# Time domain identification of loads on plate-like structures using an array of acoustic velocity sensors

Quentin Leclere and Charles Pezerat

Laboratoire Vibrations Acoustique - INSA Lyon, 25 bis avenue Jean Capelle, Bâtiment Saint-Exupéry, F-69621 Villeurbanne cedex, France
quentin.leclere@insa-lyon.fr

The FAT (Force Analysis Technique) method has been developped to identify loads on beams or plates from the knowledge of their flexural displacements. The method is based on a local discretisation of the differential operator of the studied structure : all derivatives of the displacement field are assessed at a given point from a finite difference scheme. The estimation of the operator gives as an output the value of the force distribution. Up to now, applications of the FAT method have been made in the frequency domain, scanning the studied structure with accelerometers or with a laser vibrometer, and using phase references to get the phase relation between different points. The aim of the present study is to show that the FAT method allows to identify loads in the time domain. This operation requires the simultaneous measurement of at least 13 points on the plate, that can be realized without contact using an array of acoustic velocity sensors in the very near field of the plate. The method has been applied on a plate excited by an acoustic diffuse field. The identified force distribution is compared to the parietal acoustic pressure measured in the reverberant room.

# 1 Introduction

The Force Analysis Technique (FAT also known as RIFF for the french 'Résolution Inverse Filtrée Fenetrée') is an experimental method allowing to localize vibration sources on a structure from an analytic expression of the motion equation. This approach, firstly developed for beams to identify either longitudinal excitations (compression waves) or transversal excitations (flexural waves) [1], has been extended to plates [2] and cylinders [3]. The difficulty to implement the FAT is its important needs in measurement data : the whole surface of the structure has to be scanned with a fine resolution. The possibility to use a laser vibrometer has been a first technological step allowing to finely and rapidly measure the vibration of a plane structure without any added mass, giving to FAT a real efficiency. A difficulty remains, due to the fact that measurements are not realized simultaneously. It requires the use of one or several phase references, and a frequency domain post-treatment, making impossible any time-domain identification of loads. The apparition of new sensors measuring the acoustic velocity [4] could be a new technological improvement allowing to use FAT to identify a time domain load from measurements using an array of such sensors.
FAT is briefly reminded in a first part. The second part is dedicated to an experimental application showing the use of an array of acoustic velocity sensors to identify the load generated by a diffuse field on a plate.

# 2 The Force Analysis Technique

FAT is based on the expression of the local motion equation of the structure. In the case of flexural vibration on a plate, the motion equation at a given point is expressed by :

$$D\left(\frac{\partial^4 w}{\partial x^4}+\frac{\partial^4 w}{\partial y^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}\right)+\rho h\frac{\partial^2 w}{\partial t^2}=p \qquad (1)$$

where $D = Eh^3/[12(1-\nu^2)]$, $(E, \nu, \rho)$ are material properties, $h$ is the thickness, $w$ the normal displacement, $p$ the value of the external load applied at the studied point (in Pascal).

The knowledge of the left member of equation (1) is sufficient to assess the load $p$. The displacement $w$ is easily measured, but spatial derivatives involved in equation (1) have to be determined. The principle of FAT is to assess those quantities by finite difference schemes, using for a plate like structure a 13-point scheme requiring the measurement of 12 additional points around the considered one. The scheme is given in figure 1.

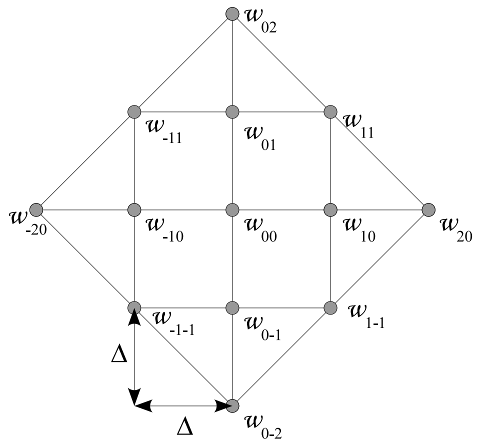


Figure 1: 13 measurement grid required to apply the FAT at the central point $w_{00}$

