Variation of mean free path length over time and its effect on room impulse response

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In their previous paper (Forum acusticum 2005, Proceedings, 2399-2404) the authors presented the local effect found in a large sports hall, described as significantly shorter reverberation time in the first 400-500 ms than that in the rest of the hall's long impulse response. To analyse this phenomenon, a new measure – short time mean free path (ST MFP) variation in time over the impulse response - was introduced. It is identified as the function of time obtained as an ensemble average in some fixed interval traced along the impulse response duration. The ray tracing simulation was used to calculate ray paths approaching the receiver in time. Analyzing ST MFP variations over time is a new insight into the structure of room impulse response, and this paper presents its main characteristics and its relation to statistically defined mean free path length (4*V*/*S*). The ST MFP variation over time can be a tool to reveal the causes of the local effect introduced in the previous paper, but it is generally useful in explaining the sound decay monotony deviations.

1 Introduction

The impulse response of the room reflects the effects of multipath propagation in the sound field. In its structure one can differentiate the early part which is gradually being transformed into late reverberation. Late reverberation is characterized by the state of the sound field which should have the attributes of diffuseness maximally possible in the given conditions.

It has been defined in the literature that the existence of the diffuse field in the spatial domain is characterized by the constant energy density in the room [1]. Schroeder has defined that the field in a certain point is completely diffuse when the distribution of sound energy flux over the solid angle is uniform [2]. On the basis of assumptions introduced in such a way, the statistical theory defines certain consequences, which can be also considered as the characteristics of the diffuse field. One of these consequences is the mean free path length determined as 4*V*/*S*. Classical theory of sound field does not take into consideration the process of transition, that is, how the state of diffuseness is achieved. The topic of statistical theory is the sound field long enough after the direct sound.

The differences between early and late part of impulse response led to introducing the initial reverberation time (EDT) as the parameter by which this difference is being described. Introducing reverberation times T10, T20 and T30 were also the ways to numerically quantify the differences between the initial part and the remains of impulse response. The efforts to define the moment when its initial part is transformed into late reverberation have been shown in the literature. Various approaches to estimation of this boundary have been demonstrated. The attempt to determine the moment of generation of diffuse field on the basis of the physical characteristics of the impulse response recorded in the point has been described in the literature [3].

In the previous paper of this authors it has been shown that the reverberation time depends on structure of energy traffic across the room [4]. The reverberation time is the function of geometric attributes of the room which participate in redirecting the sound energy. These atributes are quantified by statistical distribution of free path lengths. The shape of this distribution depends on the general geometric form of the room and the scattering on its interior surfaces.

Numerical indicator of free path length distribution shape is the mean free path (MFP) which has been defined as mathematical expectation. Based on geometrical approach it has been shown in [4] that in general MFP values differ to a lesser or greater extent from the statistically determined value 4*V*/*S* and that the amount of that deviation is determined by complex interaction between geometrical form and the scattering coefficient.

One approach in determining the moment in impulse response when late reverberation starts may be based on some consequences. One of the consequences is the expected statistics of free path lengths and the value of mean free path. In earlier paper of the same authors [5] the assumption has been introduced that variations of slope in reverberation curve may be interpreted by means of changes of energy path length statistic throughout the impulse response. Based on that an assumption is introduced here that differences in path lengths statistics in the initial part of impulse response compared to its late part may be the tool for evaluation of the boundary between them.

An attempt to analyze the changes of statistical characteristics on free path length statistics during the time is shown in this paper. The analysis was carried out by introducing the value which may be called short-time mean free path length and which represent the indicator of conditions in sound field. Changes of this value along the impulse response have been analyzed in different rooms.

2 Short-time mean free path length

Mean free path (MFP) in a room is defined as the mean value of all the possible sound paths in time and in ensemble. Ensemble of paths on which this mean value is calculated determines the sound energy traffic and it is the function of room's geometrical attributes on a macro and micro level (geometrical form and scattering). It has been shown in the literature that variations of geometrical characteristics of the room influence the shape of statistical distribution of free path lengths and also the MFP value [4]. Due to this, the MFP value in a room may be changed to a certain extent compared to 4*V*/*S*.

