Numerical simulation of acoustic wave propagation via a liquid with gas bubbles

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The two-dimensional problem of propagation of solitary pressure wave in a liquid with air bubbles is studied. The liquid is supposed to be compressible. The Herring model is used to describe the radial oscillation motion of bubbles. The numerical results demonstrate the influence of the gas volume content and of the bubble area width and of the wavelength on the wave form. The affect of heat exchange on wave propagation is taken into account. It is shown that liquid pressure in a bubbly liquid area can surpasses initial amplitude in case the liquid with bubbles area of rectangular section. The leading wave with damped wave train is formed in a gas-liquid mixture out of bell-shaped pulse.

1 Introduction

The study of bubbly liquid has old history having begun with the solution of single problems [1-2]. In our days bubbly liquid is a working medium in difference engineering applications (industrial applications of high-power ultrasound, acoustic imaging, biomedical research (test and therapy), etc.). High intensity waves, which can not be described by linear laws, are involved in such applications. There are many approaches and models used by theoretical description of processes taking place in this medium. Effects of waves damping in a bubbly liquid in the 1D cases were carefully studied (see [3-5]). 1D models, however, cannot take into account the geometrical aspects of the problem, for example, small area of bubbly liquid is surrounded liquid. In this case different modes of wave can be generating in bubbly liquid (see [6, 7]). Thus, a two-dimensional (2D) approach is required.

In this work the mechanism of pressure wave deformation at their propagation via liquid with air bubbles is investigated. It is shown that the basic cause which influences the wave deformation is the cumulative flow forming as a result of interaction of the wave with the bubble area.

This paper outline is as follows. In Section II assumptions of the mathematical model of the considered problem is presented. In Section III overview of our numerical results is presented.

2 Mathematical model

The wave propagation in the tank of rectangular section is considered. The tank is supposed to be filled by the water with a bubble area (see Fig. 1). In our cases the bubble area is the area of square section. It is also assumed that the initial disturbance does not depend on *z*, therefore the problem will be two-dimensional.

The mathematical model is considered at following assumptions. The bubbly liquid can be described as a two-phase mixture with the liquid phase as the matrix and gas bubbles as the dispersed phase. The water is assumed to be compressible liquid describing the empiric Tait equation. The size of bubbles and inter-bubbles distances are small compared to the typical length scale of wave propagating process. The direct bubble-bubble interactions are neglected. All bubbles keep their spherical shape and have equal radius at each elementary volume. The gas in bubbles is described by the ideal gas law. Processes of bubble fragmentation and coalescence are not taken into account. The fluid viscosity is significant only in the processes of interaction between phases. The heat flux between bubble and liquid is considered (for more details see [8]). The radial oscillation motion of bubbles is described by Herring equation. The mathematical model with the above mentioned assumptions is presented in [7].

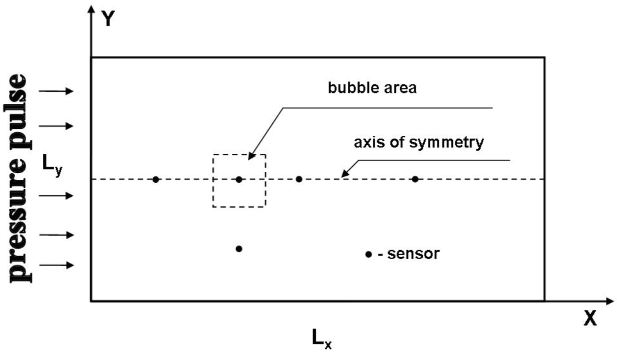

Fig. 1. Schematic diagram of field of research.

For the numerical implementation of the presented problem a two-dimensional high-order algorithm is chosen. The Herring equation is solved numerically using an explicit fourth-order Runge-Kutta scheme with automatic step length control [9].

