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Temporal response of a simplified bidimensional numerical model of the cochlea

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Within the frame of a study related to **bone conduction hearing**, numerical simulations have been driven in time domain, with the aim of comparing the cochlear partition displacement through different stimulations. An oversimplified 2D model of cochlea is used. It is first excited with pulses centered on various frequencies with a localisation of the source which is analogous to the position of the oval window. Secondly, new sets of calculations introduce different localisations and/or spatial extensions of the sources. An analogy with seismology being adequate when our purpose is to simulate the solid (cochlear partition) – fluid (perilymph) coupling, a finite difference numerical simulation of elastic waves propagation has been used to observe the movement of the simplified basilar membrane. Results of the propagation of a single pulse along the model will be presented and discussed through information available in literature.

1 Preliminaries

Since Békésy [1], it has been regarded that the cochlea has a similar response (apart from applying a complex gain parameter to the source) when excited via air (AC) or bone conduction (BC) cf. S. Stenfelt 2007 [2]. However clinical observations show that anatomical or physiological modifications from standard normality in the environment of internal ear (middle ear, vestibule, internal cranial pressure…) surely induce important differences in perception, mostly in the range of low frequencies (below a threshold varying with population between 500 and 1500 Hz).

Moreover, at higher frequencies, the whole of the skull is supposed to vibrate. S. Stenfelt & Al have conducted lots of experiments for the last 20 years [3].

The way leading to an auditory response remains an open question and a source of controversy: bone conduction, liquid transmission, resonant modes of the brain [4, 5].

These studies, being either a source of a deep disturbance or a way to rehabilitating a partial auditory function [6], are conducted in the medical field. We are not aware, though, of any such study as a physical topic.

Beside, in the ultrasonic range of frequencies, some recent studies of Lenhardt & Al (since the 80's) [7] and Nakagawa & Al (2K's) [8] tend to explain human's ear perception of a signal when a modulated ultrasound is applied on the temporal bone.

2 Purpose of our work

The purpose of this study is to get a model allowing comparison of the responses to different protocols of excitation, in terms of localization and extension of the sources. That model will be to undergo important modifications.
However, in a first approach, it is not worth considering the whole of the internal ear including the organ of Corti. Thus, only the basilar membrane displacement, as initiating the process, will be regarded. Our model will be passive and linear. As a note: The presence of the organ of Corti will be mechanically taken into account by ''loading'' the basilar membrane mass and stiffness parameters. It will be referred to as "cochlear partition".

Speculating on the principle of the internal ear being a receptor-emitter transducer, according to what is known since the oto-emissions discoveries, we set that the cochlear response will depend on its output impedances. The later will be likely to vary as boundary conditions.

The frame of the study is composed of three main lines: analytical, numerical and physical modeling.

In this paper, we will present the numerical modeling and briefly describe the two other models.

3 Geometrical representation

Roughly simplified, it is a parallelepipedic thick and rigid box. Its centered hole is filled with water and separated at middle height with an elastic membrane clamped on three edges. The fourth edge is free and supposed to be the helicotrema. Oval and round windows are represented with two membranes at abscissa $x=0$ (Fig.1). They both present an opening to the outside of the model (not shown on the figure), allowing to adapt a medium of given impedance.

