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Use of the transmission line matrix method for the sound propagation modelling in urban area

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Many works have been carried out concerning temporal methods since a few years due partly to the computer revolution that opens major possibilities of solving intricate problems. This methods are notably well suitable to model phenomena occurring in complex areas, especially in urban spaces. Indeed, the long range sound propagation in urban areas imposes to take into account atmospheric attenuation and anisotropic sound speed gradients due to vertical temperature gradients and wind, as well as diffraction, reflections and diffusion on the frontages. The combination of all these phenomena is presented in this paper in two-dimensions trough the Transmission-Line Matrix (TLM) method.

1 Introduction

Nowadays, urban noise represents a major environmental pollution. Sound propagation modelling in urban spaces should allow a better understanding of the city plannings impacts. Various energy approaches can be used, like ray or beam tracing [1]. However, such methods are restricted to middle and high frequencies. Moreover, it is often difficult to implement the multiple phenomena that occur simultaneously in such areas. Indeed, sound propagation in urban spaces combines many phenomena like reflections, diffraction, diffusion or even sound absorption effects and is affected by micrometeorological conditions. Temporal wave approaches, like finite difference in time domain (FDTD) [2] or parabolic equation solving [3], seem very promising. Recently, some authors have shown that the Transmission-Line Matrix (TLM) method is also well suitable to model sound propagation in complex areas [4, 6]. This model is based on the Huygens' principle that enunciates that a wavefront can be decomposed in a set of secondary sources emitting wavelets; these wavelets can again be broken up in a new generation of secondary sources, and so on. This principle has been numerically adapted by Johns and Beurle [7] at the beginning of the seventies for some applications in electromagnetism and has been more recently applied to acoustics [8]. The present paper deals with the applicability of this temporal method for combining all phenomena affecting sound propagation in urban spaces.

2 Principle

2.1 Basics of TLM

For simplicity, the method is outlined in two dimensions, but can be easily extended in three dimensions. The TLM method is based on the discretization of the domain of interest by means of nodes regularly spaced out. Each node is linked with adjacent nodes through transmission lines of identical length Δl. A two-dimensional cartesian grid is so formed by a transmission lines network, four branches of characteristic impedance Z_0 being connected with each node.

Considering an unitary impulse arriving to a node from one transmission line (Fig. 1(a)), the impedance discontinuity met by the impulse at the node induces the reflection of one part of the field back through this incident transmission line, whereas the other part of the field is transmitted towards the three other transmission lines connected with this node (Fig. 1(b)). This step ensures the transformation of incident impulses to a node into scattered impulses from this node, according to the Huygens' principle.

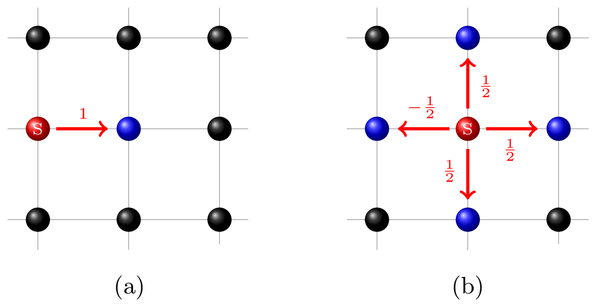

Figure 1: Simple case of (a) an incident unitary impulse from a source node (in red) and of (b) scattered impulses from the receiver node.

The field diffusion in the domain is performed using connexion laws that associate scattered impulses from a first node at a given time iteration with incident impulses to a second node at the next time iteration.
Consequently, the TLM method consists in both spatial and temporal discretizations.

2.2 TLM modelling of inhomogeneous and lossy media

The sound speed in the TLM network can be adjusted adding an open-circuited stub of length $\Delta l/2$ and of characteristic admittance $Y = \eta Y_0$ to the node [9]. A dissipative medium is modelled introducing another stub with an anechoic termination of conductance $G = \zeta Y_0$ (Fig. 2). The scattered impulses ${}_tS^n_{(i,j)}$ at branch n of the node (i,j) and at the time t are related to the incident impulses ${}_tI^n_{(i,j)}$ at the same node and at the same time t by the following equation:

$$ {}_t\widetilde{\mathbf{S}}_{(i,j)} = \widetilde{\mathbf{D}}\, {}_t\widetilde{\mathbf{I}}_{(i,j)}, \qquad (1) $$