3 Numerical results

Initially, the liquid and bubbles are in their equilibrium states: $p_{l0} = p_{g0} = 0.1$MPa pressure of liquid and gas within bubbles, respectively, $T_0 = 300$ K. At the time $t > 0$, the pressure at the boundary $x = 0$ is given by formula

$$p_l^0(t, x=0, y) = p_{l0} + \Delta p_0 \exp[\psi(t_*)], \qquad (1)$$

$$\psi(m) = -\left(\frac{t - m/2}{m/6}\right)^2, \qquad (2)$$

where Δp_0 – pressure amplitude, t_* - time duration.

At other walls conditions of a solid wall with slipping are taken into account.

For all calculations we apply the following values for the pressure pulse: $p_{l0} = 0.1$MPa, $\Delta p_0 = 0.3$MPa.

The evolution of the pressure wave of the first variant of calculation is shown in Fig. 2. The initial radius of bubbles R_0 is 10^{-3} m, and their volume content is 10^{-2}. The time duration of pressure pulse is equaled to $t_* = 1$ ms (a wave length 1,5 m). The bubble area was a square with side 0,05 m. It is placed away 1,5 m from the left boundary on an axis Ox and 0,475 m on axis Oy.

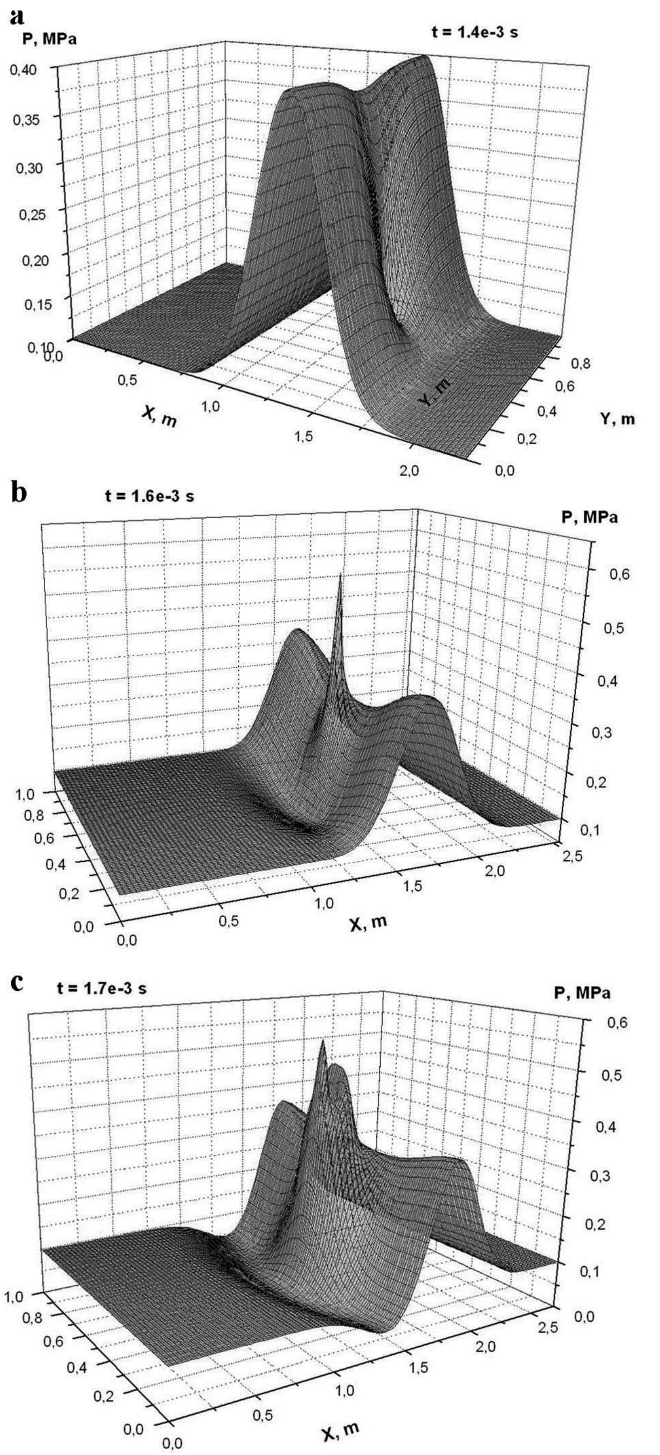

Fig. 2. Evolution of pressure wave of the first simulation variant for various instant time. The p_{l0} = 0.1MPa, Δp_0 = 0.3MPa, $\varphi_0=10^{-2}$, t_* = 1 ms. **a** – pressure diagram at t = 1,4 ms, **b** –at t = 1,6 ms and **c** –at t = 1,7 ms.