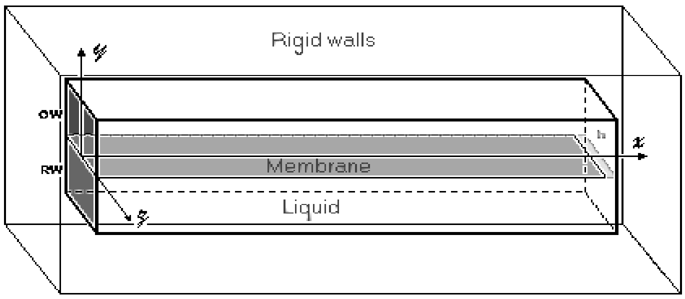

Fig.1 Simplified geometrical model

4 Analytical approach

4.1 Hypothesis

In cochlea, the fluid-structure coupling is weak. That allows separated studies of fluid and membrane behavior. For the fluid, the equations will take into account the presence of the structure as an additional force.
The very small amplitude of the basilar membrane displacement leads to make no distinction between Lagrange and Euler coordinates.
The perilymph is a non-viscous and incompressible liquid; its movement is irrotational.
The cochlear partition viscous damping is ignored for being balanced (and even overcompensated) by the active process amplification of Corti's organ. It is classically regarded as a battery of forced and coupled oscillators.

4.2 Equations summary

Fluid ↓ ***Structure*** ↓

Euler + boundaries conditions →Velocity potential →Pressure field →Applied forces →Forced oscillator equation

↑

Green's functions of the model in response to the sources

+ Green's function of the coupling of partition elements via fluid

This approach has been inspired from J.B. Allen [9], S.T. Neely [10], R. Nobili [11], C.R. Steele [12] and M.M. Sondhi [13] work for the last 25 years.

In our study, Green's functions have to be calculated for a multiplicity of sources distributed all around the geometry. As a linear model its response will be a summation of the contributions of all sources.

5 Physical modeling

5.1 Theoretical aspects

Existing experimental models are usually very small. Ours will be scaled about 40/1. The dimensional analysis is base upon Sir J. Lighthill meticulous opuscule conclusions in 1981: The whole of the cochlea is supposed to behave as presenting a critical layer for each specific frequency of excitation [14].

One of the main characteristic of a critical (resonant) layer is the dispersion curve that differs from that of a classical wave guide.

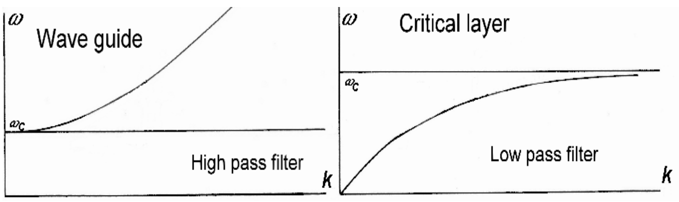

Fig. 2 Dispersion curves

In this approach, the weak coupling allows writing the equations of the vibrating partition with an additional inertia due to the movement of the fluid.

5.2 Equations

Let S be the cochlear partition volumetric stiffness and m and $m_f(k)$ the partition and fluid inertias.

The later varies greatly with the local wavenumber as described in [14].

Then the dispersion equation is:

$$\omega^2 = \frac{S}{m+m} = \omega_r^{\ 2} \frac{1}{1+{m_f}/{m}} \qquad (1)$$

where ω and ω_r are the excitation and local resonance angular frequencies, respectively. Eq.(1) leads to:

$$\frac{\omega}{\omega_r} = \left[\frac{1}{1+{m_f}/{m}}\right]^{1/2} = \frac{\textit{excitation frequency}}{\textit{natural frequency}} \qquad (2)$$

Eq.(2) is analogous to the inverse of a Strouhal number.

A classical definition of the Strouhal number is:

$$St = {L_0}/{V_0 T_0} \qquad (3)$$

when referring to a fluid (at velocity V_0) flowing around an obstacle (of length L_0) and generating turbulence (period T_0). It is the ratio: non-stationarity time / advection time or turbulence frequency / transport frequency. This is very similar to our problem.

Hence the dynamical similitude of our model as a similitude of dispersion will be based upon this so-defined "Strouhal" number.

6 Numerical model and its temporal responses

6.1 State of the art

In literature, a great number of numerical models of cochlea exist that exhibit results to an airborne conduction. Most of them also include the whole or part of the scala media.

However we found only one numerical model simulating an excitation by bone conduction. This model has been very recently developed by F. Böhnke & al [15].

They use a finite elements modeling with a compressionnal mode of excitation and the shape of the model is very realistic, 3D and coiled (according to J. Tonndorff [21], a compressionnal or an inertial mode of excitation occur by BC).

