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## Elastic characterization of ceramic balls using resonant ultrasound spectroscopy of spheroidal modes

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The use of ceramic balls, in particular silicon nitride balls, allows a substantial improvement of bearing performances. For critical aerospace and space applications, there is a need for developing new nondestructive techniques for the characterization of these balls. We propose in this work to study the possibilities offered by a specific technique of resonant ultrasound spectroscopy of spheroidal modes. As shown by the theoretical study on elastic sphere vibrations, these modes allow to characterize the whole volume of balls or only the close-to-surface layers, according to the considered frequency range. To acquire the resonance spectra of these modes, a specific measurement system composed of a piezoelectric ultrasonic probe and an optical interferometer was developed. A self-implemented numerical processing of measured spectra allows to detect the resonance frequencies and to deduce from them the propagation velocity of the spheroidal waves in each inspected subsurface layers. Then, we propose a method based on these results that permit to estimate the elastic coefficients of the balls according to various inspection depths. This method has the advantage of providing very high precision evaluations of the elastic coefficients over a wide frequency range.

## 1 Introduction

The aerospace and space sector is one of the most demanding industrial sectors in terms of the performance of materials, the quality of manufactured parts, and the reliability of testing methods, particularly non-destructive testing methods. Today, ceramic materials, composed mainly of inorganic and non-metallic materials, have gained in popularity because of their multiple advantages compared to steel. Since the 1980s, new ceramic materials have become increasingly available and have attracted more and more interest from the industrial sector [1]. This paper focuses on silicon nitride ( $\text{Si}_3\text{N}_4$ ), a ceramic material that meets the needs of the aerospace and space sector, especially for high precision bearings, which require particularly high rotation velocities or which must function in extreme environments.

Silicon nitride is one of the hardest ceramic materials [2,3]. Its cost of production, though generally higher than that of steel, has decreased considerably over the last few years, making it a viable option [3]. In addition, it has multiple advantages compared to the steels traditionally used in bearings. Its low density both permits the centrifugal strength of the rolling elements to be decreased, and elevates the rotation velocities. It does not oxidize, and has a low thermal expansion rate and permit an extended lubricant life. It also has a high fatigue resistance, thus increasing bearing lifetime significantly. Moreover, using ceramic material lowers the acoustic noise of bearings.

In addition to its numerous assets, ceramic balls made of silicon nitride have one principal disadvantage: an inherent brittleness resulting from defects that can occur in the material's surface and sub-surface areas, two areas that are subject to severe contact pressure. Defects can be created at any stage of production, and fall into two broad categories: "fissure" defects (fissure, crack, C-crack, pore, groove, etc.) that are generally restricted to a specific area, and "material" defects (variations in elasticity, homogeneity, color, etc.) stemming from undesirable variations in the properties of the material near the surface.

To assure the quality and, thus, the lifespan of these silicon nitride balls, potential defects in the finished products must be detected in a non-destructive manner. Since surface and

near-surface defects in ceramic balls can have more disastrous consequences than similar defects in steel balls, the testing method must be particularly well-suited to analyzing the sub-surface layer at a depth of about one hundred microns. Given that safety standards today are not always met through statistical parts controls, a rapid method that can inspect 100 % of the parts produced is required. Such verification is becoming more and more necessary; in fact, it is essential in order to guarantee an extended lifespan for engine bearings.

We used a theoretical study of the elastic vibrations of balls to demonstrate that, given the similarity of one specific vibration mode to overall surface wave propagation in the balls, ball quality can be characterized by characterizing the surface waves. By using both an ultrasonic probe (piezoelectric transducer) and a heterodyne optic probe (interferometer), we were able to take spectroscopic measurements for a large frequency range (100 kHz - 45MHz) in a continuous regime. Given that the ultrasonic probe was also used to support the ball, these measurements could be taken without coupling and without rotating the ball. Using resonance spectrums, we determined the velocity of the surface waves and characterized the elasticity parameters of the balls, paying special attention to the high frequencies that allowed the zone near the ball's surface (cortical zone) to be analysed. This system, called "RUSSAW" has been jointly developed by SKF Group (Rouvignies) and IEMN-DOAE laboratory (University of Valenciennes).

In the case of spheres, the classic resonant methods used for evaluating the Poisson coefficient are generally employed within the framework of the simultaneous estimation of elastic parameters and supposing that the density and the curvature are known. Generally, a method of optimisation is needed in which the theoretical resonance frequencies are calculated for different elastic coefficients and are compared with the measured frequencies. The end result is that the method of optimisation retains the combination of elastic coefficients giving theoretical frequencies closest to the measured frequencies [4].

