Two dimensional finger-string interaction in the concert harp

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The sound of the concert harp partly depends on the way the string is plucked. The vibrating string is brought into a state of initial conditions by the finger-string interaction and then oscillates according to two transverse planes.
In order to understand the plucking action of the concert harp, a one-dimensional model of this interaction has been developed in a previous paper [1]. The parameters of this model were deduced from measurements of the string's and finger's trajectories. The aim of the present paper is to extend this model to a more realistic one, including a two-transverse trajectory for each one of the interaction's elements. To do so, a special experiment with a high-speed camera, which films the interaction, is set up. Specific image processing, based on edges detection, helps to automatically track both objects' positions. The results show that the finger-string interaction takes place in two planes and permit to obtain the parameters of the two-dimensional model for the finger-string interaction.

1 Introduction

The sound of string instruments is not only due to their intrinsic characteristics but also to the way in which they are played. In the family of plucked instruments, the excitation systems are either manual, with a direct contact of the finger, or mechanical, by use of a plectrum or of a hammer. When the instrument is plucked with the finger, as in the case of the classical guitar or the harp, the excitation is dependant on its morphology producing a sound characteristic of the player [1, 2]. In a previous paper [1], a one-dimensional model of the plucking action in the concert harp has been developed. Parameters experimentally obtained seem to depend on the harpist's morphology and therefore explaining the characteristic sound each player produces. The aim of the present paper is to extend this model to a two-dimensional one, including two transverse trajectories for the finger and for the string, whose parameters are evaluated from repeatable measurements.
The paper is organised as follows: after a description of the finger-string interaction in the concert harp, a two-dimensional model of the plucking action is proposed. Then, an experimental investigation leads us to obtain the model parameters.

2 Description of the finger-string interaction

The finger-string interaction starts when the finger touches the string and ends when the string is released to vibrate freely. For the concert harp, the classical plucking action can be decomposed into three phases [1] depending on three characteristic moments (t_c when the finger touches the string, t_s when the finger starts to slip on the pulp of the finger and t_r when the string is released). These three phases, linked to string's and finger's trajectories (see figure 1), can be described as follows :

1. sticking phase [t_c;t_s]: the finger pulls the string ($x_f(t)$ and $x_s(t)$ in the same orientation);
2. slipping phase [t_s;t_r[: the string slips on the finger ($x_f(t)$ and $x_s(t)$ in opposite orientation);
3. free oscillation phase [t_r;∞[.

The frictional force between the fingertip and the string determines the moment when the slipping phase starts and the moment when this force reaches a maximum. It also determines the velocity of the string at the plucking position during the slipping process. Therefore, the friction between the string and the pulp of the finger is similar to that of the violin's string and bow.