where ${}_t\widetilde{\mathbf{I}}_{(i,j)}$ and ${}_t\widetilde{\mathbf{S}}_{(i,j)}$ are respectively the vectors composed of the incident impulses ${}_tI^n_{(i,j)}$ and by the scattered impulses ${}_tS^n_{(i,j)}$ through each transmission line n, expressed as

$$ {}_t\widetilde{\mathbf{I}}_{(i,j)} = \left[{}_tI^1;\, {}_tI^2;\, {}_tI^3;\, {}_tI^4;\, {}_tI^5\right]^{\mathrm{T}}, \qquad (2a) $$

$$ {}_t\widetilde{\mathbf{S}}_{(i,j)} = \left[{}_tS^1;\, {}_tS^2;\, {}_tS^3;\, {}_tS^4;\, {}_tS^5\right]^{\mathrm{T}}. \qquad (2b) $$

$\widetilde{\mathbf{D}}$ is a scattering matrix given by

$$ \widetilde{\mathbf{D}} = \frac{2}{\eta+\zeta+4}\begin{bmatrix} a & 1 & 1 & 1 & \eta \\ 1 & a & 1 & 1 & \eta \\ 1 & 1 & a & 1 & \eta \\ 1 & 1 & 1 & a & \eta \\ 1 & 1 & 1 & 1 & b \end{bmatrix}, \qquad (3) $$

with

$$ a = -\left(\eta/2 + \zeta/2 + 1\right), \qquad (4a) $$

and

$$ b = \eta/2 - \left(\zeta/2 + 2\right). \qquad (4b) $$

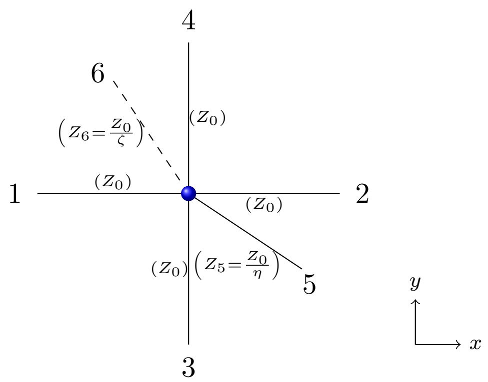

Figure 2: Two-dimensional TLM complex node consisting of four main transmission lines and of two extra branches: the line 5 is used to vary the sound speed in the TLM network (inhomogeneous atmosphere), and the line 6 is employed to introduce dissipation (atmospheric attenuation).

The diffusion of the acoustic field in the domain is obtained by the following connexion laws that expressed the incident impulses at time $t + \Delta t$ according to the scattered impulses at adjacent nodes at time t, Δt corresponding to the time required for an impulse to travel a transmission line (i.e. a distance Δl):

$$ {}_{t+\Delta t}I^1_{(i,j)} = {}_tS^2_{(i-1,j)}, \qquad (5a) $$

$$ {}_{t+\Delta t}I^2_{(i,j)} = {}_tS^1_{(i+1,j)}, \qquad (5b) $$

$$ {}_{t+\Delta t}I^3_{(i,j)} = {}_tS^4_{(i,j-1)}, \qquad (5c) $$

$$ {}_{t+\Delta t}I^4_{(i,j)} = {}_tS^3_{(i,j+1)}, \qquad (5d) $$

$$ {}_{t+\Delta t}I^5_{(i,j)} = {}_tS^5_{(i,j)}. \qquad (5e) $$

The total pressure ${}_tP_{(i,j)}$ at node (i,j) is given by:

$$ {}_tP_{(i,j)} = \frac{2}{\eta+\zeta+4}\left(\sum_{n=1}^{4} {}_tI^n_{(i,j)} + \eta\, {}_tI^5_{(i,j)}\right). \qquad (6) $$

The combination of Eqs (1), (5) and (6) leads to a finite difference form of the wave equation that permits to define the propagation speed of the wave front in the TLM network according to the sound speed c_0:

$$ c_{\mathrm{TLM}} = \sqrt{\frac{2}{\eta+4}}\, c_0. \qquad (7) $$

In the three-dimensional case, Eq.(1) remains valid. The vectors ${}_t\widetilde{\mathbf{I}}_{(i,j,k)}$ and ${}_t\widetilde{\mathbf{S}}_{(i,j,k)}$ are then composed of seven impulses, adding the impulses in the third direction. The scattering matrix $\widetilde{\mathbf{D}}$ becomes a 7×7 matrix, similar to Eq.(3), with coefficients a and b a bit different than ones given by Eqs.(4). Two connexion laws must also be added to Eq.(5). To limit the extra numerical cost related to the three-dimensional modelling, another spatial discretization form based on a tetrahedral mesh structure enables to reduce the size of the vectors composed of the incident impulses and of the scattered impulses, as well as the size of the scattering matrix, to the two-dimensional case ones [5].