The expression of finite differences for spatial derivatives used in equation (1) are given by :

$$\begin{aligned}
\frac{\partial^4 w}{\partial x^4} &\approx \delta_w^{4x} = \frac{w_{-20}-4w_{-10}+6w_{00}-4w_{10}+w_{20}}{\Delta^4}\\
\frac{\partial^4 w}{\partial y^4} &\approx \delta_w^{4y} = \frac{w_{0-2}-4w_{0-1}+6w_{00}-4w_{01}+w_{02}}{\Delta^4}\\
\frac{\partial^4 w}{\partial x^2 \partial y^2} &\approx \delta_w^{2x2y} = \frac{1}{\Delta^4}\left[4w_{00}\ \dots\right.\\
&\quad +w_{11}+w_{-11}+w_{-1-1}+w_{1-1}\\
&\quad \left.-2(w_{01}+w_{10}+w_{0-1}+w_{-10})\ \right]
\end{aligned}$$

The computation of the load acting at the central point seems straightforward, but the finite difference operations are highly sensitive to measurement noise, and a specific regularization is required. If the structure displacement is measured on a M*N grid, the load can be assessed at (M-4)*(N-4) points, the finite difference scheme being not assessable for a 2-point width external border of the grid. A specific operation of the FAT technique is filter the resulting load distribution in the k-space to obtain smooth load distributions with a physical meaning [2]. This operation is absolutely required in low frequency, but not always necessary above a frequency depending of several things (measurement noise, spacing between grid points, wavelength of the structure). The aim of this paper is to present a single-point load identification from 13 simultaneous measurement points. Thus, the windowing - filtering operation is not available for our experiment, so we will focus on the frequency range for which this operation is not necessary.

# 3 Time domain identification of the parietal acoustic pressure generated by a diffuse field

## 3.1 Experimental setup

A rectangular aluminum plate (1m*1.5m, 3mm thickness) is placed in the opening of a reverberant room ($411 m^3$, $RT_{60} = 10s$ at 125Hz). The Experimental setup is illustrated in figure 2. The velocity of the plate

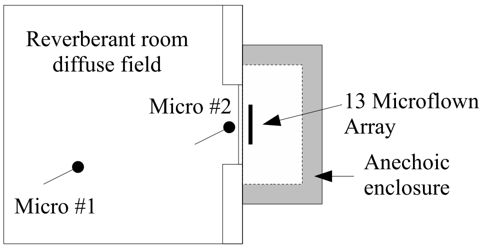


Figure 2: Experimental setup.

is measured using an array of 13 microflown sensors in the very near field (less than 1cm). The normal acoustic velocity is assumed for continuity reasons to be equal to the plate velocity. The spacing $\Delta$ between microflowns is 4cm, a picture of the array in front of the plate is shown in figure 3. The parietal acoustic load applied to

Figure 3: 13 microflown array designed for FAT applications.

the plate at the center of the array is measured with a microphone placed in the near field of the plate on the other side (in the reverberant room, microphone #2 on figure 2). All measurements are realized simultaneously to make possible the time domain identification. The procedure is repeated for 16 different array positions. Each measurement is realized during 20s with a 8192Hz sampling frequency.

## 3.2 Post-treatment of measured velocities

Measured velocities have to be expressed in the frequency domain for a calibration procedure and for time integrations and derivations required to apply the FAT. The signals are thus expressed in the frequency domain using a single classical FFT algorithm realized on the whole length of signals. The load is identified in the frequency domain (using linear properties of the FFT) using the following expression :

$$p(\omega) = \frac{D}{j\omega}\left(\delta_v^{4x}(\omega) + \delta_v^{4y}(\omega) + 2\delta_v^{2x2y}(\omega)\right) + j\omega\rho h v_{00}(\omega) \tag{2}$$

with $\delta_v^{4x}(\omega)$, $\delta_v^{4y}(\omega)$, and $\delta_v^{2x2y}(\omega)$ the finite difference approximations of the spatial velocity derivatives, and $v_{00}(\omega)$ the velocity measured a the center of the array in the frequency domain. Finally, the resulting load $p(\omega)$ can then be expressed in the time domain using an IFFT algorithm.