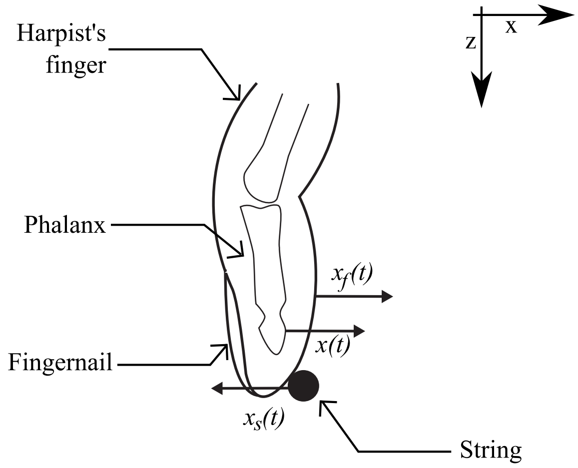

Figure 1: Description of the finger

3 Two dimensional interaction model

The finger-string interaction model presented thereafter is an extention of a previous one-dimensional model [1, 2] based on previous works on the classical guitar [3]. The two-dimensional model takes into account the two transverse vibrations of the string in planes (y, z) and (y, x) as defined in figure 2.
The string is considered perfectly flexible of linear mass ρ, of length l, of tension T uniform along the y-axis and of air damping β supposed independent in frequency. Moreover, the string's fixings on the soundboard and on the arm are both supposed perfectly rigid. The pulp of the harpist's fingertip, of width w, plucks the string at position y_0. The string moves along the x-axis and along the z-axis at magnitude of $x_s(t)$ and of $z_s(t)$. The finger of the harp player is modeled by a mass m_f connected to two stiffnesses k_f^x and k_f^z and to two dampings c_f^x and c_f^z. The control of the harpist's finger, $x(t)$ and $z(t)$, can be different from the effective displacement of the pulp of the finger $x_f(t)$ and $z_f(t)$ because of the mechanical behaviour of the skin and of the flesh. The harpist's control can be represented by the movement of the phalanx along the x-axis and the z-axis.

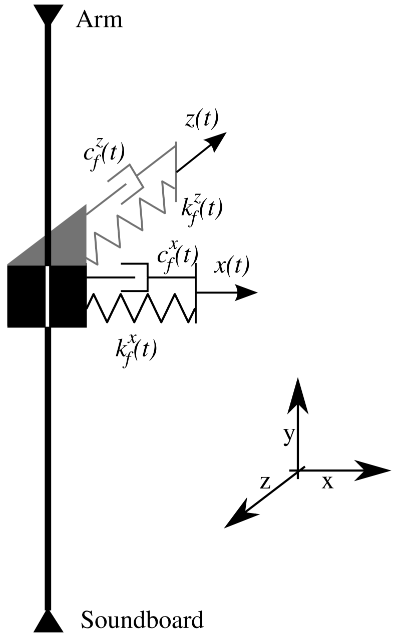

Figure 2: Two-dimensional finger-string model.

The frictional force existing between the pulp of the finger and the string is, to our knowledge, a quantity which has not yet been studied experimentally. We suppose that the force is a function of the relative velocity, V_{rel}^x and V_{rel}^z, between the pulp of the finger and the string, similar to the function describing the bow-string interaction in the violin [4]. We choose to use a simplified frictional force, decomposed along two axis Φ_x and Φ_z, defined by the relation $\forall t \in [t_s; t_r]$:

$$\Phi^x(t) = \left\{ \begin{array}{l} -f_v^x \left(\frac{dx_s(t)}{dt} - \frac{dx_f(t)}{dt} \right) + \Phi_{max}^x(t) \text{ if } V_{rel} < 0 \\ +f_v^x \left(\frac{dx_s(t)}{dt} - \frac{dx_f(t)}{dt} \right) - \Phi_{max}^x(t) \text{ if } V_{rel} > 0 \end{array} \right. , \tag{1}$$

where the coordinate x can be substituted by the coordinate z. The $f_v^{x,z}$ and $\Phi_{max}^{x,z}$ parameters are respectively called the frictional characteristic slope and the maximum frictional force.

Applying Newton's second law on the string element in contact with the finger and the finger element in contact with the string during the sticking phase ($\forall t \in [t_c; t_s]$), we obtain the two following coupled equations of motion:

$$\left\{ \begin{array}{lcl} \rho\omega \frac{\partial^2 x_s(t)}{\partial t^2} & = & Tw^x \frac{\partial^2 x_s(t)}{\partial y^2} + \Phi(t) - \beta\rho w^x \frac{\partial x_s(t)}{t} \\ m_f \frac{dx_f(t)}{dt} & = & k_f^x \left(x(t) - x_f(t) - l_f \right) - \Phi(t) - c_f^x \frac{dx_f(t)}{dt} \end{array} \right. . \tag{2}$$