3 Urban application

3.1 Anisotropic sound speed gradients

Despite the fact that no experimental studies have been carried out to estimate the micrometeorological effects on the sound propagation in urban areas, it can be assumed that vertical temperature gradients and wind can have a non negligible impact.
An inhomogeneous atmosphere (see §2.2) can be modelled with the TLM approach from the effective sound speed definition [6]:

$$ c_{\mathrm{eff}\,(i,j)} = \sqrt{\gamma R T_{(i,j)}} + \mathbf{W}_{(i,j)}.\mathbf{u}_{(i,j)}, \qquad (8) $$

where γ corresponds to the specific heats ratio, R is the perfect gas constant, $T_{(i,j)}$ represents the temperature at node (i,j), $\mathbf{W}_{(i,j)}$ is the wind vector and $\mathbf{u}_{(i,j)}$ is a unit vector giving the acoustic sound waves direction of propagation. The η parameter is then given by:

$$\eta_{(i,j)} = 4\left[\left(\frac{c_0}{c_{\mathrm{eff}\,(i,j)}}\right)^2 - 1\right]. \tag{9}$$

The direction of the sound propagation is determined by means of the ratio

$$\mathbf{u}_{(i,j)} = \frac{\mathbf{I}_{\mathrm{moy}\,(i,j)}}{\left|\left|\mathbf{I}_{\mathrm{moy}\,(i,j)}\right|\right|}, \tag{10}$$

where $\mathbf{I}_{\mathrm{moy}\,(i,j)}$ is the averaged intensity vector that is expressed as an arithmetic mean of the intensity vector around the central point (i,j), so that:

$$\mathbf{I}_{\mathrm{moy}\,(i,j)} = \frac{1}{(2\varepsilon+1)^2}\sum_{i-\varepsilon}^{i+\varepsilon}\sum_{j-\varepsilon}^{j+\varepsilon}\mathbf{I}_{(i,j)}, \tag{11}$$

with ε the span of the average. The intensity vector is calculated by the scalar product of the sound pressure and the particle velocity vector, as

$$\mathbf{I}_{(i,j)} = p_{(i,j)}.\mathbf{v}_{(i,j)}. \tag{12}$$

The x and y directional components of the particle velocity vector can be evaluated by the expressions [10]:

$$_t v_{x\,(i,j)} = \frac{_tI^1_{(i,j)} - {_tI^2_{(i,j)}}}{\rho_0\, c_{\mathrm{TLM}}}, \tag{13a}$$

$$_t v_{y\,(i,j)} = \frac{_tI^3_{(i,j)} - {_tI^4_{(i,j)}}}{\rho_0\, c_{\mathrm{TLM}}}. \tag{13b}$$

3.2 Atmospheric attenuation

Atmospheric attenuation can not be neglected when dealing with long range sound propagation, like it is the case in urban areas. The atmospheric absorption coefficient is frequency-dependent and can be exactly modelled adding digital filters to the original TLM configuration [11]. However, this technique seems very expensive in term of computational burden. As a first approach, the atmospheric attenuation can be implemented by means of the dissipative parameter ζ (Fig. 2), which can be calculated according to the atmospheric absorption coefficient α expressed in dB.m^{-1}:

$$\zeta = -\alpha\sqrt{2(\eta+4)}\,\Delta l\,\frac{\ln(10)}{20}. \tag{14}$$

3.3 Complex impedance boundaries

Boundary conditions are most often formulated in the frequency domain by a complex impedance. This kind of condition can be implemented in time domain models using a polynomial representation of the complex impedance written as [12]:

$$Z_b(\omega) = \sum_{n=-\infty}^{\infty} a_n\,[j\omega]^n. \tag{15}$$

Given that[1]

$$\frac{\partial f(t)}{\partial t} \underset{\mathrm{IFT}}{\overset{\mathrm{FT}}{\rightleftharpoons}} j\omega F(\omega), \tag{16a}$$