Some modifications at the windows show basilar membrane responses that are compatible with clinical observations. For example an opening of the oval window (removal of the stapes footplate) reveals an increase of the basilar membrane displacement by 9dB at 1500Hz.

This model does not include the surrounding bone and: "the actual stimulation at the outer bone is in phase for all areas and has equal amplitude along the complete outer wall. The stimulating signal which arrives at the outer wall will have a frequency dependent phase shift and might differ in its amplitude from place to place" (quotation).

This is precisely what is planned to do in our model with an excitation being applied directly onto the external surface of the bone.

Most of existing models, including the one mentioned above are excited in frequency providing a modal response of the membrane. To our knowledge there is no physical or numerical model response in the time domain. On the contrary, there exist some experimental data about the transient response of the basilar membrane such as that of Robles & Al [16].

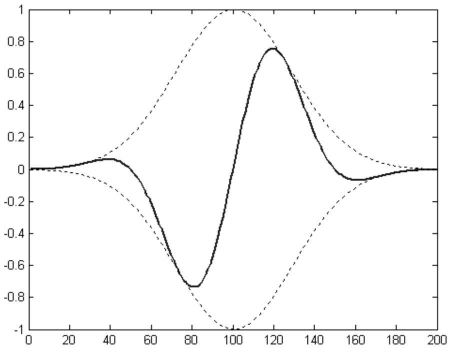

Fig. 4 Excitation at
central frequency f = 10 kHz

6.2 Model of cochlea

We use a finite differences method. The software was developed by M. Tanter [17] and N. Dominguez [18] in order to compute the propagation of waves in anisotropic media. The fluid-structure coupling being a typical situation encountered in seismology, the Virieux scheme for propagation of elastic waves has been used.

In order to introduce many variants in our model, it was necessary to shorten the computation time. Thus, our very simplified system is similar to fig.(1), but in two dimensions only: x, y plane.

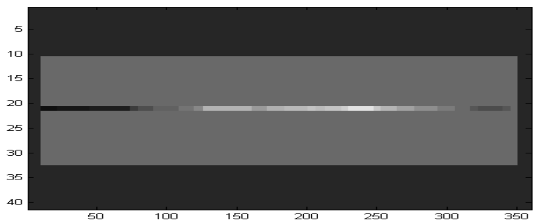

Fig.3 2D system
The cochlear partition shows variable
parameters all along the model
The black surrounding part simulates the temporal bone
while the grey medium represents the perilymph

The windows are simulated as internal boundary conditions with a reflection at the oval window and a zero pressure at the round window.

For the excitation and duration of simulation, we refer to Robles & Al observations [16]: a single pulse of 150 µs induces a response of the cochlea for 15 to 25 cycles.

Our signal is a sine windowed by a Gaussian for about 2 periods and all simulations last 8ms. Amplitude is normalised to 1. As a consequence, when frequency decreases more energy is provided (Fig. 5).

The model has not yet been excited with low frequency signals. It will be done for bone conduction excitation.

6.3 Temporal response to AC excitation

These simulations aim to validate the model by comparing its response to the classical one.

The source, localised at the base is acting as a piston with a longitudinal velocity. It is centered on frequencies 20, 10, 5 and 2 kHz.

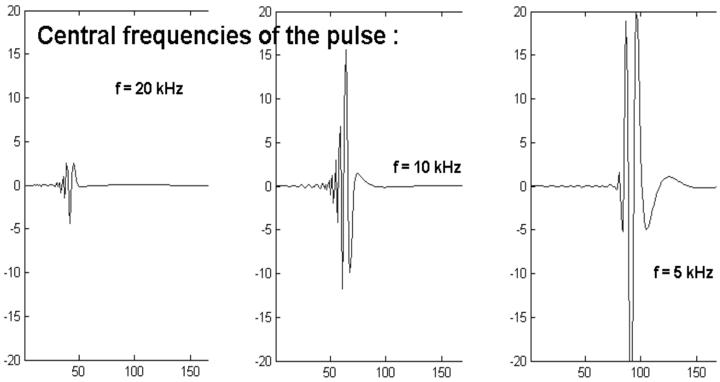

Fig. 5 Maximal displacement of the partition
Its position varies with central excitation.
Though it doesn't fit perfectly the tonotopic
map it is a good approximation
(Play attached movie: central f = 20 kHz)