These two equations can be simplified by supposing that the finger movement is slowly variable in the sense that the finger-string physical system moves slowly compared with the characteristic movement of the string free oscillations. This quasi-static hypothesis leads to the cancellation of the differential terms in Eq (2). Moreover, we assume that the string's displacement induced by the finger is smaller than the length of the string, leading to no variation of the tension along the y-axis. Thus, the bending force which tends to bring back the string at its resting position can be simplified by using a geometrical relation [5]:

$$\begin{array}{c} F_f = -k_s x_s(t) \\ \text{with } \; k_s = T \left(\frac{1}{y_0 - w/2} + \frac{1}{L-(y_0+w/2)} \right), \end{array} \tag{3}$$

where k_s is homogeneous to a stiffness. Considering the natural length l_f of the spring as the origin for $x_f(t)$, the system of Eq. (2) describing the movement of the string, of the finger's phalanx and of the finger's pulp can thus be simplified as follows:

$$\left\{ \begin{array}{rcl} -k_s x_s(t) + \Phi^x(t) & = & 0 \\ k_f^x \left(x(t) - x_f(t) \right) - \Phi^x(t) & = & 0 \\ -k_s z_s(t) + \Phi^z(t) & = & 0 \\ k_f^z \left(z(t) - z_f(t) \right) - \Phi^z(t) & = & 0 \end{array} \right. \quad \forall t \in [t_c; t_r]. \tag{4}$$

All parameters of the model, described by this previous equations system Eq. (4), can be evaluated by characterizing the pulp of the finger and the string movements fitting by an appropriate polynomial approximation obtained in the least mean squares sense [6]. With these polynomial representation the maximal frictional force (Φ_{max}^x, Φ_{max}^z), the frictional characteristic slope (f_v^x, f_v^z) and the finger stiffness (k_f^x, k_f^z) can be evaluated [1, 6] as follows:

- At the beginning of the slipping phase, the string displacements reach their known maximum x_{smax} and z_{smax}. At this instant t_s, the first and third equations of the system Eq. (4) take the following forms:

$$\Phi_{max}^x = k_s x_{smax} \;\text{ and }\; \Phi_{max}^z = k_s z_{smax}. \tag{5}$$

- During the slipping phase, the string covers a distance h_f (h_f^x and h_f^z according to the x and z axis) from the initial position to the fingertip which can be measured on the harpist's finger, implying:

$$x_f(t_r) = x_s(t_r) + h_f^x \;\text{ and }\; z_f(t_r) = z_s(t_r) + h_f^z. \tag{6}$$

  Considering the polynomial estimation of $x_s(t)$ ($x_s(t) = \sum_{i=0}^{n_s} a_i^s t^i$) at the release moment t_r, the frictional characteristic slope during the slipping phase can be determined by

$$f_v^x = \frac{\sum_{i=1}^{n_s+1} -\frac{k_s a_i^s}{i} \left(t_r^i - ts^i \right) + \Phi_{max}^x \left(t_r - ts \right)}{x_s(t_r) + h_f^x - x_{smax} - \sum_{i=1}^{n_s} \frac{a_{i+1}^s}{i} \left(t_r^i - ts^i \right)}. \tag{7}$$

  An equivalent expression of Eq. (7) along the z axis can also be found by substituting the coordinate x by z.

- During the slipping phase, the equivalent finger stiffness (k_f^x and k_f^z) can be estimated from the knowledge of the harpist's control $x(t)$ and the string displacement $x_s(t)$ by using these equations [6]:

$$k_f^x = \frac{k_s x(t)}{x(t) - x_s(t)} \;\text{ and }\; k_f^z = \frac{k_s z(t)}{z(t) - z_s(t)}. \tag{8}$$

4 Experimental results

4.1 Experimental setup

As explained in section 3, the displacement of the phalanx and the displacement of the string at the plucking position have to be known in order to obtain the model's parameters. These measurements were successfully done, in a previous paper [1], with a particular set up using a high speed camera. For this experiment, we supposed that the phalanx displacement was equivalent to that of the fingernail or of the fingertip. In the present paper, we extend this experimental set up for the case of the two-dimensional measurements of the string's and of the phalanx' displacements. For this purpose, a mirror is added near the string and positioned at 45° in relation to the strings plane's, whereas the camera is oriented perpendicular to the string's plane. A picture and the description of the experimental setup is proposed figure 3. In order to obtain the phalanx displacements according the two axes, two black dots are drawn on the fingertip of the player, each dot is tracked along each axe. In the following subsection, the image processing method that is used to track these dots and the string to obtain the two trajectories is explained.