Our experiments on the subject have shown, on the one hand that the convergence of the routine is not always reached according to the high frequency resonance groups used and on the other hand, that the results obtained are relatively incoherent: a single coefficient is adjusted, the

final value of the second strongly depends on the starting values of the routine. Moreover, it is very difficult to obtain a correct estimation at high frequency using this method.

To palliate these limits, an innovative estimation method has been developed. Contrary to classic methods, it allows the Poisson coefficient to be calculated independently of Young's modulus, and the ball's density and radius. Moreover, this method can be used for high frequencies. These advantages allow, on the one hand high accuracy estimations of the Poisson coefficient since the uncertainties of Young's modulus, and the ball's density and radius do not have any consequence. On the other hand, the estimation on a large range of frequencies allows the characteristics of the Poisson coefficient of the overall volume of the ball to be estimated, but also that of the sub-surface layers.

## 2 Resonant ultrasound spectroscopy of balls

Concerning the resonances of an elastic sphere, Lamb [5] was one of the first to show the two types of vibrations possible in an isotropic case: toroidal (tangential displacements of the spherical surface) and spheroidal (displacements whose radial component is not zero). In this study, spheroidal modes are considered. Indeed, in order to have free vibrations of balls (without contact force), a laser interferometer (sensitive to radial displacement) is used for the detection of these vibrations. The resolution of Helmholtz's equations, with spherical coordinates for an isotropic sphere whose surface stress is zero, provides the following characteristic equation for spheroidal modes [5-7]:

$$\begin{aligned}
 & j_n(kx) \cdot j_n(x) \cdot [x^2 \cdot (2(n-1)(2n+1) - x^2)] \dots \\
 & \dots + j_n(kx) \cdot j_{n+1}(x) \cdot [2x \cdot (x^2 - 2n(n-1)(n+2))] \dots \\
 & \dots + j_{n+1}(kx) \cdot j_n(x) \cdot [4kx \cdot (x^2 - (n-1)(n+1)(n+2))] \dots \\
 & \dots + j_{n+1}(kx) \cdot j_{n+1}(x) \cdot [4kx^2(n-1)(n+2)] = 0
 \end{aligned} \tag{1}$$

where  $x = k_T \cdot R$ ,  $k = k_L/k_T$ ,  $R$  is the ball's radius,  $k_L$  and  $k_T$  are the longitudinal and shear wave numbers respectively and  $j_n$  is the spherical Bessel function of order  $n$  ( $n \geq 0$ ) of the first kind.

The appearance of resonance is closely linked to the behaviour of the surface waves [8]. Resonance appears in the  $n^{\text{th}}$  order of the vibration mode every time a surface wave propagates over the ball's circumference in an integer number of wavelengths. The phase variation for a complete revolution is therefore a multiple of  $2\pi$ . For each value of  $n \geq 1$ , an infinity of solutions  $x_n$  can be calculated (i.e. an infinity of frequencies  $f_n$  obtained from the relation:  $f_n = (x_n \cdot V_T)/(2 \cdot \pi \cdot R)$  where  $V_T$  is the velocity of the shear wave. The first solution is called "Rayleigh mode" since it penetrates the surface of the sphere by approximately one wavelength. Subsequent solutions, called whispering gallery mode solutions, have oscillations that are even more concentrated inside the sphere. When  $n$  is large enough, the slopes of whispering-gallery-mode dispersion curves become similar to one another, while the Rayleigh mode maintains a distinct slope of its own. This distinctive character makes it easy to distinguish the Rayleigh mode

from the other modes in the spectrum. When  $n$  tends towards 1, the solution of the Rayleigh mode tends towards zero. In this case, the Rayleigh mode does not exist; the centre of the sphere merely undergoes a simple translation. The resolution of this equation for  $n = 0$  can be treated separately; this case implies a particular mode called "radial resonance", in which the entire ball successively contracts and expands. In the case of perfectly spherical, homogeneous and isotropic balls, the shear wave velocity  $V_T$  and the radius  $R$  of the ball are constant parameters in the theoretical expression of resonance frequencies  $f_{iSn}$ , whatever the mode  $i$  and the order  $n$  of the resonances. Consequently, the ratio  $\eta$  between two resonance frequencies of different modes and/or orders theoretically allows the influence of the radius  $R$  and the shear wave velocity  $V_T$  to be eliminated. The ratio between two resonance frequencies of modes  $i_1$  and  $i_2$  and of order  $n_1$  and  $n_2$  respectively, indeed depends only on the solutions of the frequency equation for these modes and orders ( $x_{i_1Sn_1}, x_{i_2Sn_2}$ ). Between resonances of different modes and/or orders, only the solutions of the frequency equations  $x_{iSn}$  vary. The value of these solutions depends on the elastic properties of the ball, by means of the variable  $k$  present in the frequency equation. This variable corresponds to the ratio of the shear and longitudinal wave velocities ( $k=V_T/V_L$ ), and consequently depends only on Poisson coefficient (fig.1).