To analyze the process of establishing the sound field, one can observes the changes of the mean free path length over time for rays which reach the receiver. The function $\overline{l_{ST}}(t)$ is introduced representing short-time mean free path (ST MFP). It is defined as the mean value of all free paths of rays reaching the receiver in some interval of time Δt. Sliding this interval along the time axis the ST MFP becomes time function. Thus defined value may be represented as:

$$\overline{l_{ST}}(t) = \frac{1}{\sum N^{(i)}} \left[\sum_{t-\Delta t < t^{(i)} < t} \left(\sum_{j=1}^{N_i} l_j^{(i)} \right) \right] \tag{1}$$

where $l_j^{(i)}$ represents the path lengths of ray i which reaches the receiving point in a interval of time Δt, N_i represents the number of free paths of the i ray before hitting the receiver including the path from the last reflection to the receiving sphere, and $N^{(i)}$ is the total number of paths of all rays which reached the receiving point in the interval Δt. It is seen that $\overline{l_{ST}}(t)$ is the function of time and Δt. ST MFP value defined by this equation represents the geometrical approach in the analysis of the sound field transition. In the ray tracing analysis this type of mean free path is easily achieved on the basis of data about history of all the rays which hit the receiving sphere. In order to meet the needs of this analysis the special software for ray tracing analysis has been made.

The analysis of ST MFP value change in time in the room of different geometrical characteristics has been performed by means of that software. Its results are shown in the following part. The value of ST MFP is not the general indicator of the room, but it reflects the state in impulse response which is registered at the specified receiver.

3 Experiment

Three different rooms have been selected for the experimental analysis of ST MFPL characteristics, and software models have been created for these rooms. The rooms have the same volumes, but different global geometrical form. The forms of rooms have been selected accoriding to the results obtained in the previous paper of these authors, where it has been shown that the acoustic response in these rooms has sufficiently different reactions to the scattering coefficient change which is uniformly arranged along the inner surfaces [4]. The analyzed rooms are presented in Figures 1, 2 and 3.

Room 1 is regular parallelepiped, applied in this analysis as a common shape of many rooms, but also a great number of the concert halls (″shoe box″). Rooms 2 and 3 were generated by introducing some modification of the basic parallelepipedic shape, with deformations in the symmetry and parallelism of its sides. Difference between room 2 and 3 is in a scale of introduced deformations. In all three rooms it is assumed that the absorption is minimal. The absorption coefficient on all interior surfaces in the rooms had the same value, set at 0.1. Their volumes are about 2200 m^3.

The geometrical characteristics at the micro level of the three analyzed rooms were modelled by controlling the scattering. The amount of diffusion was modeled by the value of scattering coefficient g. As with the absorption coefficient, it was assumed that the scattering coefficient is uniform along at all interior surfaces, and the Lambert cosine law was assumed. In the analysis the scattering coefficient has been assigned with seven different values: 0, 0.1, 0.25, 0.35, 0.5, 0.75 and 1.

The preliminary analyses of the acoustic response in these three rooms were done by ODEON software. Calculation of the reverberation time by ray-tracing simulation was repeated for seven different values of the scattering coefficient g applied on the interior surfaces. Calculated reverberation time in the rooms occurred in the interval of 2.3 – 3.1 s, depending on the form of the room and the scattering coefficient value. Besides the ray-tracing simulation, in all the rooms the value of the reverberation time was also calculated by the statistical theory as a function of volume V, interior surface S and the default absorption coefficient $\overline{\alpha} = 0.1$. The calculations were made using the Sabine formula. In each room this value was used for normalization of the reverberation time obtained by the ray-tracing simulation. The value 1 of the normalized reverberation time means that the reverberation time measured in the simulated sound field is identical to the statistically expected value, and the values of normalized reverberation time greater or smaller than 1 indicate the occurrence of deviation in the acoustic response calculated by the simulation.