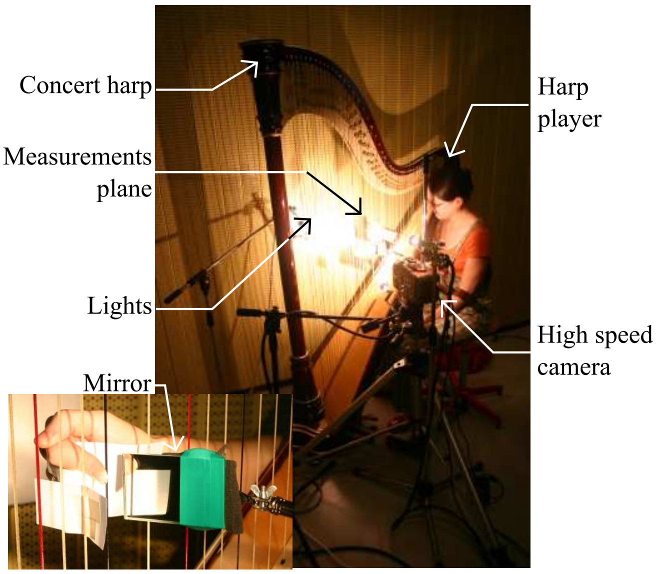

Figure 3: Description of the experimental setup with a focus on the string plucking.

4.2 Image Processing

The aim of image processing is to automatically detect the position of the finger and of the string through time. Processing basically consists of four steps:

1. Region of interest definition
2. Edge detection
3. Object detection
4. Position estimation

As the finger cannot be traced directly by its contour, we put black markers on the fingertip. We use two markers in order to handle occlusions.

The camera is directed perpendicularly to the string's plane and the mirror is positioned close to the string. Thus the right part of the image gives us a lateral view on the string and the finger. When regarding the image sequence (figure 4), regions where the movements take place are manually defined. This decreases the chance of false-detections as well as computing time.

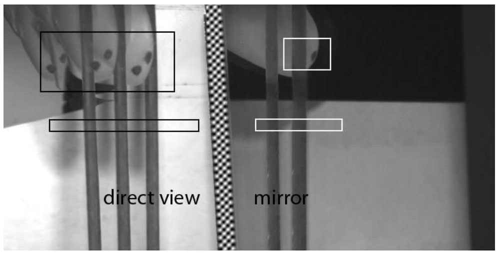

Figure 4: Superposition of some representative images. The rectangles define the regions of interest.

The second step, 'edge detection', is a standard method used in image processing in order to find the borders we see between objects. The Canny Edge Detector [7] creates quite well suited smooth contours (figure 5). This method replaces the luminance-based object detection used in the former one-dimensional experiment [1] which had great problems when lighting distribution was not perfectly uniform.

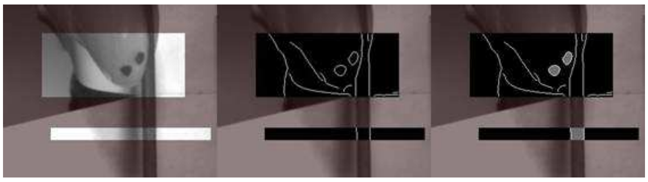

Figure 5: Regions of interest, edge detection and object detection.

For object detection we apply an algorithm [8] which finds connected regions in the contour-image. Not every closed region corresponds to what we are looking for, thus we have to apply some constraints. The constraints for finger markers are a certain object size (number of pixels) and a measure of circularity that has to be met. Also for the string, a size constraint is applied. The other constraint is that the diameter has to be constant along the vertical axis.
This combination of methods yields a very reliable detection of objects that allows for calculating their (visual) centre of gravity with a sub-pixel resolution. Knowing the physical dimensions of the image the position values can now be converted from pixels to millimetres.

