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Analysis of porous plate/water layered structures by means of the transition terms method

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This paper is devoted to the study of water-saturated porous plate/water layered structures by means of the transition terms defined from the reflection and transmission coefficients. Transition terms are obtained from the eigenvalues of the scattering matrix of the water-immersed structure and exhibit the symmetric or antisymmetric resonances of the structures. The N porous plates associated in our structures obey Biot's theory. The reflection and transmission coefficients of a unique water-saturated plate being calculated, an induction on N process allows to find the reflection and transmission coefficients of a given N plate/water-layer structure. The plates used in the experiments at normal incidence are 5mm thick. The reflection and transmission coefficients of sets of 1, 2, 3, and 4 water immersed plates, separated from each other by a 1cm water gap, are measured. There are good agreements between the calculated and experimental transition terms. Which obey the Breit-Wigner resonant form which characteristics can be obtained.

Introduction

The study of the transition terms of water-saturated porous plate/water layered structures is way to obtain resonance characteristics of the structures. This paper is devoted to the study at normal incidence of both theoretical and experimental transition terms of sets of periodic water-saturated porous plates. Up to four identical porous plates separated by water layers are investigated experimentally.

The constituting material named QF-20 is produced by Filtros®, and obeys Biot's theory.

Experiment

The plates (350×200×5 mm) are carefully slid parallel into a machined plates holder and stand vertically between two transducers in a 2000-litre water tank (see the experimental set-up in Fig.1). The distances between the transducers and the plates are about 50cm ; the diffraction is negligible and a normal incident plane pulse is repeatedly launched by the emitter onto the incident face, denoted A, in the sets of plates.

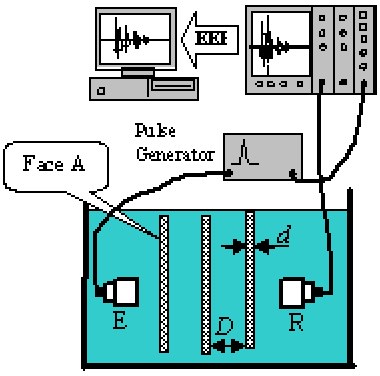

Fig.2 experimental set-up

The transducers are identical (Panametrics® non-focused, diameter of the active element: 1,5in., central frequency: 500 kHz; the frequency range runs approximately from 150 kHz to 850 kHz. The signals are not amplified, and the data are stored after the electronic perturbations have been removed thanks to an average of 300 acquisitions. The sampling frequency is 100 MHz, and the recorded signals have at least 20,000 samples with no reflected signals from the walls of the tank. The reflected and the transmitted signals from the plates are normalized with the direct signal, passing from the emitter to the receiver, recorded after the plates have been removed for the same locations of the transducers. The zero-padding technique is used to obtain experimental reflection and transmission coefficients with an effective resolution of the order of 200Hz.

Theory

Reflection and transmission coefficients of a set of N periodic porous plates

Let us consider N identical porous plates with thickness *d* separated by water layers with thickness *D*. The face A of the set is always considered as the phase reference plane. The reflection and transmission coefficients of the whole structure obey at normal incidence to the relation Eq.(1), Eq.(2). These coefficients only depend on D and C_1 (the velocity of sound in water). The reflection and transmission coefficients of unique plate denoted R_1 and T_1 respectively. R_N and T_N will be calculated thanks to Biot's theory in the following. Proceeding by induction on $N \geq 2$, the general expressions of the reflection and transmission coefficients of N layers separated by $(N-1)$ water layers obey the formulas:

$$R_N = R_{N-1} + \frac{R_1 \left(T_{N-1}\right)^2 \exp\left(-i\omega \frac{2D}{c_1}\right)}{1 - R_1 R_{N-1} \exp\left(-i\omega \frac{2D}{c_1}\right)}, \qquad (1)$$

$$T_N = \frac{T_1 T_{N-1} \exp\left(-i\omega \frac{D}{c_1}\right)}{1 - R_1 R_{N-1} \exp\left(-i\omega \frac{2D}{c_1}\right)}, \qquad (2)$$

where ω is the angular frequency. The basic principle of this method is to consider, first, that R_1 and T_1 contain the complete acoustic behavior of a plate. Second, two successive plates and their separating water layer can be replaced by a unique plate with coefficients R_2 and T_2, and located where the first one was. In this way, the heavy matrix formalism proposed by Gordon et *al.* is not necessary [1].

