfrequencies are also easily obtained as

$$f = f_0, \quad f_0 \pm \sqrt{\left|\frac{C_b}{C_a}\right|^2 + 1}\,\frac{|C_a|}{2\pi}. \tag{8}$$

Coupled-mode equations for an arbitrary number of defects were similarly constructed, and their eigenvalue equations were obtained recursively. The eigenvalue equation of a degree $2n$ requires us to find, in general, n roots of a polynomial of a degree n with real coefficients. The eigenvalue equation of a degree $(2n+1)$ has always a solution of $f = f_0$. So, it requires us to find similarly n roots of a polynomial of a degree n with real coefficients. A general recursive expression of the eigenvalue equation of a degree N is

$$\Re_0(f - f_0) = 1, \tag{9}$$

$$\Re_1(f - f_0) = 2\pi(f - f_0), \tag{10}$$

$$\begin{aligned}\Re_{na\ldots\alpha_{n-1}}(f - f_0) &= \Re_1(f - f_0)\Re_{(n-1)a\ldots\alpha_{n-2}}(f - f_0)\\ &\quad - |C_{\alpha_{n-1}}|^2\Re_{(n-2)a\ldots\alpha_{n-3}}(f - f_0)\\ &\quad \text{for } n \geq 2,\end{aligned} \tag{11}$$

where α_n denote the n-th letter of the English alphabet. Then, it is easy to find eigenfrequencies for an arbitrary number of defects at least numerically, for example, by the Newton-Raphson method.

In case of identical coupling coefficients, the eigenfrequency equations for one to seven coupled modes are simple, and they are given by

$$\sqrt{X} = 0, \tag{12}$$

$$X - 1 = 0, \tag{13}$$

$$\sqrt{X}(X - 2) = 0, \tag{14}$$

$$X^2 - 3X + 1 = 0, \tag{15}$$

$$\sqrt{X}(X^2 - 4X + 3) = 0, \tag{16}$$

$$X^3 - 5X^2 + 6X - 1 = 0, \tag{17}$$

$$\sqrt{X}(X^3 - 6X^2 + 10X - 4) = 0, \tag{18}$$

where $X = \left\{\frac{2\pi}{|C|}(f - f_0)\right\}^2$. These equations are all solved analytically by means of the root formulae for the algebraic quadratic and cubic equations.

4.2 Comparison between theoretical results and numerical transmission spectra

The eigenfrequencies are calculated from the coupled-mode theory using the coupling coefficient $|C| = 4.85 \times 10^3$ [s^{-1}] which was determined from the rate of the alternative energy transfer between the coupled two point-defects by a FDTD simulation of the internal excitation in Sec. 3.1. They are plotted in Fig. 5 together with the frequency-peaks of the transmission spectra which are obtained in Sec. 3.2. The central frequencies of chains of point-defects in the crystal are moved slightly to the higher frequency side due to the unsymmetrical structure of the full band-gap in which the defect-modes exist. The frequency differences between the highest and

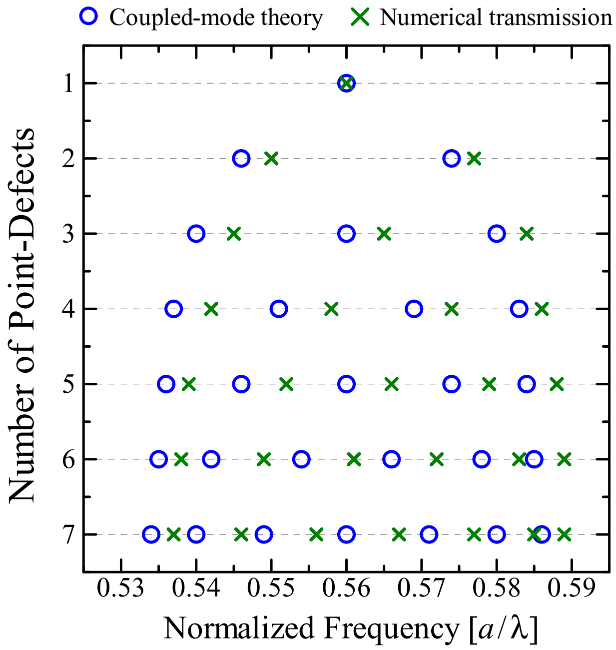


Figure 5: Numerical comparison between the coupled-mode theory and the transmission spectra of the DMWG by the FDTD method.