4.3 Finger's and string's trajectories

The image processing method previously explained is applied on 3 measurement sequences of the finger-string interaction. The high speed camera is located at 31 cm from the string's plane and its sampling rate is adjusted at 4197 frames per second in order to obtain the better compromise between a good resolution and a high sampling frequency (the resolution of the picture is 768 px × 384 px). The string plucked by the player is a Fb of fundamental frequency of 164.8 Hz. A superposition images of some typical frames obtained by the camera is shown in figure 4 where the left part corresponds to the x-axis trajectory, and the right part corresponds to the z-axis trajectory in the mirror.

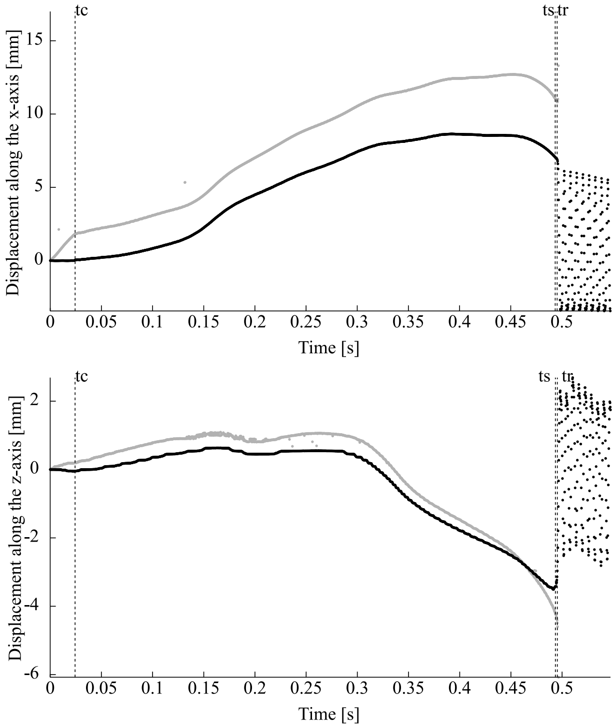

Figure 6: Displacement of the string (in black) and the finger (in gray) along the x-axis and along the z-axis.

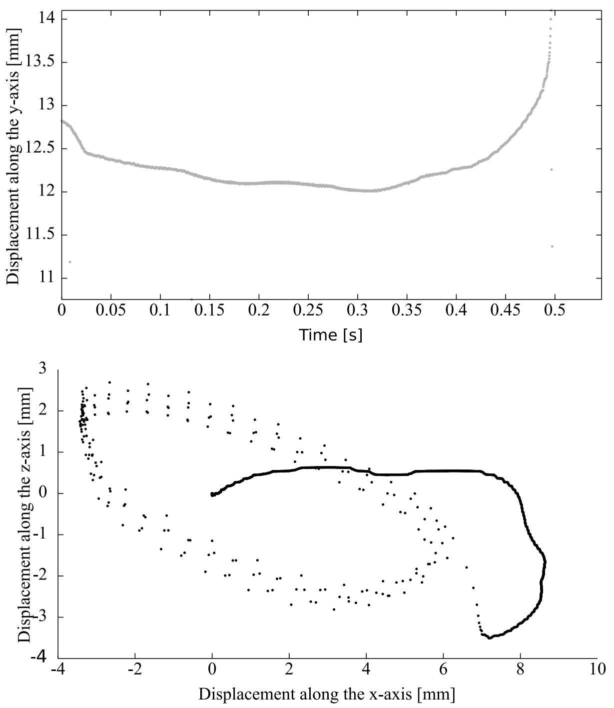

Figure 7: Displacement of the finger (in gray) along the y-axis and of the string (in black) in the plane perpendicular to the string length, on the (x, z) plane.