Reflection and transmission coefficients of a porous plate obeying Biot's theory

At normal incidence, only two longitudinal waves propagate in the porous material [2, 3]. The velocities of the so-called fast and slow waves are the solutions of a biquadratic equation presented by Stoll [4]. R_1 and T_1 are the solutions of a (6×6) matrix equation obtained from the boundary conditions satisfied by the scalar potentials at the interfaces of the immersed plate. We follow the procedure proposed in [5] to calculate them. The experimental conditions are taken into account to give to the parameters of the plates the values leading to the weakest discrepancy between the theoretical and experimental transmission coefficients for the four studied layered structures. The viscosity of the tap water we use is stronger than this of pure water.

As the viscosity governs the possibility of the fluid to flow more or less easily in the connected pores of the porous media, it influences the coefficient of permeability introduced in the empirical law of Darcy, used in Biot's theory, in order to state the complex relation between the moving fluid and the solid part [2, 3]. As a result, the permeability taken here is smaller than that of the pioneering papers using QF-20, see in the accompanying Table 1 the physical constants of water and QF-20. On the whole, the values of the two columns are close. However, it must be noticed that the discrepancy between the experimental and calculated transmission coefficients is reduced when an imaginary part is added to the dried bulk modulus. So is established the viscoelasticity of the solid part of the plates.

Transition terms

The scattering matrix of the structure is a 2x2 matrix [6]. Its diagonal elements are equal to the reflection coefficient R_N, while the off-diagonal elements are equal to the transmission coefficient T_N. The two eigenvalues of the scattering matrix then take the form

$$\mu_S = R_N + T_N\ , \tag{3}$$

$$\lambda_A = R_N - T_N\ . \tag{4}$$

The physical meaning of the eigenvalues is contained in the transition terms T_{μ_S} and T_{λ_A} defined by:

$$1+2iT_{\mu_S} = \mu_S\ , \tag{5}$$

$$1+2iT_{\lambda_A} = \lambda_A\ . \tag{6}$$

		Values in pioneering papers.	Experimental values used in this paper.
Bulk modulus of grains	K_r (Pa)	$36.6\cdot 10^9$	$36.6\cdot 10^9$
Dried frame bulk modulus	$\overline{K}_b$ (Pa)	$9.47\cdot 10^9$	$(10+0.4\boldsymbol{i})\cdot 10^9$
Dried frame shear modulus	$\overline{\mu}$ (Pa)	$7.63\cdot 10^9$	$(9+0.5\boldsymbol{i})\cdot 10^9$
Solid density	ρ_s (kg m^{-3})	2760	2760
Bulk modulus of water	K_f (Pa)	$2.22\cdot 10^9$	$2.22\cdot 10^9$
Water density	ρ_f (kg m^{-3})	1000	1000
Water sound velocity	c_f (m s^{-1})	1478	1478
Saturating water viscosity	η (kg m^{-1}s^{-1})	$1.14\cdot 10^{-3}$	$1.5\ 10^{-3}$
Porosity	β	0.402	0.38
Permeability	K (m^2)	$1.68\cdot 10^{-11}$	$1.5\cdot 10^{-11}$
Pores radius	a_p (m)	$3.26\cdot 10^{-5}$	$3.2\cdot 10^{-5}$
Tortuosity	α	1.89	2.15

Table 1 Physical constants of water and QF-20

Results and discussion

The modules of the transmission coefficients, and those of the symmetric and antisymmetric transition terms of the systems of 2 and 4 plates separated with a gap between plates of $D = 1\text{cm}$ are presented below in the *fd* range 0.5 to 4.5 MHz mm.