the lowest, which is the bandwidth of the DMWG filter, well coincide numerically with each other in Fig. 5. They saturate enough with a chain of seven point-defects. We can predict from these results the pass-band width of long DMWGs to be approximately $2 \times 1.85 \times \frac{|C|}{2\pi}$ [Hz], namely 1.85 times the split of the resonant frequencies of the coupled two point-defects. These results have been discussed for an identical coupling coefficient between all point-defects in the DMWG. There will be no practical broadening of the passband with non-uniform coupling coefficients, being considered from the detailed coefficients of the eigenfrequency equations.

5 Conclusion

Good transmission characteristics of the defect-mode waveguide as an acoustic bandpass waveguide filter was considered due to linear mode-coupling of a chain of point-defects which are embedded in a sonic/phononic crystal and compose a defect-mode waveguide. We have developed a linear coupled-mode theory for two coupled modes to a general one for an arbitrary number of linearly coupled resonant modes, whose couplings are restricted between adjacent modes. General formulae of eigenvalue equations have been derived to determine the eigenfrequencies.

The transmission spectra of defect-mode waveguides previously obtained by numerical simulations are quantitatively compared with the eigenfrequencies calculated from the eigenvalue equations. Their frequencies agree quite well with each other, except a slight deformation of the central frequencies of the DMWG to the higher frequency side due to the unsymmetrical structure of the full band gap. The analytically solved eigenfrequencies were shown up to the seventh order, although they can be quite easily solved numerically up to an arbitrary order. However the higher-order solutions are unnecessary, because the frequency width of the eigenfrequencies proved to saturate around at the seventh order.

Therefore the bandwidth of the DMWG in a sonic/phononic crystal with an identical coupling coefficient is estimated to be around 1.85 times the resonant frequency split of the coupled two point-defects. Adjustment of the coupling coefficient will be the easiest way to control the bandwidth of the DMWG bandpass filter.

Acknowledgments

This work was partially supported by a Grant-in-Aid for Scientific Research (B) No. 18360176 from the Ministry of Education, Culture, Sports, Science and Technology of Japan.

References

[1] J. D. Joannopoulos, S. G. Johnson, J. N. Winn, and R. D. Meade, *Photonic Crystals, Molding the Flow of Light*, 2nd Ed., Princeton University Press, Princeton, NJ, 2008.

[2] T. Miyashita, "Full Band Gaps of Sonic Crystals Made of Acrylic Cylinders in Air –Numerical and Experimental Investigations–, " *Jpn. J. Appl. Phys.* **41**, 3170–3175 (2002).

[3] T. Miyashita and C. Inoue, "Sonic-Crystal Waveguides by Acrylic Cylinders in Air –Experimental Observations based on Numerical Analyses," *Proc. IEEE Ultrasonics Symposium*, 615–618 (2001).

[4] A. Khelif, P.A. Deymier, B. Djafari-Rouhani, J.O. Vasseur, and L. Dobrzynski, "Two-dimensional phononic crystal with tunable narrow passband: Application to a waveguide with selective frequency," *J. Appl. Phys.* **94**, 1308–1311 (2003).

[5] A. Khelif, A. Choujaa, R. Laihem, M. Wilm, S. Ballandras, V. Laude and M. Solal, "Experimental Study of Band Gaps and Defect Modes in a Two-Dimensional Ultrasonic Crystal," *Proc. IEEE Ultrasonic Symposium*, 377–380 (2003).

