Plate waves in phononic crystals slabs

Jiu-Jiu Chen and Bernard Bonello

CNRS and Paris VI University, INSP - 140 rue de Lourmel, 75015 Paris, France
chen99nju@gmail.com

We have computed the dispersion curves of plate-mode waves propagating in periodic composite structures composed of isotropic aluminum cylinders embedded in an isotropic nickel background. The phononic crystal has a square symmetry and the calculation is based on the plane wave expansion method. Along Γ*X* or ΓM directions, SH modes do not couple to the Lamb wave modes which are polarized in the sagittal plane. Whatever the direction of propagation between Γ*X* and ΓM, SH modes convert to Lamb wave modes and couple with the flexural and dilatational modes. This phenomenon is demonstrated both through the mode splitting in the lower-order symmetric band structure and through the calculation of all three components of the particle displacements. The phononic case is different from the pure isotropic plate case where SH waves decouple from Lamb waves whatever the propagating direction.
PACS number(s): 43.20.+g, 63.20.-e, 43.40.+s,41.20.Jb,

In the last past decade, the existence of forbidden gaps in the band structure of acoustic and elastic waves propagating in periodic composite materials has received a great deal of attention. For frequencies within the band gap, the propagation of acoustic or elastic waves is forbidden regardless the direction, suggesting numerous technological applications such as acoustic filters, ultrasonic silent blocks, acoustic mirrors etc… Such composite structures for bulk, surface, or plate-mode waves have been studied both theoretically and, in a less extent, experimentally [1] by many groups. In particular, there has been a growing interest for Lamb waves and plate modes which can be used for a variety of high frequency applications such as physical, chemical, and biological sensors.

A plate can support a number of Lamb waves which depends on the value of the ratio h/λ, where h and λ are respectively the thickness of the plate and the acoustic wavelength. The thin plates only support lowest-order antisymmetric and symmetric Lamb waves and lowest- order SH waves. Recently, Chen *et al*. have used a plane waves expansion method (PWE) to calculate the band structures of lowest-order Lamb waves propagating perpendicularly to the alternating layers of a one-dimensional (1D) phononic crystal thin plates.[2] A similar result was found recently in the 2D case by Sun *et al.* who studied the propagation of Lamb waves along Γ*X* in 2D phononic crystal plates with a square lattice.[3] In homogeneous plates, SH waves exist only if the material is isotropic. In that case, three types of the free plate modes must be considered, namely the pure shear horizontal mode witch polarization is parallel to the free surfaces; this SH mode is uncoupled from the two others modes: the dilatational and the flexural modes. This greatly simplifies the investigation of Lamb wave motion in the isotropic materials. When the material is anisotropic, SH modes are still solutions for the equations of motion, but only along some particular directions of high symmetry. Outside these particular solutions, there is no longer a family of pure shear horizontal modes independent from the dilatational and flexural modes: all partial waves are coupled and the free plate modes can only be classified as symmetric or antisymmetric modes with respect to the median plane.

However, in the isotropic/isotropic phononic crystals, the SH wave cannot exist in the same way that in the pure isotropic plates. Sun *et al.*[3] have demonstrated by using a finite-difference time-domain technique, that in 2D phononic crystals thin plates consisting of an array of isotropic steel cylinders embedded in an isotropic epoxy matrix, the SH waves decouple from Lamb waves along the ΓX propagation direction. To our knowledge, this part has not been studied in other directions of the irreducible Brillouin zone so far.

In this paper, we analyze the relationship between SH waves and Lamb waves in 2D phononic crystal plates consisting of a square array of isotropic aluminum (Al – material *A*) cylinders embedded in an isotropic nickel (Ni – material *B*) background, for propagation along any direction of the irreducible Brillouin zone. Our calculations are based on a new PWE method, details of which are given elsewhere[4].Compared to the full 3-D PWE method. This is more effective to search roots of plate waves since it does not require explicitly the boundary conditions on the free surfaces.