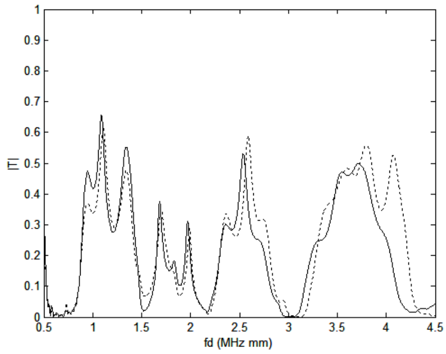

Fig.2 Transmission coefficient of two porous plates separated by water layer. Experiment (solid curve) and theory (dotted curve).

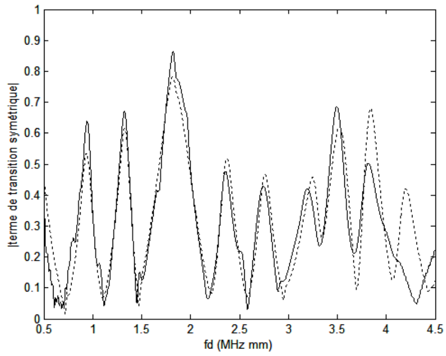

Fig.3 Symmetric transition term of two porous plates separated by water layer. Experiment (solid curve) and theory (dotted curve).

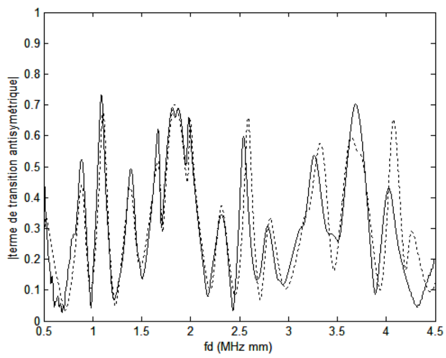

Fig.4 Antisymmetric transition term of two porous plates separated by water layer. Experiment (solid curve) and theory (dotted curve).

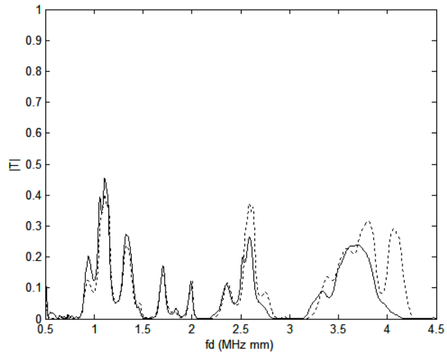

Fig.5 Transmission coefficient of four porous plates separated by water layers. Experiment (solid curve) and theory (dotted curve).

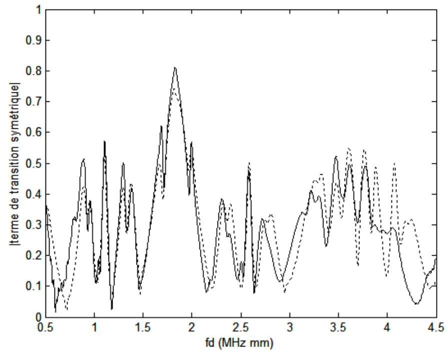

Fig.6 Symmetric transition term of four porous plates separated by water layers. Experiment (solid curve) and theory (dotted curve).

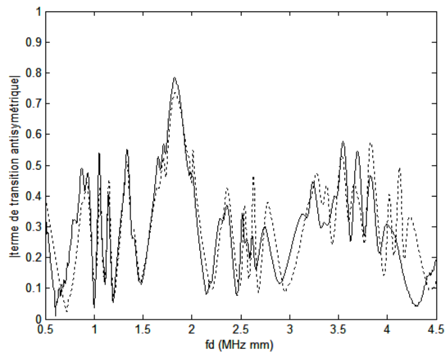

Fig.7 Antisymmetric transition term of four porous plates separated by water layers. Experiment (solid curve) and theory (dotted curve).