## 3.3 Results

The load identified using FAT is compared to the parietal acoustic pressure measured in front of the central velocity sensor on the diffuse field side. FAT result identifies the resulting load on both sides of the plate, but it is admitted for this work that the acoustic pressure outside the reverberant room in negligible compared to the pressure inside the room. Measured and identified loads on a first point are drawn in figure 4 in function of the frequency.

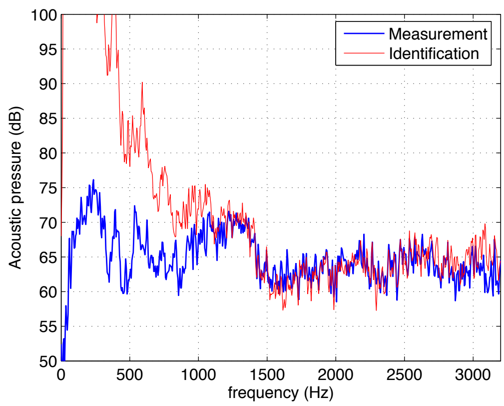


Figure 4: Measured and identified acoustic pressures for the first array position.

The level of the identified sound pressure is largely overestimated below 1200Hz, but really concordant to direct measurements between 1200 and 3200Hz, the difference in this frequency range never exceeds 2dB.
Averaged spectra over the 16 different array positions are given in figure 5. The two members of the FAT expression of the load corresponding to stiffness and mass terms are drawn separately to understand their respective contributions :

$$
\begin{aligned}
p(\omega) &= p^K(\omega) + p^M(\omega) \\
p^K(\omega) &= \frac{D}{j\omega}\left(\delta_v^{4x}(\omega) + \delta_v^{4y}(\omega) + 2\delta_v^{2x2y}(\omega)\right) \\
p^M(\omega) &= j\omega\rho h v_{00}(\omega)
\end{aligned}
$$

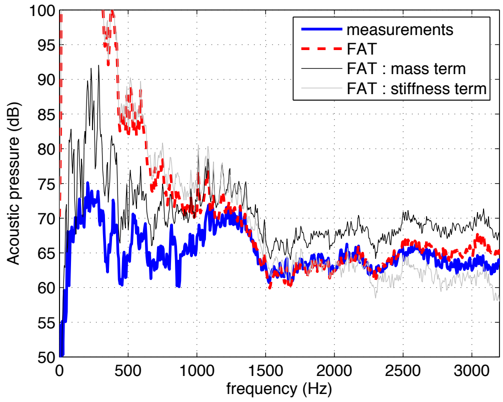


Figure 5: Measured and identified acoustic pressures averaged over the 16 array positions.

The frequency range for which the method is valuable without filtering is the same for the 16 array positions. At this frequency, the wavelength of the plate is about 16cm, it corresponds to a ratio $\lambda/\Delta$ equal to 4, or in other words we do not need any regularization when the size of the array is greater than one structure's wavelength. Identified pressures are lightly overestimated above 2700Hz. At this frequency, the structure's wavelength is equal to 10.5cm ($\lambda/\Delta = 2.6$).
It can be seen on figure 5 that the overestimation in low frequency is due to the stiffness term, that contents finite difference computations. The mass term, directly given by the velocity measured at the center of the array, is always greater than the real acoustic pressure. The stiffness term acts as an efficient correction to the mass term above 1200Hz. It is interesting to note that above this frequency, the stiffness term is always smaller than the mass term. It could be a good indicator (for application cases for which the load is not directly measured) to highlight the frequency range for which FAT results are valuable without filtering.
Measured and identified pressures in the time domain for the first array position are drawn in figure 6 during 50ms, band-pass filtered on the optimal frequency range [1200-2700]Hz. It is quite difficult to compare time traces, but it can be seen that the levels, timings and shapes of pressure bursts are quite concordant. To quantify the quality of the identified load, a transfer function H1 is computed between identified and measured loads, as well as the coherence function. Those estimators are computed over 20 seconds using non filtered signals, with a Hanning-weighted time window of 100ms, 90% overlap. The transfer function is drawn in figure 7. The H1 estimator, that should be equal to 0 in amplitude (dB) and phase (rad), is quite satisfying between 1500 and 2500 Hz, that corresponds to a $\lambda/\Delta$ between 3.5 and 2.7. In this frequency range, the amplitude varies between -2 and 2dB, and the phase between 0 and -25 degrees. The corresponding coherence