Three pressure plots for various value of t are illustrated derivative pattern. In Fig. 2a the initial stage of interaction of the wave with bubble area is shown when wave absorption is began due to bubbles collapse. The pressure undershoot in the wave is clearly seen. It's width in y - direction more than width of bubble area. It is related by that the wave tend to equalize the fallen pressure in bubble area at the expense of the sides. In Fig. 2b the instant time is presented when the pressure outburst is formed behind the wave crest and has a great value. The pressure peak reaches the value of 7 atmospheres that exceeds initial pressure amplitude in 2 times. It is seen that that part of wave which encloses bubble area is taken part in formation of the peak. Two-dimensionality of the problem is exhibited in it. The disturbance wave leaves bubble area, moving forward, and the requirements supporting existence of peak are broken. That immediately brings to its breakup (Fig. 2c) and to formation cylindrical divergent compression wave.

To understand the gained effect, lets see velocities in the liquid (Fig. 3) at the time t = 1,4 ms. This time corresponds to the initial stage of interaction of the wave with bubble area. The liquid velocity on the wave crest reaches value of 0,2 m/s, in bubble area it reaches values of 0,27 m/s.

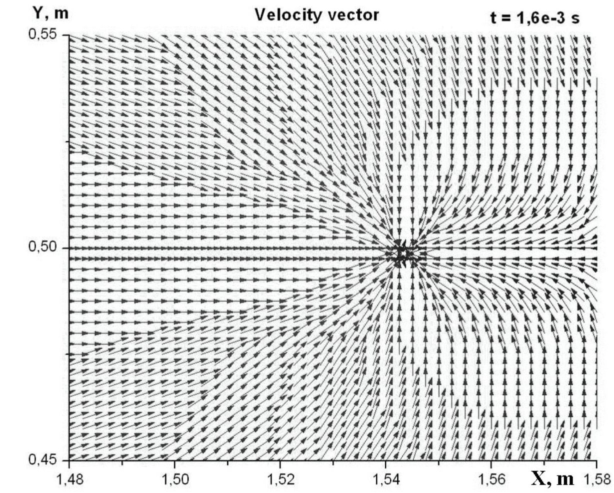

Fig. 3. Velocity field at the instant time t = 1,6 ms.

From Fig. 3 it is seen, that velocity around bubble area is directed to its centre. It is shown that the basic cause of formation pressure outburst is the cumulative flow which leads to significant increase of pressure in the cumulative point. The cumulative point is displaced and when it reached a pure liquid the peak began breakup because the existence conditions of the cumulative flow is possible only in bubble area.

In Fig. 4 the evolution of the pressure wave of the second variant of simulation is shown. The time duration of pressure pulse is 0,1 ms (wave length 0,15) that less order of magnitude than in the previous case. The bubble area was dislocated away 0,5 m from the left boundary of the tank. Other parameters are the same. The first pressure plot corresponds to the time t = 0,4 ms The disturbance wave has partially entered in bubble area. In the given variant bubble area is like a suppressing barrier, and the interfacial of heterogeneous phases is similar to the free surface due to the bubbly liquid density is less than pure water density. It explains pressure drop in the wave to initial value of the unperturbed bubble area. The reflected rarefaction wave with amplitude of 2 atmospheres propagates from the left boundary of bubble area that is less the than initial pressure amplitude. The bubble area absorbs a part of the pressure wave. There is the very small pressure outburst (Fig. 4b). In this case the velocity of cumulating flow is too small to generate a pressure peak. In Fig. 5 the data of sensors are presented. Sensors are placed on the symmetry axis away 0,4 m (the solid curve or the 4th sensor), 0,525 m (the dashed curve or the 5th sensor) and 0,6 m (the dash-dot-doted curve or the 6th sensor) from boundary x = 0. It is seen, that disturbance wave propagates through the 4th sensor, not having undergone modification, and the next pressure peak corresponds to the dispersing rarefaction wave. The 5th sensors placed at the centre of bubble area. It practically does not feel a pressure modification when the disturbance wave propagates thought bubble area (time from 0,33 ms to 0,46 ms). The following increase of pressure corresponds to small pressure outburst. The 6th sensor already registers a wavefront reconstruction.