No viscous damping has been included in the model either in solid, liquid or partition. However, while propagating, the amplitude of the response decreases, after passing the characteristic frequency position.

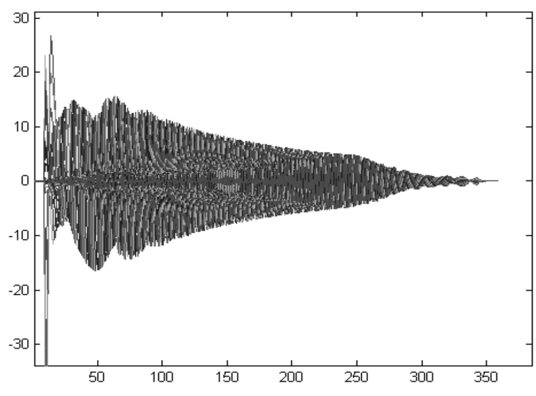

Fig. 6 Envelope of the travelling wave
(central f = 10 kHz)

6.4 Discussion

We are likely to expect that the envelope of the response shows a small slope before the characteristic frequency then a sharp slope after that point has been reached. This would be the case if the cochlea were excited via the oval window with a sinus signal [19].

In our model the aspect of the temporal response is different. First of all, the slopes are reversed. Secondly, as the signal delivered is a single pulse, the response occurs after its extinction.

Some mechanical parameters used in the model have been chosen to agree with Young's modulus and Poisson coefficient for soft to rigid biological tissues. They may need some adjustments due to the partition complexity.

Anyway, our purpose is not to get a realistic model of the cochlea but to compare two different modes of stimulation.

6.5 Comparison with literature

Experimental data by Robles & Al [16] are reported below. Our results (Fig. 4, Fig. 6) seem in good agreement with theirs.

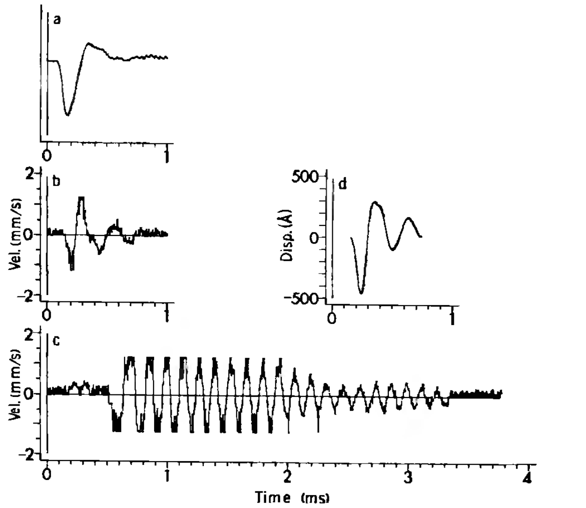

FIG. 10. Impulse responses of the malleus and basilar membrane in the same animal. (a) Acoustic click used as stimulus with a peak SPL of 103 dB. (b) Velocity curve showing the click response measured at the malleus using the click stimulus shown in (a) attenuated by 14 dB. (c) Click response measured at the basilar membrane using the stimulus in (a). (d) Displacement response of the malleus obtained by integrating the velocity response in (b).

Fig. 7 (from [16])
Stimulus (a) and basilar membrane response (c)

From another point of view, our final objective is the realization of a very simplified physical model for experimentation and we refer to Lighthill's work (see 5.1). He stipulates that scala vestibuli and scala tympani are analogous to elastic tubes of varying diameter.