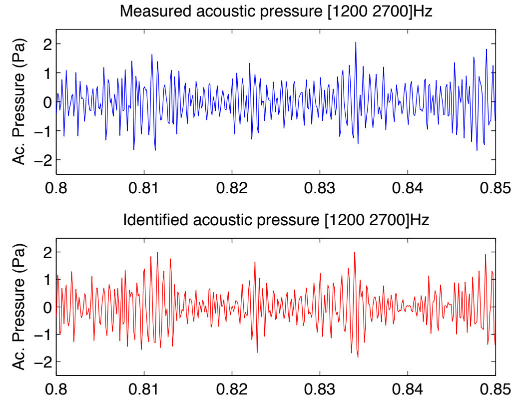


Figure 6: 50ms of measured and identified acoustic pressures in the time domain. Band pass filter [1200 2700Hz].

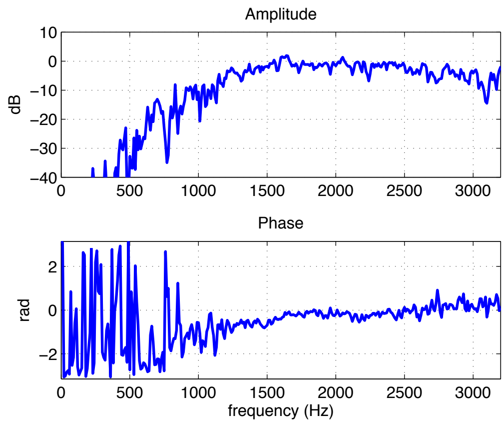


Figure 7: Transfer function H1 between measured and identified acoustic pressures.

function, drawn in figure 8, is between 50 and 75 % in this frequency range. This coherence function should be equal to 100%, but this result is impossible to perfectly reach because of the nature of the excitation, that is a diffuse field. The coherence between the measured parietal pressure in the diffuse field side and the velocity just at the other side of the plate is also given for comparison in figure 8. This coherence, around 30% on the whole frequency range, difficultly exceeds 50 %. The fact that the coherence between measured and identified pressures be significantly higher is an indirect validation of the success of the identification.

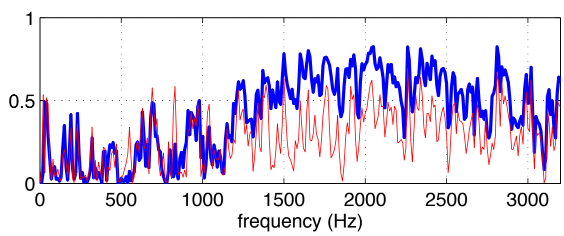


Figure 8: Coherence functions between measured and identified acoustic pressures (blue thick line), and between measured acoustic pressure and plate velocity at the same point (red thin line).

# 4 Conclusion

In its classic form, FAT is a technique allowing to identify vibration excitations using several velocity measurements. With asynchronous acquisitions, FAT necessitates to be regularized, that increases the number of measured velocities. In this work, a first experiment shows that 13 synchronous measurements corresponding to the used finite difference scheme are sufficient. Also, an acoustic velocity probe array is used, showing that this kind of new acoustic sensors can be used for vibration source identification, since they are close enough to the structure.

# References


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