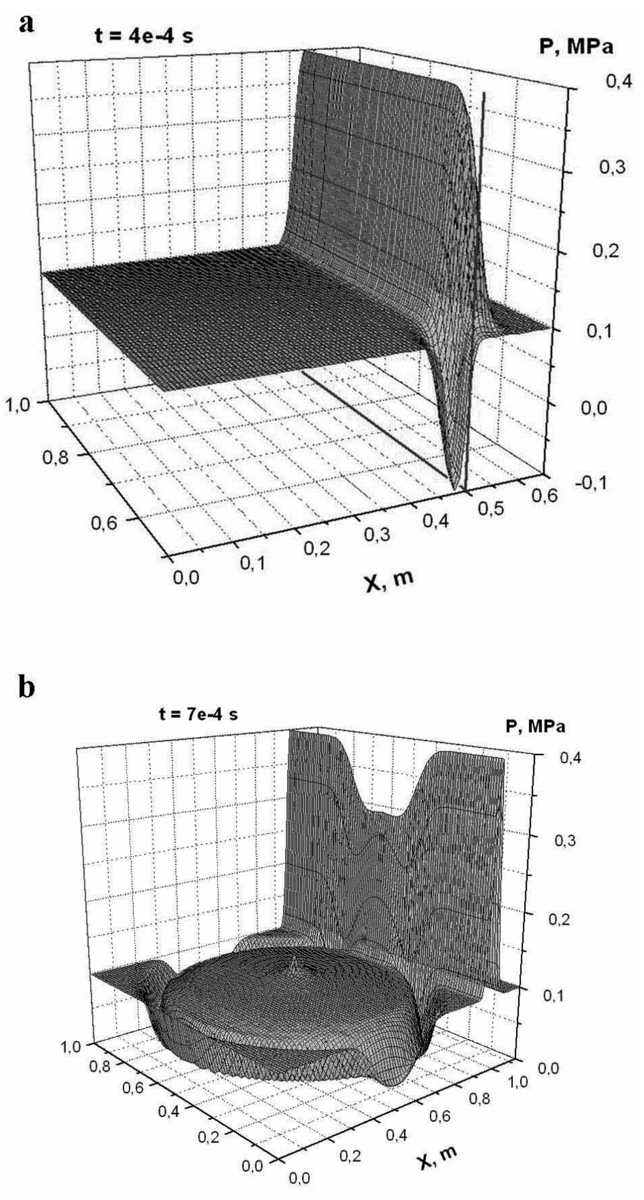

Fig. 4. Evolution of pressure wave of the second simulation variant for various instant time. The p_{l0} = 0.1MPa, Δp_0 = 0.3MPa, $\varphi_0=10^{-2}$, t_* = 0,1 ms. **a** – pressure diagram at t = 0,4 ms, **b** –at t = 0,7 ms.

But if the gas volume content is decreased to value 0,001 the pressure peak is occurred again. It is illustrated in Fig 6.