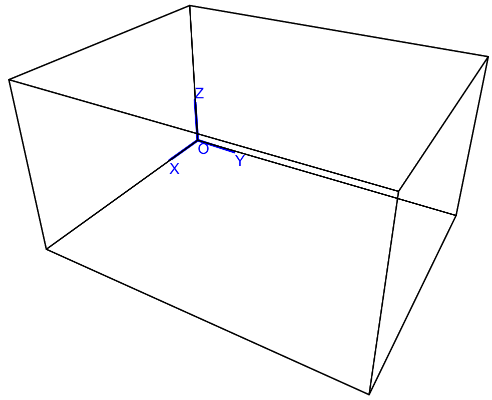

Fig.1 Room 1 in the shape of regular parallelepiped

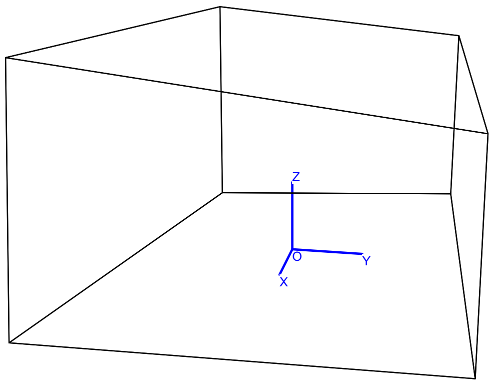

Fig.2 Room 2 in the shape of slightly deformed parallelepiped

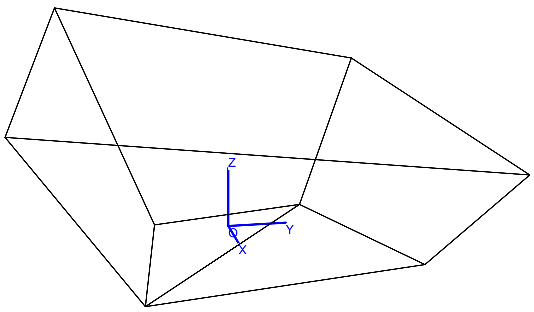

Fig.3 Room 3 in the shape of highly deformed parallelepiped

The values of normalized reverberation time calculated in three rooms for seven different values of the scattering coefficient g are presented in Fig. 4. It may be seen that these three rooms react differently to the changes in scattering. In the room 1 and 2 there is a significant negative deviation of the normalized reverberation time for small values of the scattering coefficient between 0.2 and 0.3. In the room 3 normalized reverberation time as a function of the scattering coefficient has a concave shape with a local maximum in the interval between 0.5 and 0.75.

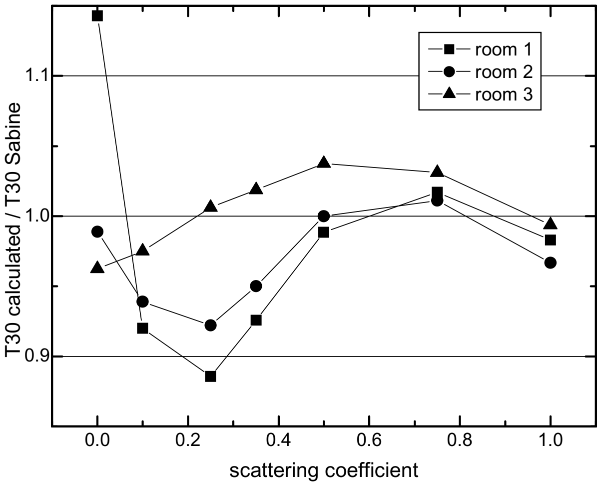

Fig.4 Calculated normalized reverberation time in the three rooms as a function of scattering coefficient

4 Results

In Figures 5, 6 and 7 are shown the results of calculating ST MFP in rooms 1, 2 and 3, respectively, for all the seven values of scattering coefficients. The interval Δt was 30 ms. Diagrams are drawn for the first 3 seconds of response, which is comparable to the duration of reverberation time in these rooms. Statistical mean free path $4V/S$ is also marked on all three diagrams for comparison.

Some characteristics of ST MFP may be seen from the diagrams on Fig. 5, 6 and 7. Its value is always asymptotically approach the final value which is reached when $t \to \infty$. Accordingly, one can conclude that the diffuse field in the room is characterized by $\overline{l_{ST}}(t) = const$, and in the period of early reflections is characterized by variability of $\overline{l_{ST}}(t)$. For the various scattering in the same room final value $\overline{l_{ST}}(\infty)$ may be higher or lower than the statistically expected value $4V/S$.