Figure 1 depicts the lowest-order dispersion curves for the plate waves propagating along the boundaries of the irreducible part of the Brillouin zone. The filling fraction of the Al/Ni square lattice is $f = 0.6$ and the thickness-lattice spacing ratio is $h/a = 0.80$. The vertical axis is the normalized frequency $\omega^* = \omega a / C_t$, where C_t is equal to $(\overline{C}_{44}/\overline{\rho})^{1/2}$; $\overline{C}_{44} = fC_{44}^{A} + (1-f)C_{44}^{B}$ and $\overline{\rho} = f\rho_A + (1-f)\rho_B$ are the effective elastic stiffness tensor and density, respectively. The horizontal axis is the reduced wave number $k^* = ka/\pi$. In the calculations, x and y axes are parallel to edges of the square unit cell. To insure very good convergence of the computations, we considered 25 reciprocal vectors in the propagation plane and five Fourier components along the out-of-plane direction. Moreover, we have only considered the low frequency part of the Brillouin zone ($\omega^* \le 5$ in reduced units) where the lowest-order

SH waves in the ΓX and ΓM propagation direction are the fundamental first modes in the band structure.

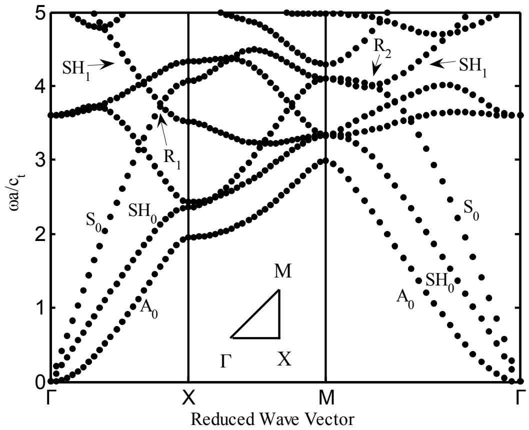

Fig.1: Dispersion curves of plate waves in a phononic crystal with the square lattice symmetry. (Al cylinders in a Ni background; f=0.6, $h/a = 0.8$).

When the plate waves propagate along ΓX, the SH wave and Lamb waves are decoupled and the fundamental symmetric Lamb wave (S_0) crosses over the SH_1 wave at point denoted R_1 in Fig. 1. To further examine whether this intersection is real or apparent, we have calculated the displacement fields associated to SH_1 and S_0 modes, propagating along ΓX. The results are displayed in Fig. 2 where we show the magnitudes of the component u_x for S_0 and u_y for SH_1. For any wave number in ΓX, both components continuous and R_1 is actually a real cross point.

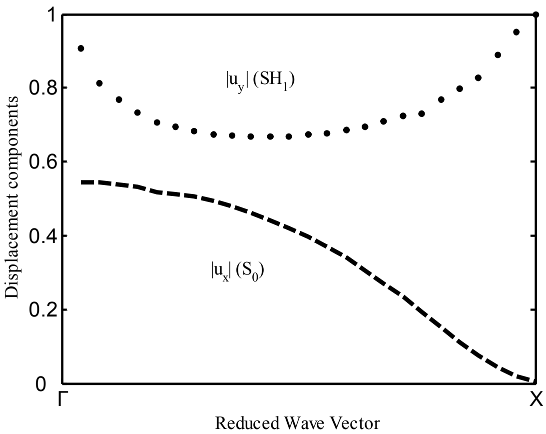

Fig.2: Displacement components of SH_1 and S_0 along ΓX.

We have then calculated the lowest-order dispersion curves for the plate waves with **k** vector making an angle $\varphi = 15°$ with respect to the direction ΓX in the reduced Brillouin zone. The results are shown in Fig. 3. As soon as the angle φ

departs from $0°$, and because of the anisotropy of the effective velocity in the isotropic/isotropic phononic crystal[5], the sagittal plane is no longer a plane of mirror symmetry. As a consequence, there is no longer a family of SH modes independent from the flexural and dilatational modes: all partial waves are coupled and the free plate modes can only be classified as symmetric or antisymmetric with respect to the mid plane of the plate. In that case, as shown in Fig. 3, a splitting occurs at points T_1, which are equivalent to the crossing point R_1 in Fig. 1 ($\varphi = 0°$).When the plate waves propagate along ΓM ($\varphi = 45°$), the sagittal plane is again a plane of mirror symmetry, just as for propagation along ΓX ($\varphi = 0°$). Therefore, the dispersion curves of the free plate vibrations can again be classified into the three families: flexural, dilatational, and SH, as show in the right panel in Fig. 1 where the shear horizontal mode SH_1 intersects the fundamental dilatational mode S_0 at point R_2, similar to point R_1 in the left panel .