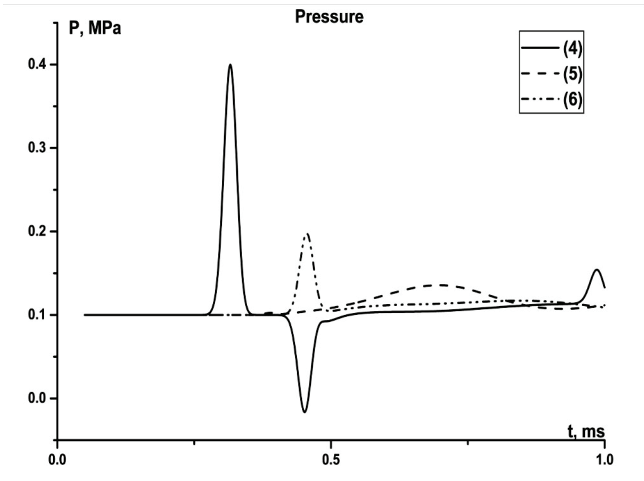

Fig. 5. Pressure in liquid which registers by sensors placing on the symmetry axis. The solid curve is data of sensor which is placed away 0,4 m from boundary x=0. The dashed curve is data of sensor which is placed away 0,525 m from boundary x=0. The dash-dot-dot curve is data of sensor which is placed away 0,6 m from boundary x=0.

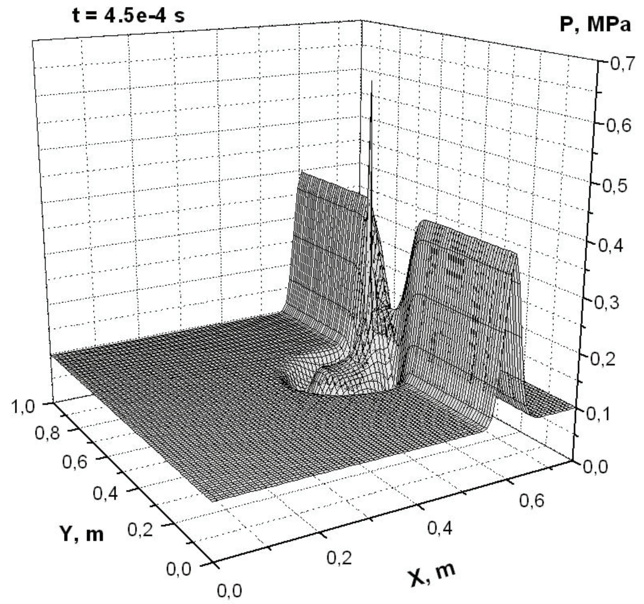

Fig. 6. Pressure diagram at the time t = 0,45 ms. Parameters are p_{l0} = 0.1MPa, Δp_0 = 0.3MPa, $\varphi_0=10^{-3}$, t_* = 0,1 ms.

Let's consider one more problem.

The pressure wave propagates thought the tank filled by a bubbly fluid with constant gas volume content. The disturbance wave is described by the follow formula:

$$p_l^{\,0}(t,y) = p_{l0} + \Delta p_0 \exp[\psi(t_*)]\exp[\varphi(y_*)], \quad (3)$$

$$\psi(m) = -\left(\frac{t-m/2}{m/6}\right)^2, \varphi(m) = -\left(\frac{y}{m}\right)^2. \quad (4)$$

At other walls conditions of a solid wall are taken into account.

Two different variants are shown in Fig. 6. The top pressure plot corresponds to case where t_* = 1 ms, y_0 = 0,4 m and $\varphi_0=10^{-3}$. The bottom pressure plot corresponds to case where t_* = 0,1 ms, y_0 = 0,04 m and $\varphi_0=10^{-3}$. It is seen that the leading wave with damped ripple is formed in the top picture. In the bottom picture there is the leading wave with damped wave train formed.