[6] T. Miyashita, "Defect Modes for a Promising Acoustic Waveguide in Two-Dimensional Sonic/Phononic Crystals," *Proc. IEEE Ultrasonics Symposium*, 281–284 (2005).

[7] T. Miyashita, S. Takubo and K. Waki, "Experimental and theoretical analyses of mode coupling between single-defects in a 2-D sonic crystal," *Proc. Symposium on Ultrasonic Electronics*, **28**, 195–196 (2007).

[8] T. Miyashita, W. Sato, Y. Nakaso, and R. Mukuda, "Experimental Studies on Two-Dimensional Defect-Mode Waveguides in a Sonic/Phononic Crystal," *Jpn. J. Appl. Phys.* **46**, 4684–4687 (2007).

[9] T. Miyashita, "Acoustic defect-mode waveguides made in a sonic crystal - Numerical analyses by an elastic finite-difference time-domain method," *Jpn. J. Appl. Phys.* **45**, 4440–4447 (2006).

[10] T. Miyashita and N. Doi, "Mode Coupling Behavior of Neighboring Single-Defects in a Sonic Crystal Made of Acrylic Resin Cylinders in Air," *Proceedings of the IEEE Ultrasonics Symposium*, 3–6 October 2006, Vancouver, Canada, pp.321–324.

[11] C. Rubio, D. Caballero, J. V. Sánches-Pérez, R. Martínes-Sala, J. Sánchez-Dehesa, F. Meseguer, and F. Cervera, "The existence of full gaps and deaf bands in two-dimensional sonic crystals," *J. Lightwave Technol.* **17**, 2202–2207 (1999).

[12] T. Miyashita, "Experimental study of a sharp bending waveguide constructed in a sonic-crystal slab of an array of short aluminum rods in air," *Proceedings of the IEEE International Ultrasonics* (Ferroelectrics, and Frequency Control 50th Anniversary Joint Conference), 24–27 August 2004, Montreal, Canada, pp.946–949.

[13] T. Miyashita, "Sonic crystals and sonic waveguides," *Meas. Sci. Technol.* **16**, R47–R63 (2005).

[14] T. Miyashita and M. Nagase, "Two-dimensional sonic crystals, sonic-crystal slabs and sonic waveguides for sound control and measurement," *Proceedings of the Eleventh International Congress on Sound and Vibration* (ICSV11), 5–8 July 2004, St. Petersburg, Russia, pp.1487–1494.

A Coupled-mode equations for different resonant frequencies with energy loss

(a) A point-defect deep in a sonic/phononic crystal.

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\varphi_1(t) = j2\pi f_1\,\varphi_1(t) - \gamma_{\mathrm{i}}\varphi_1(t). \tag{19}$$

The nonzero γ_{i} is due to acoustic power dissipation in the scatterers which fabricate the crystal and by acoustic radiation to the outer free space. The internal Q is given by $Q_{\mathrm{i}} = 1/\gamma_{\mathrm{i}}$

(b) A point-defect shallow in a sonic/phononic crystal.

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\varphi_1(t) = j2\pi f_1\,\varphi_1(t) - \gamma_{\mathrm{i}}\varphi_1(t) - \gamma_{\mathrm{IO}}\varphi_1(t). \tag{20}$$

Here γ_{IO} describes the input or output coupling. The external Q is given by $Q_{\mathrm{e}} = 1/\gamma_{\mathrm{IO}}$.

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\varphi_1(t) = (j2\pi f_1 - \gamma_{\mathrm{i}} - \gamma_{\mathrm{IO}})\,\varphi_1(t) + C_a\,\varphi_2(t), \tag{21}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\varphi_2(t) = (j2\pi f_2 - \gamma_{\mathrm{i}})\,\varphi_2(t) - C_a^*\,\varphi_1(t) - C_b^*\,\varphi_3(t), \tag{22}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\varphi_3(t) = (j2\pi f_3 - \gamma_{\mathrm{i}} - \gamma_{\mathrm{IO}})\,\varphi_3(t) + C_b\,\varphi_2(t). \tag{23}$$