T .Bryant Moodie & Al [20] have conducted a theoretical calculation of the propagation of waves in such tubes, as well as the dispersion effects. The liquid filling the tube is incompressible and inviscid. Their results are reported Figure 8.

ρ is the fluid density,

α and γ are arbitrary non dimensional parameters;

t_0 stands for the propagation time.

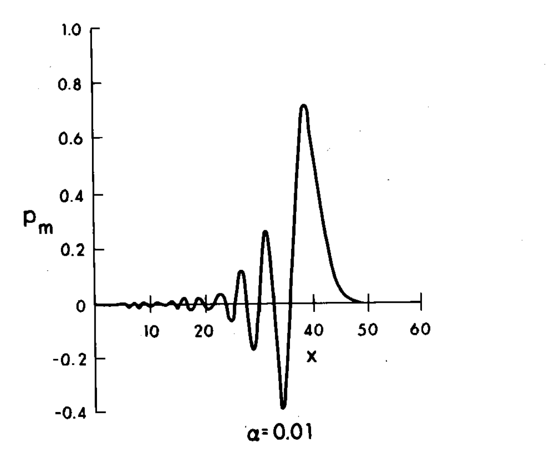

FIG. 9. Dispersed pulse for $\gamma/\rho = 1.1$, $\alpha = 0.01$, $t_0 = 40$.

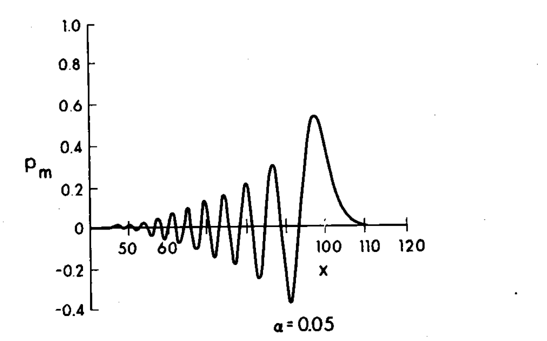

FIG. 11. Dispersed pulse for $\gamma/\rho = 1.1$, $\alpha = 0.01$, $t_0 = 100$.

Fig. 8 (from [20]) Dispersion of the pulse.

Our model shows a similar dispersion (See attached movie).

6.6 Bone conduction excitation

Different position and extensions of the source of excitation have already been experimented. New ones are to be tested. Figure 7 gives a summary of all the positions.

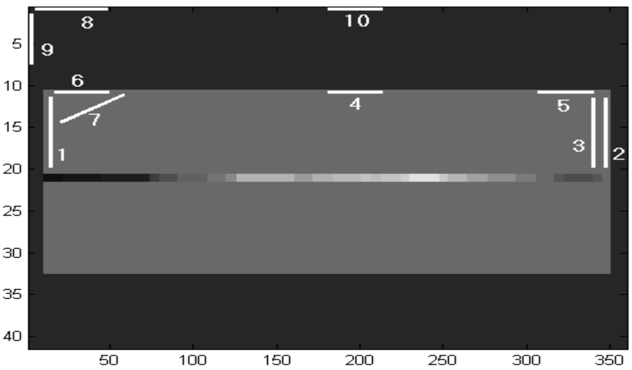

Fig. 9 Localisation of sources

Simulations with positions 1 to 4 and different extensions have been processed. Some results for position 1 are shown on Figure 5.

The reason for choosing such positions as number 2 and 4 to 6 is to validate Békésy's assertion that the response is independent of the localisation of the source.

Positions 8 to 10 are typically representative of bone conduction: excitation will be propagated in the whole of the bony box and distributed around the scalae.

Conclusions

The model has still to be refined. Its main fault is a lack of precision in the definition of the different media. For example, the cochlear partition is simulated by only one pixel and that may introduce calculation errors. At present, we are still experimenting to get a better model and for reasons of computation time, we maintain that low definition. That will not be the case for the definitive model. The progress of our study has been satisfactory till now.

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