$$k = \sqrt{\frac{1-2\nu}{2 \cdot (1-\nu)}} \tag{2}$$

$$\text{and } \eta_{((i_1,n_1),(i_2,n_2))} = \frac{f_{i_1Sn_1}}{f_{i_2Sn_2}} = \frac{x_{i_1Sn_1}}{x_{i_2Sn_2}} \tag{3}$$

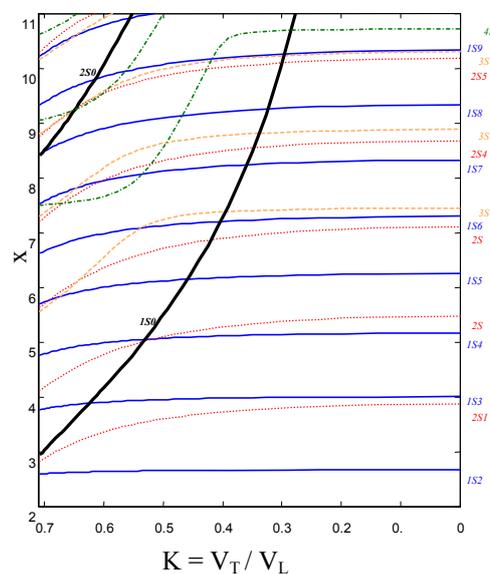


Fig 1: Evolution of the values of the frequency equation solutions  $x_{iSn}$  for a ball as function  $k$  versus the resonance modes ( $i$ ) and orders ( $n$ ).

In order to estimate the Poisson coefficient  $\nu$ , an iterative routine was implemented. This routine varies the Poisson coefficient until a theoretical ratio as close as possible to the measured ratio for each resonance frequency pair is obtained. The routine is executed for each resonance pair in order to estimate the Poisson coefficient over the whole frequency range. For an accurate and rapid estimation, the iterative routine calls upon an optimisation method based on Levenberg-Marquard's least squares minimisation

algorithm [9]. This choice allows a compromise between rapidity and reliability of the estimation, whatever the initial value set at the beginning of the routine. The method proves firstly relatively fast: with the laboratory's computer equipment, the estimation with a resonance pair takes on average 2 seconds. Also, it was verified that the inversion algorithm was independent of the initial value fixed. For example, by choosing  $v_0 = 0$  as the initial parameter, the number of iterations is of course important, but the algorithm gives the same result to within  $10^{-9}$ . Secondly, the method proves to be very accurate. With a tolerance below  $10^{-6}$  on the deviation between the theoretical and measured values, the sensitivity of the Poisson coefficient estimation using this resonance pair is inferior to  $10^{-5}$ . Here, we talk of the sensitivity as this value only takes into account the accuracy of the algorithm, and not the measurement and detection errors that may influence the two frequencies measured. In other words, this value corresponds to the accuracy of the estimation assuming that the two frequencies measured correspond exactly to the resonance frequencies of the ball.

The inversion algorithm developed proves therefore sufficiently accurate to estimate the Poisson coefficient from a resonance spectrum of the ball. The selection of the resonance pairs for this estimation is however important as the choice influences the accuracy of the Poisson coefficient estimation. Amongst all the resonances, those that correspond to the Rayleigh mode present several advantages. Firstly, the Rayleigh mode resonances have the highest amplitudes and are consequently the easiest to identify. Secondly, these high frequency modes that are similar to Rayleigh waves (penetration depth of approximately one wavelength) allow the whole volume of the ball at low frequency and a low depth zone under the surface of the ball at high frequency to be characterised. However, the breathing or whispering gallery modes prove to be particularly sensitive to variations in the Poisson coefficient. Thus, to satisfy the requirements for the estimation of the Poisson coefficient (accuracy, reliability and control of the inspection depth) the estimation algorithm is based on the ratios between the Rayleigh mode resonance frequencies and the frequency  $f_{ref}$  of a reference resonance.