$$\int_{-\infty}^{t} f(\tau)\,d\tau \underset{\mathrm{IFT}}{\overset{\mathrm{FT}}{\rightleftharpoons}} \frac{1}{j\omega}F(\omega), \tag{16b}$$

the sound pressure at the boundary can be evaluated according to the particle velocity v_n normal to the boundary and a second order approximation of the complex impedance (Eq.(15)) by

$$\begin{aligned} p(t) = {} & a_0 v_n(t) + a_{-1}\int_{t_0}^{t} v_n(\tau)\,d\tau \\ & + a_{-2}\int_{t_0}^{t}\int_{t_0}^{t'} v_n(\tau)\,d\tau dt'. \end{aligned} \tag{17}$$

The resulting pressure on the boundary and the particle velocity vector component normal to the boundary can be expressed by means of the scattered impulses from the nodes located on both sides of the boundary, $_tS_{\mathrm{in}}$ standing for the scattered impulse from the node situated inside the simulated domain, and $_tS_{\mathrm{out}}$ standing for the scattered impulse from a virtual node placed outside. This leads to:

$$\begin{aligned} _tS_{\mathrm{out}} = \frac{1}{1+B_{0,1,2}}\Bigg[& {_tS_{\mathrm{in}}}\left(-1+B_{0,1,2}\right) \\ & + (B_{1,2})\sum_{\tau=t_0}^{t-\Delta t}\left({_\tau S_{\mathrm{in}}} - {_\tau S_{\mathrm{out}}}\right) \\ & + A_{-2}\sum_{t'=t_0}^{t-\Delta t}\sum_{\tau=t_0}^{t'}\left({_\tau S_{\mathrm{in}}} - {_\tau S_{\mathrm{out}}}\right)\Bigg], \end{aligned} \tag{18}$$

with

$$B_{0,1,2} = A_0 + A_{-1} + A_{-2}, \tag{19a}$$

$$B_{1,2} = A_{-1} + A_{-2}, \tag{19b}$$

$$A_0 = a_0\frac{c_{TLM}}{c_0}, \tag{19c}$$

$$A_{-1} = a_{-1}\frac{c_{TLM}}{c_0}\Delta t, \tag{19d}$$

$$A_{-2} = a_{-2}\frac{c_{TLM}}{c_0}(\Delta t)^2. \tag{19e}$$

[1] The acronyms FT and IFT stand respectively for Fourier Transform and Inverse Fourier Transform.

3.4 Absorbing boundaries

The simulation of sound propagation in open space imposes the use of absorbing boundaries (AB) to reduce unwanted reflections on opened extremities. A first technique consists in a Taylor's series expansion of the sound pressure at the boundary ${}_tp^{AB}$ according to the sound pressure ${}_{(t-\Delta t)}p_1$ at time $t-\Delta t$ and at the node adjacent to the boundary node located on the normal to the front, namely [13]:

$$
{}_tp^{AB} = {}_{(t-\Delta t)}p_1 + \Delta t \frac{\partial_{(t-\Delta t)}p_1}{\partial t} + \frac{\Delta t^2}{2!}\frac{\partial^2{}_{(t-\Delta t)}p_1}{\partial t^2} + \frac{\Delta t^3}{3!}\frac{\partial^3{}_{(t-\Delta t)}p_1}{\partial t^3} + o\left(\Delta t^3\right). \quad (20)
$$

A second order approximation of Eq.(20) leads to:

$$
{}_tp_2^{AB} = \frac{5}{2}{}_{(t-\Delta t)}p_1 - 2{}_{(t-2\Delta t)}p_2 + \frac{1}{2}{}_{(t-3\Delta t)}p_3, \quad (21)
$$

where ${}_{(t-2\Delta t)}p_2$ and ${}_{(t-3\Delta t)}p_3$ are the sound pressures calculated at previous time iterations and at adjacent nodes on the normal to the boundary, as presented at Fig. 3.

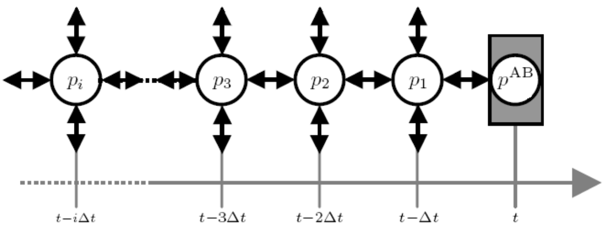

Figure 3: Taylor's series expansion of the sound pressure at the boundary according to the sound pressures at adjacent nodes on the normal to the front and at previous time iterations.