The rate by which the curve $\overline{l_{ST}}(t)$ approaches its final value during the time depends on the geometrical attributes of the room: its form and scattering. There are circumstances when this process taked place relatively gradualy (for example, room 1, $g = 0.25$ in Fig. 5), and also when it goes very rapidly (room 1 and 2, $g = 1$ in Fig. 5 and 6). In some combinations of the form and the scattering the curve in the initial part of the response demonstrates the values greater than the value $\overline{l_{ST}}(\infty)$, so the curve tends asymptotically toward its final value from the upper side.

In rooms whose geometrical form doesn't guarantee the diffuseness for low scattering (room 1 and 2), $\overline{l_{ST}}(t)$ demonstrate some fluctuations in time. The largest fluctuations are in room 1. In room 2, which has a slightly deformed form, these fluctuations are smaller and in the room 3 they are the smallest. With the increase of scattering fluctuations disappear in all the rooms.

The normalization of all the curves have been performed to its value for $t = \infty$. The normalization has been performed to the value achieved after 6 s, which is approximately twice as longer than the reverberation time. After that time the value has been practically reached the final value (the differences are less than 0.1%). Diagrams with such normalized curves for the rooms 1, 2 and 3 are shown in Fig. 8, 9 and 10, respectively. The interval of the first second of response is shown. The curves are drawn for the scattering coefficient from 0.25 to 1, assuming that the values 0 or 0.1 have only the theoretical significance.

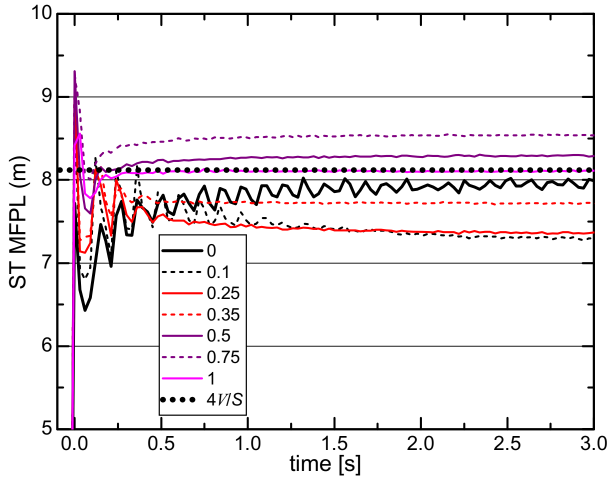

Fig.5 ST MFP for room 1 calculated for different uniformly distributed values of scattering coefficient

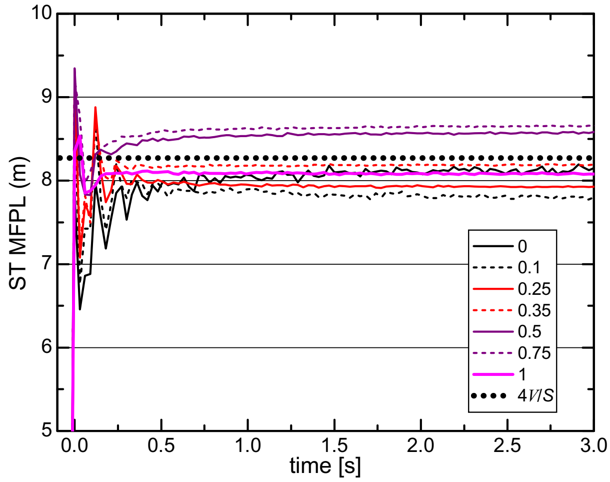

Fig.6 ST MFP for room 2 calculated for different uniformly distributed values of scattering coefficient

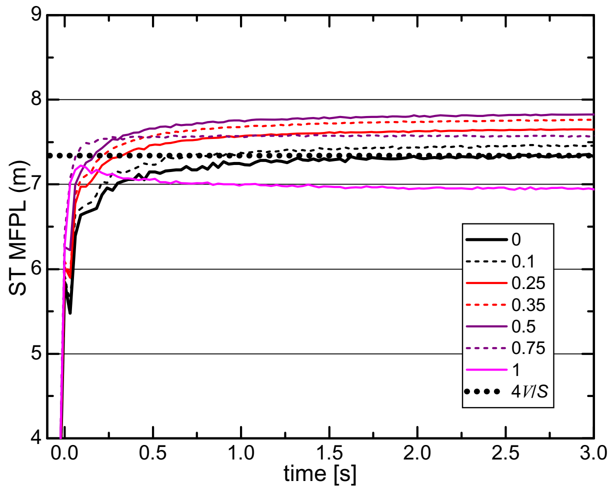

Fig.7 ST MFP for room 3 calculated for different uniformly distributed values of scattering coefficient