$$\eta_{(ISn,ref)} = \frac{f_{ISn}}{f_{ref}} = \frac{x_{ISn}(v)}{x_{ref}(v)} \quad (4)$$

The reference resonance is taken from the modes for which the frequency equation solutions are most sensitive to variations in the Poisson coefficient. That is to say either the breathing mode, or certain orders of whispering gallery modes, for example the first order of the second whispering gallery mode (3S1). For a given reference frequency, the estimation is carried out by taking into account in succession different order Rayleigh mode resonance frequencies. This allows the Poisson coefficient to be estimated for different inspection depths.

The expression of Young modulus is obtained from Rayleigh resonance frequencies  $f_{ISn}$  detected and identified on the spectrum:

$$E = 2 \cdot \rho \cdot (1 + \nu) \cdot \left( \frac{f_{ISn}}{x_{ISn}(v)} \cdot 2\pi \cdot r \right)^2 \quad (5)$$

Thus, from resonance spectrum and the Poisson coefficient, the ball's Young modulus can be estimated if the ball's

radius and the ball's density are known.

### 3 Results and discussions

Ultrasonic Spectroscopy consists of analysing a system's response to an ultrasonic excitation in the frequency domain. Many spectroscopy techniques are used in the impulse mode, and the spectrum is calculated from the FFT of the temporal response [10]. These techniques are rapid, but measurement accuracy is limited [11]. Our method subjects the ball to a continuous sinusoidal signal (harmonic regime) whose frequency varies relatively slowly with time, thus producing a highly accurate spectrum. The displacement amplitudes are measured in terms of the excitation frequency. Consequently, the spectrum presents different peaks for the ball resonance frequencies that correspond to the in-phase combination of ultrasonic waves at the detection point. Schematically, the measurement set up has four distinct parts: sinusoidal signal generation, ultrasonic wave emission, ultrasonic laser detection, and signal demodulation [12,13].

Experimentally, by using both an ultrasonic probe for the emission (piezoelectric transducer) [14] and a heterodyne optic probe for the reception (interferometer), it was possible to take spectroscopic measurements of spheroidal vibrations over a large frequency range (100 kHz – 45 MHz) in continuous regime. But, a certain number of precautions must be taken to guarantee measurement quality, particularly in terms of avoiding any mechanical vibrations that could make the ball oscillate on its support. In the same way, dust and grease must not be allowed to collect on the cylindrical support because the contact quality is critical for obtaining good quality results.

Figure 2 gives an example of a resonance spectrum between 100 kHz and 5 MHz obtained on a silicon nitride ball with a diameter of 11.112 mm. The Rayleigh mode resonances have been detected and identified over the whole spectrum. Considering the first breathing resonance mode as the reference resonance frequency (fig. 3), the Poisson coefficient was estimated for the whole frequency range (fig. 4).

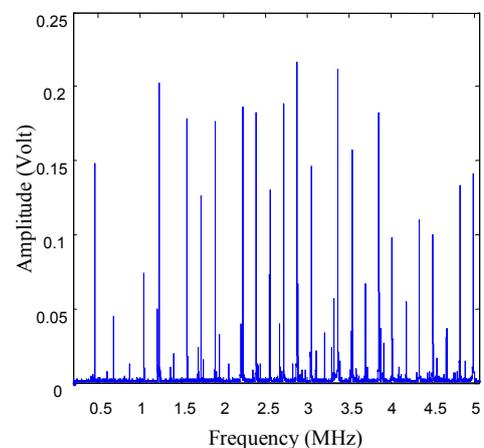


Figure 2: Resonance spectrum (100 kHz – 5 MHz).

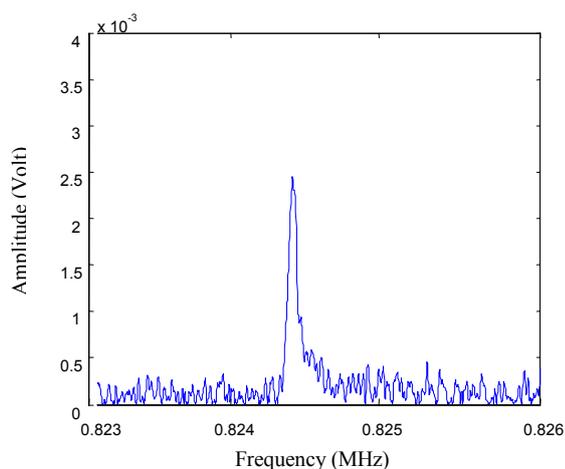


Figure 3: First breathing resonance mode.