The major advantage of this technique is, in opposition to the Perfectly Matched Layer (PML) technique [14], that the simulation area is not enlarged. Indeed, initially proposed by Bérenger to solve unbounded electromagnetic problems [15], PML consist in an absorbing medium extending the spatial domain of interest.

4 Simulations

The two-dimensional TLM model presented above has been implemented with Matlab®. The connexion laws (Eq.(5)) are programmed using Toeplitz matrices [16] that permit to shift scattered impulses calculated at a given time iteration to obtain incident impulses to adjacent nodes for the next time iteration.
A canyon street section of 4 m width and 6 m height, illustrated at Fig. 4, has been simulated. The spatial discretization criterion is chosen as $\Delta l = \lambda_{\min}/40$, with $\lambda_{\min}$ the minimal wavelength of interest ($\lambda_{\min} = 48\ 10^{-2}$ m, i.e. a maximal frequency $f_{\max} = 1$ kHz), corresponding to a distance of $12\ 10^{-3}$ m between two adjacent nodes. The ground is implemented by means of a first order polynomial approximation of the complexe impedance condition (see §3.3). The polynomial coefficients are chosen in order to reproduce the ground effect obtained with the Delany-Bazley impedance model for a flow resistivity σ=1000 c.g.s.Rayls.m^{-1}, namely $a_0 = 12.4$ and $a_{-1} = 6.32\ 10^{-4}$ [17]. The street walls are perfectly reflective and the top is an absorbing boundary of second order (see §3.4). The atmosphere is homogeneous and non-dissipative. The source is located at 1 m high and 1 m from the left side wall (Fig. 5(a)).

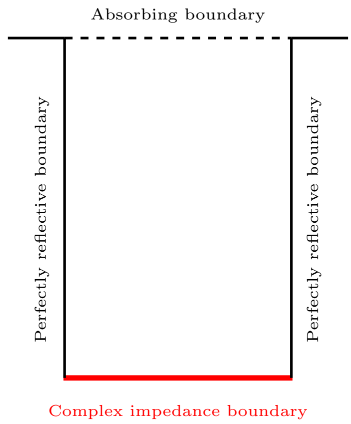

Figure 4: Street section geometry.

Different kind of sound sources, like sinusoidal or gaussian sources, can be easily implemented. Simulations presented below have been carried out with an unitary impulse emitted at a source node. The results obtained for the field of sound pressure at successive time iteration are presented at Fig. 5, showing that reflections on the street walls, as well as diffraction in the street corners, are accurately modelled. Moreover, the top of the street section absorbs the field of acoustic pressure entirely. The Delany-Bazley condition effect does not appear really in the simulations. However, the numerical scheme is stable, what is already promising.

5 Conclusion

The TLM method is a well suitable method to model sound propagation in confined domains as well as in open spaces. Diffraction, reflection and diffusion are well-modelled, and anisotropic sound speed gradients, atmospheric attenuation and spe-

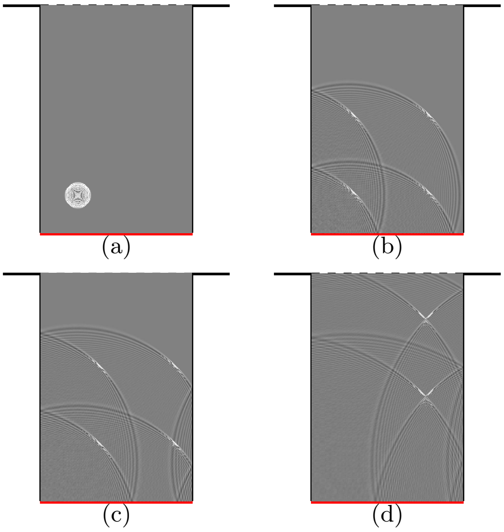

Figure 5: Sound pressure obtained for an unitary impulse emitted in the left down corner of the street section, at successive time iterations: after (a) 0.95 ms, (b) 8.80 ms, (c) 10.68 ms and (d) 15.63 ms.

cific boundary conditions can be implemented with this method. A two-dimensional TLM model has been presented and a simple example is given for modelling the sound propagation in a canyon street. The next step will consists in comparing a three-dimensional TLM model with numerical and experimental results.

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