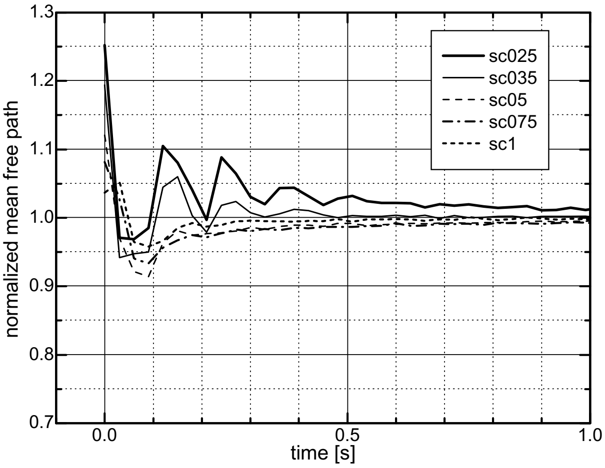

Fig.8 Results for room 1 for the values of scattering coefficient (0.25;1)

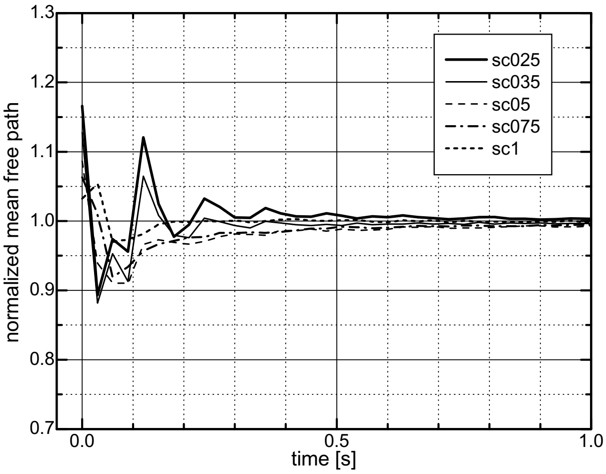

Fig.9 Results for room 2 for the values of scattering coefficient (0.25;1)

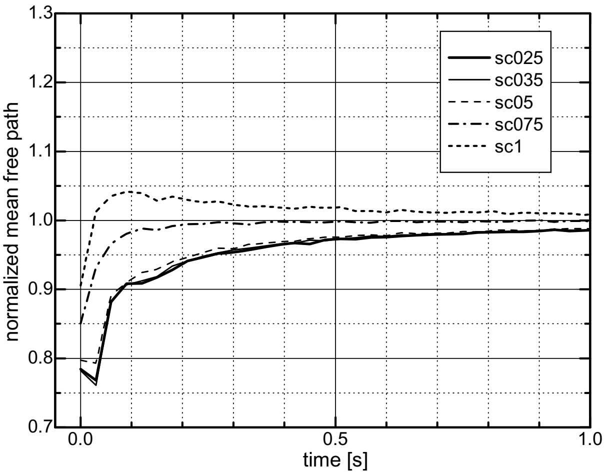

Fig.10 Results for room 1 for the values of scattering coefficient (0.25;1)

Due to the fact that reverberation time in three analyzed rooms is not the same, and that these values vary with the changes of scattering coefficient, the normalization on the time axis has been introduced also. The time is normalized to the value of reverberation time determined from the impulse response for the given (source – receiving sphere) pair. The normalized curves for all three rooms are shown in Figure 11. These curves are also drawn for values of scattering coefficient from 0.25 to 1

5 Discussion

Stationary state of sound field in the room is manifested by the stationary value of ST MFPL. This value is achieved after sufficiently long interval of time after the beginning of impulse response. These stationary values differ for each room and it is also the function of scattering. In order to look at this dependency in more detail, in Fig. 12 $\overline{l_{ST}}(\infty)$ are shown normalized to the statistically expected value $4V/S$.