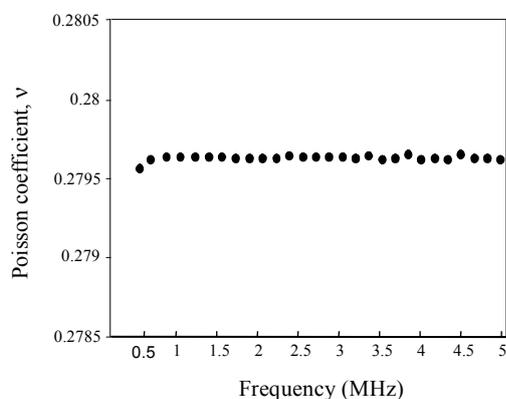


Figure 4: Example of estimation of Poisson coefficient on the frequency range [100 kHz-5 MHz].

The estimation of the Poisson coefficient, over this range of frequencies, presents values that are quasi-identical from one Rayleigh resonance mode to another. Only the values estimated from the first two resonances are slightly lower than the general trend. These values can be explained by the fact that, despite the care taken, the scan speed is too fast for these very low frequencies and induces a slight shift in the resonance frequencies. The mean value of the estimations in between is 0.2796. From the density ( $\rho = 3240 \text{ kg/m}^3$ ) and the results obtained on spectrums, the Young modulus has been estimated on the frequency range [100 kHz - 45 MHz]. The mean value of the estimations in between is 304.54 GPa. These estimations are completely coherent with values published in the literature for this nuance of silicon nitride [15].

To quantify the accuracy of the Poisson coefficient estimations by the proposed method, two sources of error must be considered: the accuracy of the measurement of the resonance frequencies and the repeatability errors observed on the resonance frequencies measured. The accuracy of the resonance frequency measurements depends on the coefficient of quality of the resonance peaks, identified on the spectra, and the resolution of acquisition. In this case, the inherent accuracy error is:  $\Delta f/f = \pm 0.015 \%$ . This error influences the Poisson coefficient estimation as:  $(\Delta\nu)_f = \pm$

$4.3 \cdot 10^{-4}$  and the Young modulus as:  $(\Delta E)_f = \pm 0.6 \text{ GPa}$ , in the case of silicon nitride balls. The repeatability errors generate an uncertainty of the resonance frequencies measured. Consequently they affect the Poisson coefficient estimations. To characterise this uncertainty, thirty measurements on the same ball were carried out, with the ball being randomly repositioned on the equipment after each measurement (fig 6).