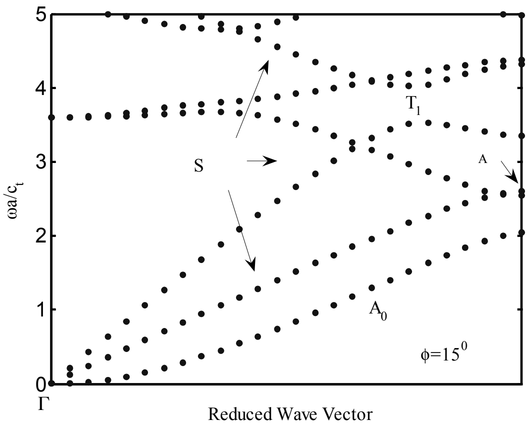

Fig.3: Dispersion curves of plate waves for $\varphi = 15°$ angle.

To further understand the peculiar behavior of the plate modes in T_1, we have computed, for $\varphi = 15°$, the displacement fields in the thickness of the plate, below a selected point in the unit cell: the center of the Al cylinder. Shown in Fig. 4 are the relative amplitudes of the displacement fields calculated at points A. In Fig. 4, dotted-, dashed- and full-lines refer respectively to u_x, u_y and u_z. It is clear from this figure that the displacements are either symmetric or antisymmetric with respect to the mid plane of plate. Indeed, at points A both real and imaginary parts of u_z have antisymmetric variations, whereas u_x and u_y exhibit symmetric behaviors. This analysis confirms that the vibrational mode at points A is symmetric Lamb waves.

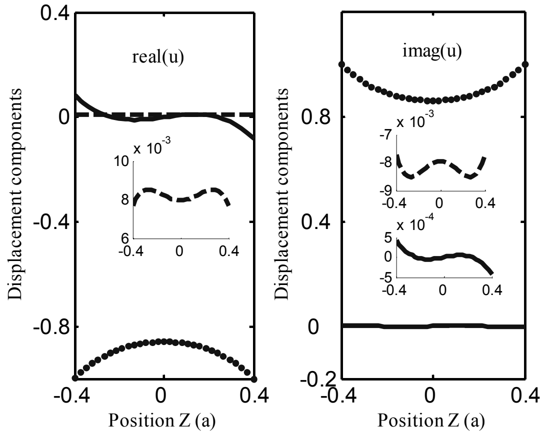

Fig.4: Displacement components of plate waves propagating in a Al/Ni phononic crystal (f=0.6, $h/a = 0.8$) calculated at points A in Fig.3, as a function of the distance from the mid plane. The unit of the horizontal axis refers to the lattice parameter a; dotted-, dashed- and solid lines correspond respectively to u_x, u_y and u_z.

In conclusion, we have investigated the propagation of plate waves along the all directions of the irreducible Brillouin zone of a phononic crystal thin slab. Along the ΓX or ΓM directions, SH modes do not couple to the Lamb waves polarized in the sagittal plane. Between ΓX propagation and ΓM propagation direction, SH modes convert to Lamb wave modes and couple with the flexural and dilatational modes, giving rise to a splitting of the mode.

a) Corresponding author
Email address: a), chen99nju@gmail.com

References

[1]B. Bonello, C. Charles, and F. Ganot, Appl. Phys. Lett. **90,** 021909 (2007).

[2]J. J. Chen, K. W. Zhang, J. Gao, and J. C. Cheng, Phys. Rev. B **73,** 094307 (2006).

[3]J. H. Sun and T.T. Wu, Phys. Rev. B **76**, 104304 (2007).

[4]Z. L. Hou and B. M. Assouar, Phys. Lett. A **372,** 2091.

[5]Z. L. Hou, F. G. Wu, X. J. Fu, and Y. Y. Liu, Phys. Rev. E **71**, 037604 (2005).