The shapes of curves are the function of the value of uniformly distributed coefficient g. Diagram shows that the shapes of the curves for three rooms are equal to the shapes of the curves for normalised reverberation time from Figure 4. In room 1 and 2 there is negative deviation of the normalized ST MFP for small values of the scattering coefficient, between 0.2 and 0.3, and positive deviation for scattering coefficient larger or equal 0.5. In room 3 normalized ST MFP has a concave shape with a local maximum at 0.5. It is obvious that deviation of ST MFP from statistical values exhibit the same dependence on scattering as the reverberation time.

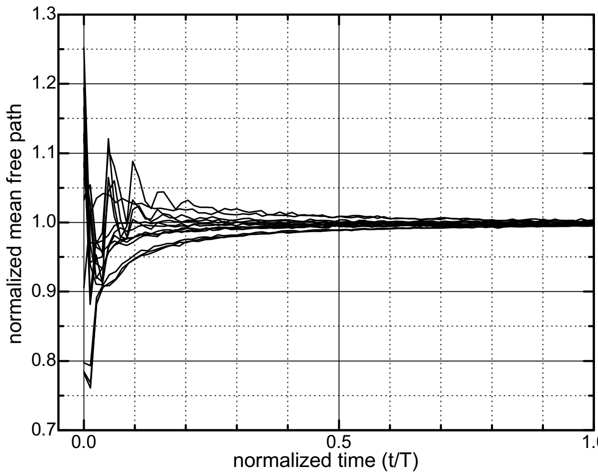

Fig.11 Normalized mean free path from Figures 8, 9 and 10 for scattering coefficient (0.25;1), normalized on reverberation time in the room

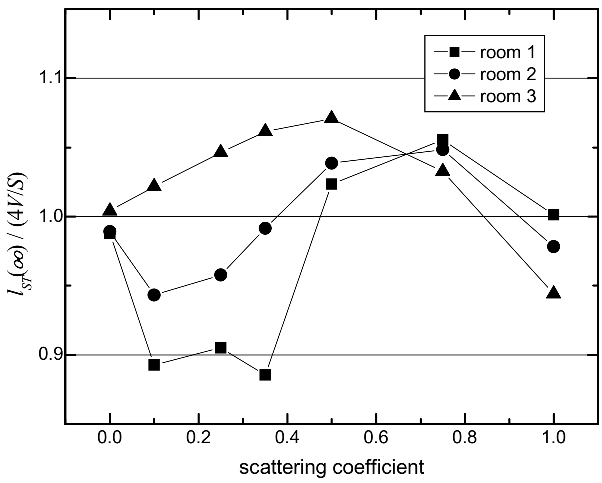

Fig.12 Normalized values $\overline{l_{ST}}(\infty)$ in three rooms as the function of scattering coefficient

There is a statement in literature that 20 – 40 successive reflections are necessary for establishing diffuse field [8]. To compare the obtained results with this statement time axis in Figures 8, 9 and 10 is transformed in the number of reflection using for each room its statistically estimated frequency of reflection number per second $cS/4V$. Results for all the three rooms are shown together in Figure 13. As in Fig. 11, the curves are drawn for values of scattering coefficient from 0.25 to 1. From these diagrams it is possible to estimate the average number of reflections needed for the ST MFP value to reach its final value.

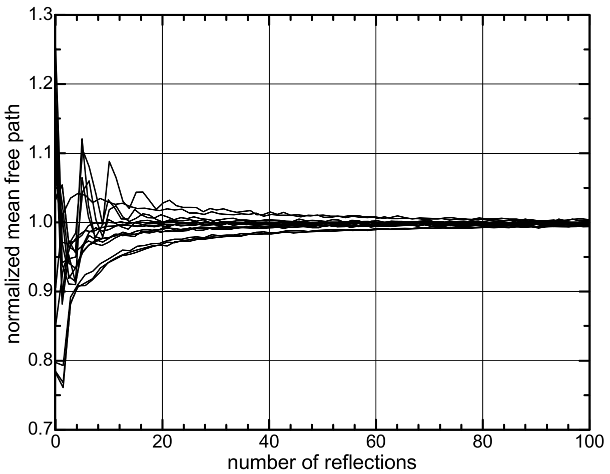

Fig.13 Diagrams from Figure 11 calculated in each room on total number of reflections $cS/4V$.