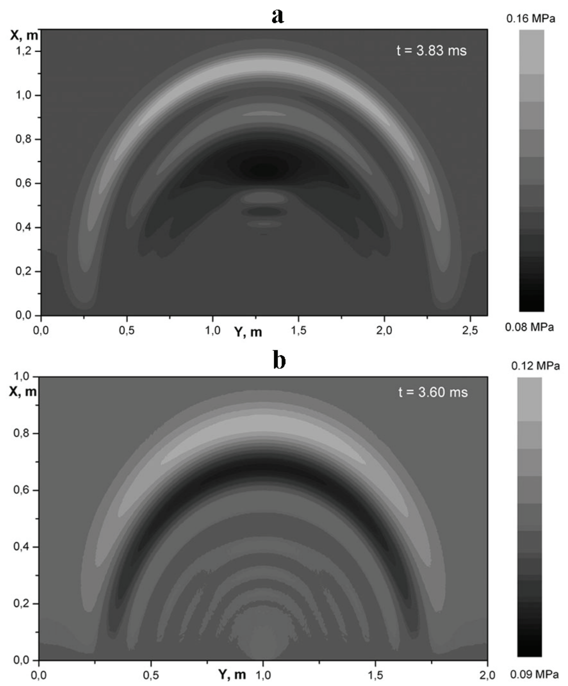

Fig. 6. Pressure diagrams. **a** - at the time t = 3,83 ms, parameters are p_{l0} = 0.1MPa, Δp_0 = 0.3MPa, $\varphi_0 = 10^{-3}$, t_* = 1 ms, y_* = 0,4 m; **b** - at the time t = 3,6 ms, parameters are p_{l0} = 0.1MPa, Δp_0 = 0.3MPa, $\varphi_0 = 10^{-3}$, t_* = 0,1 ms, y_* = 0,04 m.

4 Conclusion

Analysing the results of numerical simulation of the pressure wave propagation in the liquid with the bubble area we get that the pressure outburst is formed in the bubble area.

It was shown that the basic cause which influences the wave deformation is the cumulative flow forming as a result of interaction of the wave with the bubble area.

It was also shown that velocities of cumulative flow depend on pressure amplitude, gas volume content, the bubble area width and on the wavelength.

In some cases this flow can lead to generation a significant pressure outburst in the bubble area and in some cases cumulative flow generate a “plateau” of pressure.

It was shown that the leading wave with damped ripple or with damped wave train was formed when the pressure wave propagated in bubbly liquid mixture.

Acknowledgments

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References

[1] L. Rayleigh On the pressure developed in a liquid durring collapse of a spherical cavity. Philos. Mag., 34, 94-98 (1917)

[2] W. Nusselt Die Oberflachenkondensation des Wasserdampfes.- VDI-Z., N 27. Teil 1, 541-546; N 28. Teil 2, 569-576 (1916)

[3] V.E. Nakoryakov, V.G. Pokusaev and I.R. Shreiber, *Propagation of waves in gas- and vapour-liquid media.* Institute of Thermorphysics, Novosibirsk, 280 pp. (1983)

[4] T.G. Leighton, S.D. Meers and P.R. White, “Propagation through nonlinear time-dependent bubble clouds and the estimation of bubble populations from measured acoustic characteristics”, Proc. R. Soc. Lond A, 2521-2550 (2004)

[5] R.I. Nigmatulin, *Dynamics of multiphase media.* Vol.1 & 2. Hemisphere, Washington (1990)

[6] R.I. Nigmatulin, V.S. Shagapov, I.K. Gimaltdinov, M.N. Galimzyanov, “Two-dimensional pressure waves in a fluid with bubbles”, Dokl. Akad. Nauk., 378, **№6**, 763-768 (2001)

[7] N. A. Kudryashov and N. A. Teterev, “Numerical modeling of the solitary pressure wave propagation in a liquid containing bubble area”, Journ. Mathematic modeling, in press (2008)

[8] R.I. Nigmatulim Mathematical modeling of bubbly liquid motion and hydrodynamical effects in wave propagation phenomenon // Appl. Scientific Research, 38, 267-289 (1982)

[9] E. Hairer, S.P. Norsett, and G. Wanner, “Solving ordinary differential equations nonstiff problems”, Springer-Verlag: Berlin and New York (1987)