As the number of plates increases, the stopbands become more clearly defined and the number of the resonances increases. The agreement is quite good. Since each plot of T_{μ_S} and T_{λ_A} is devoted to only one kind of vibration mode (symmetrical or antisymmetrical), the resonance peaks are less numerous than in the plot of the transmission coefficients and therefore easily separated [7]. They obey the Breit-Wigner form. A resonance will be located in the Argand diagram of a given frequency rang of the transition terms by its circular shape; the resonance frequency fd_0 is located at the peak of the derivative of the curvilinear abscissa. The resonance width Γ and the value of the diameter are simply estimated [8]. This method is successively applied for the calculated and the experimental transition terms. For example, some Argand n are presented in Fig.8 and Fig.9. The characteristics of the resonances are given in Table 2. The results are rather close for the symmetrical modes.

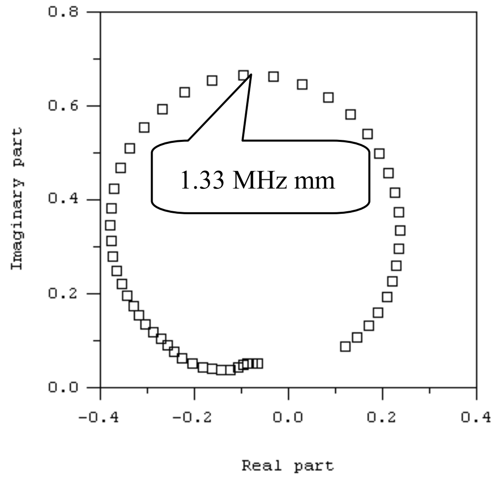

Fig.8 Argand diagram of T_{μ_S} experimental of two porous plates separated by water layer. Frequency range 1.15-1.45 MHz mm

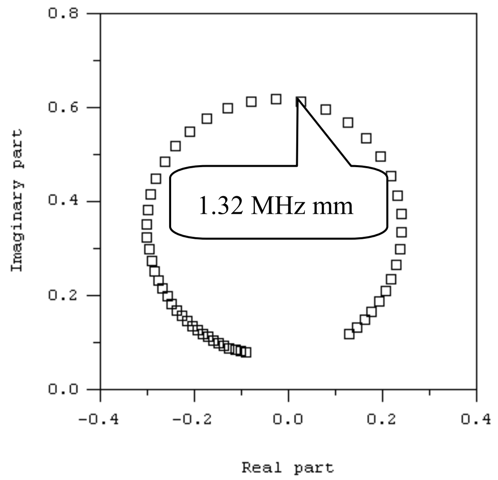

Fig.9 Argand diagram of T_{μ_S} theoretical of two porous plates separated by water layer. Frequency range 1.15-1.45 MHz mm

Experiment symmetric resonances		Theoretical symmetric resonances	
fd_0 (MHz mm)	Γ (MHz mm)	fd_0 (MHz mm)	Γ (MHz mm)
0,95	0,09	0,93	0,13
1,33	0,13	1,32	0,14
1,82	0,30	1,81	0,31
2,35	0,12	2,34	0,18
2,74	0,22	2,74	0,19
3,19	0,19	3,25	0,20
3,49	0,17	3,51	0,25
3,78	0,18	3,80	0,19

Table 2 Position and width of resonance. Symmetrical case.

5 Conclusion

It has been experimentally set in evidence that transition terms can be exhibit only one kind of the vibration modes of the stacks made with porous plates separated by water layers. Their amplitudes are greater than those of the transmission coefficients. The resonant behavior of the water-saturated porous plate/water layered structures is clearly established *via* the properties of the Argand diagram at the vicinity of a resonance. There are good agreements with the corresponding calculated transition terms. The other outcome of this work is that the acoustic attenuation of stack of the plates can be rigorously quantified by measuring the amplitudes of the transition rather than those amplitudes of the reflection or transmission coefficients.

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