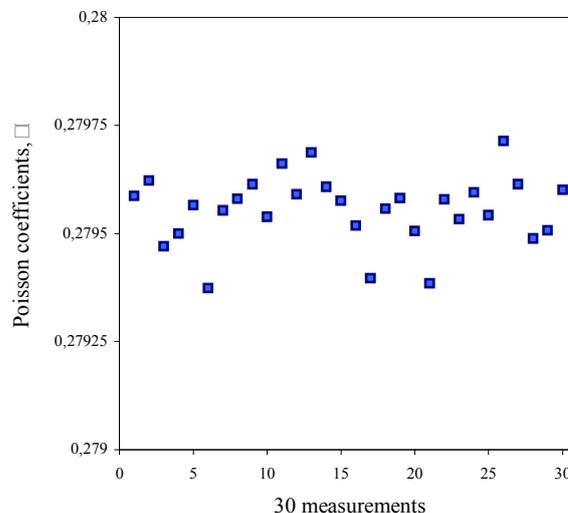


Figure 6: Repeatability of Poisson estimation of coefficient

The estimation of the Poisson coefficient is carried out on each measurement. The reference frequency chosen is that of a first order second whispering gallery mode ( $f_{3S1}$ ), because amongst the reference frequencies that can be used, it is the most affected by repeatability errors ( $\Delta f_{3S1}/f_{3S1} = 0.025\%$  and  $\Delta f_{iS0}/f_{iS0} = 0.01\%$  with  $i=1,2$  on the thirty measurements). A mean value of 0.27955 and a standard deviation ( $\sigma$ ) of  $0.8 \cdot 10^{-4}$  have been evaluated from the thirty Poisson coefficient estimations. To evaluate the uncertainties caused by repeatability errors, the statistical confidence interval of 99.73% ( $\pm 3 \sigma$ ) calculated from these results was retained. Finally, the overall uncertainty of the estimation of the Poisson coefficient of a ball, by the method proposed here, and by measuring one resonance spectrum of the ball, reaches a maximum value of:  $\Delta\nu = \pm (4.3 + 2.4) \cdot 10^{-4} = \pm 6.7 \cdot 10^{-4}$ . The same repeatability errors affect the Young modulus and finally the estimation of the ball's Young modulus, by the method proposed here, and by measuring one resonance spectrum of the ball, reaches a maximum value of:  $\Delta E = \pm (0.6 + 1.3) = \pm 1.9 \text{ GPa}$ .

## 5 Conclusion

The estimation of elastic parameters developed here has therefore proved to be very accurate, repeatable and rapid [16]. Compared with static mechanical techniques (bending or compression, nano- or micro-indentation), it proves more accurate and totally non destructive [17]. It is not limited to low frequencies [4]. Also, tests here were carried out on silicon nitride balls, but the estimation of elastic coefficients on steel balls has also been successfully experimented. This method therefore enables balls of different materials to be characterised.

## References

- [1] R.N. Katz, "Friction and wear of ceramics", Ed. Marcel Dekker, New York, 313-328 (1994)
- [2] S. Jahanmir, "Friction and wear of ceramics", Ed. Marcel Dekker, New York, 3-12 (1994)
- [3] I. Krepchin, "Ceramic bearings on a roll", Design News December, 7, (1998)
- [4] A. Migliori, J.L. Sarrao, W.M. Visscher, T.M. Bell, M. Lei, Z. Fisk, R.G. Leisure, "Resonant ultrasound spectroscopic techniques for measurement of the elastic moduli of solids", Physica B, Condensed Matter, **183**, 1-24 (1993)
- [5] H. Lamb, "On the vibrations of an elastic sphere", Proceedings of the London Mathematical Society, **13**, 189-213 (1882)
- [6] A. Yaita, T. Adachi, A. Yamaji, "Determination of elastic moduli for a spherical specimen by resonant ultrasound spectroscopy", NDT&E Int., **38**, 554-560 (2005)
- [7] D. Royer, Y. Shui, X. Jia, E. Dieulesaint, "Surface waves on spheres", 3<sup>rd</sup> Western pacific acoustics conf. (1988)
- [8] L. Flax, G. Gaunaurd, H. Uberall, W.P. Mason, R.N. Thurston (Eds.), Physical Acoustics, XV, Academic Press, New York (1981)
- [9] J. More, "The Levenberg-Marquardt algorithm: implementation and theory" Mathematics, **630**, Ed. G.A. Watson, Springer Verlag, 105-117 (1977)
- [10] D. W. Fitting, L. Adler, "Ultrasonic spectral analysis for nondestructive evaluation", Plenum press, New York (1981)
- [11] X. Jia, D. Royer, G. Quentin, "Pulsed laser excitation and optical detection of Rayleigh waves on a metallic sphere", Journal de Physique IV, **2**, 789-792 (1992)
- [12] S. Petit, M. Duquennoy, M. Ouaftouh, F. Deneuille, M. Ourak, S. Desvaux, "Non-destructive testing of ceramic balls using high frequency ultrasonic resonance spectroscopy", Ultrasonics, **43**, 10, 802-810 (2005)
- [13] C.P. Hsieh, B.T. Khuri-Yakub, "One-point contact measurement of spherical resonances" Appl. Phys. Lett., **62**, 3091-3093 (1993)
- [14] M. Ourak, F. Lefebvre, M. Duquennoy, G. Nassar, T. Devaux, "Métrologie des champs ultrasonores haute fréquence", 6<sup>th</sup> ECNDT, 249-250 (1994)
- [15] H. Wang, A.A. Wereszczak, "Mechanical responses of silicon nitrides under dynamic indentation", Ceram. Eng. Sci. Proc., **26**, 275-283 (2005)
- [16] Deneuille, M. Duquennoy, M. Ouaftouh, F. Jenot, M. Ourak, S. Desvaux, "Wide band resonant ultrasound spectroscopy of spheroidal modes for high accuracy estimation of Poisson coefficient of balls", Review of Scientific Instruments, **79**, 044901 (2008)
- [17] M. Radovic, E. Lara-Curzio, L. Riester, "Comparison of different experimental techniques for determination of elastic properties of solids", Sci. Eng. A, **368**, 56-70