Process of asymptotically approaching the final value which $\overline{l_{ST}}(t)$ has for $t = \infty$ may be quantified by the moment when it reaches sufficiently near value. Here, it is assumed that this be the time when the curve reaches 95% of its final value. The progress of $\overline{l_{ST}}(t)$ was shown here in three ways: in time domain (Figures 8, 9 and 10), relatively to the reverberation time (Figure 11) and in the domain of average number of reflections (Figure 13). The moment when the curve reaches 95% of its final value is determined in all three domains and the results are shown in Table 1. The results are shown for all three analyzed rooms and for five values of uniformly distributed scattering coefficient g.

From data shown in Table 1, on can see that in the analyzed rooms $\overline{l_{ST}}(t)$ reach 95% of the final value after time interval of 20 ms, for large values of scattering coefficient, up to 280 ms for small scattering. For more strictly condition, moment of reaching the final value would be even later.

6 Conclusion

The short-time mean free path is time function and represents the geometrical approach in quantifying the transition of sound field to diffuse state. It asymptotically reaches the value characteristic for diffuse field in the room. Variations of $\overline{l_{ST}}(t)$ in its initial part depend on geometric properties of the room. There are the differences in the way the curve reaches its final value, and that process is the function of the geometrical form of the room and the scattering in it.

The moment when the initial part of the response transforms to late reverberation is the issue of the criterion of sufficiently small difference. For the three analyzed rooms the criterion of reaching 95% has been established, and this state is reached for 11% of reverberation time for small scattering, and for 2-3% of large scattering.

room	g	$t_{95\%}$	$(t/T)_{95\%}$	$n_{95\%}$
1	0.25	0.28 s	0.11	12
	0.35	0.16 s	0.06	6
	0.5	0.11 s	0.05	5
	0.75	0.11 s	0.04	4
	1.0	0.09 s	0.03	2
2	0.25	0.14 s	0.06	6
	0.35	0.15 s	0.05	5
	0.5	0.12 s	0.04	4
	0.75	0.11 s	0.04	4
	1.0	0.03 s	0.02	2
3	0.25	0.26 s	0.11	12
	0.35	0.25 s	0.10	12
	0.5	0.25 s	0.10	12
	0.75	0.04 s	0.02	2
	1.0	0.02 s	0.02	1

Table 1 moments when the curve $\overline{l_{ST}}(t)$ reaches 95% of its final value (when $t \to \infty$) in all three rooms and for five values of scattering coefficient g, expressed in time duration of impulse response $t_{95\%}$, in time relative to reverberation time $(t/T)_{95\%}$ and in the average number of reflections $n_{95\%}$.

References

[1] H.Kuttruf, ″Room acoustics″, Spon Press, London, 2000

[2] M.Schroeder, Measuremen od sound diffusion in reverberation chamber, JASA, Vol 31 (1959) 1407-1414

[3] T.Hidaka, Y.Yamada, T.Nakagawa, ″A new definition of boundary point between earlz reflections and late reverberation in room impulse responses″, JASA, Vol 122, No 1 (2007) 326-332

[4] D.Šumarac-Pavlović, M.Mijić, ″An insight into the influence of geometrical features of rooms on their acoustic response based on free path length distribution″, Acta Acustica, Vol 92, No 6 (2007) 1012-1026

[5] M.Mijić, D.Šumarac Pavlović, ″Acoustic design of the Belgrade Arena hall″, FORUM ACUSTICUM, Budapest, 2005. Proceedings on CD

[6] J.H.Rindel, “Computer Simulation Techniques for Acoustical Design of Rooms”, Acoustics Australia 1995, Vol. 23 p. 81-86

[7] C.L.Christensen, J.H.Rindel, “A new scattering method that combines roughness and diffraction effects”, Forum Acousticum 2005, Budapest

[8] F.V.Hunt, “Remarks on the mean free path problem”, JASA, Vol 36, No 3 (1964) 556-564