In figure 6, the displacement of the string and of the finger are shown on the (x, z) plane. The three characteristic times (t_c, t_s and t_r), defined in section 2 are also indicated. After t_r, the string is released to vibrate freely as shown by the oscillations of the string. The shape of the curves is representative for the plucking action of the player. The slipping duration is almost the same for the 3 measurements, around 5 ms, which is very similar to others players [1]. An interesting result of these displacement measurements concerns the better understanding of the harpist movements during the sticking phase. Indeed, as shown in figure 6, the finger first pulls the string along the x-axis (the z-axis remains quite stable) and then, after the time 0.3 s, along the z-axis. The sticking phase can be decomposed into 2 phases in relation to the decomposition of its displacement. This result can be also found in figure 7 where the displacement of the string in the (x, z) plane is shown. Moreover, we can see in this figure, the characteristic elliptical movement of the string when released. In this figure, the finger's trajectory along the y-axis is also shown. This trajectory reveals that the finger principally moves in the (x, z) plane since the displacement along the y-axis is quite stable during the plucking.

4.4 Model's parameters

From the measurements of the finger's and string's trajectories, parameters of the finger-string interaction can be obtained by an appropriate polynomial fitting, as explained in section 3. The polynomial of lowest order which fits the experimental curves given in figure 6 is of 9$^{\text{th}}$ order for the sticking phase and of 3$^{\text{rd}}$ order for the slipping phase. Model parameters are computed from these polynomials according to its two dimensions and synthesized in tables 1 and 2. In these tables, the initial conditions (obtained with a geometrical relation [5, 6]) of the free oscillation phase, corresponding to the displacement and velocity of the string at the plucking position, are also added.

Model's parameters are obtained for a single harpist playing the same string three times. These results can be use to evaluate the repeatability uncertainty of each parameter. Parameters $\Phi^{x,z}_{max}$ and k^x_f are found quite stable (with less than 20% of variation) whereas parameters $f^{x,z}_v$ and k^z_f are not repeatable. This problem is due to the estimation of polynomials in the slipping phase. We will therefore not be able to use these parameters for the comparison of harp players.

In tables 1 and 2, initial conditions for the 3 measurements are given. Results show that the harp player seems to release the string at the same spacial location. For the velocity, results vary more and are therefore not repeatable. Nevertheless, as previously found [1], the string leaves the finger with a non-negligible velocity.

5 Conclusion

In this paper, a straightforward 2 dimensional model describing the finger-string interaction in the concert harp has been developed. To obtain the model's parameters, measurements of the finger and string displacements have been carried out in two dimensions. A particular experimental setup was developed and an advanced image processing method was applied to track these two

	Φ^x_{Max} [N]	$f^x_v \times 10^{-4}$ [N.m^{-1}.s]	k^x_f [N.m^{-1}]	$x_s(t_r)$ [mm]	$\frac{x_s}{dt}(t_r)$ [m.s^{-1}]
1	57	0.99	21039	6.9	-0.05
2	56	9.3	20739	6.7	-0.01
3	53	7.6	24054	6.3	-0.05

Table 1: Model's parameters and initial conditions of the free oscillations phase for 3 measurements along the x-axis.

	Φ^z_{Max} [N]	$f^z_v \times 10^{-4}$ [N.m^{-1}.s]	k^z_f [N.m^{-1}]	$z_s(t_r)$ [mm]	$\frac{z_s}{dt}(t_r)$ [m.s^{-1}]
1	27	0.66	5797	-3	0.68
2	32	0.93	7102	-3.7	0.21
3	39	1.6	2964	-3.6	1.03

Table 2: Model's parameters and initial conditions of the free oscillations phase for 3 measurements along the z-axis.

objects. Some parameters are found repeatable and thus let us to compare harp players in future works. A result coming from the model is the determination of the initial shape and the velocity distribution of the string when releases. Thus, it has been shown that the string leaves the finger at a same spatial location for the harpist and with a non-null velocity.

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