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On the scattering of a plane wave by porous sound-absorbing strip

Alex De Bruijn

Independent Acoustical Consultant, Schonenvaardersstraat 18014, 7418 CC Deventer,
Netherlands
alex.bruijn@planet.nl

The edge effect of sound absorbing materials is a well-known phenomenon in room acoustics. It is the result of diffraction of sound waves on the edges of an absorbing sample. Without careful consideration of this effect it leads to inaccurate values for the absorption coefficient as measured in a reverberation chamber. Mathematical analysis has given a satisfactory picture of the phenomenon and the numerical data for the additional absorption as a function of frequency explain much of the deviation in the experimental data in the reverberation room. Somewhat surprising was that the extra-absorption is significant for oblique incidence. This means that diffusivity of the sound field is most important.

This paper deals with some specific mathematical details about the diffraction field, such as the field direct above the absorbing strip and in general the diffraction pattern in the space above the plane of the strip using FDTD methods.

1 Introduction

The scattering and absorption of the sound waves are relevant for an accurate assessment of the absorption coefficient of absorbing samples made by porous materials. The most appropriate facility in this respect is the reverberation room, in which finite patches of material can be tested using the reduction of the reverberation time. Already in the years fifty some doubts were cast on the method since the absorption coefficients found were quite variable, but worse, above the 100 %. It was clear that the sizes of the sample played some role. Through the work of *Kosten*[1] and *Kuhl*[2], it became clear that the edge length was important and that the diffraction on the edges of the impinging sound waves was responsible for an increased absorption. *Kosten* proposed a manageable formula to quantize the effect

$$\alpha_E = \alpha_{stat} + \beta \cdot E, \quad (1)$$

in which α_E denotes the absorption coefficient measured in a reverberation room with the aid of, for example, Eyring's reverberation formula; α_{stat} is the absorption coefficient for an infinite sample for random incidence of sound; E = relative edge length, i.e. the ratio between perimeter and area, and β is a parameter indicating the increase of absorption by the diffraction around the edge. See also *Ten Wolde* [3]. This latter factor is about 0.25 m^{-1} and frequency-dependent with a maximum at 400-500 Hz.

The additional absorption has attracted much attention of research, even already in the years fifty [4, 5]. Several mathematical models have used to obtain some insight into the effect and to get some quantitative data on the magnitude of the effect. The most well-known paper is due to *Northwood*[5].

Since it was evident from experimental data that it is by and large an true edge effect, *De Bruijn*[6] elaborated the model using an absorbing half-plane. The data for the additional absorption in the form β agreed quite well with the experimental data.

This paper discusses a better model: the strip of absorbing material embedded in a sound-hard infinite plane. The additional absorption can be calculated with the aid of an efficient algorithm. Especially the interaction of the two scattered edge waves can be evaluated. The boundary condition of the absorbing surface has been expressed into the Robin boundary condition. This is, however, a debatable condition. For this reason the scattered field has been observed via the FDTD method using a true porous material.

2 Mathematical background

The model starts with the assumption of a plane wave incident upon an absorbing strip. See Fig.1. A plane sound wave is impinging under arbitrary angle θ . The total field is written as the superposition of three contributions: the incident wave field $\Phi^{(i)}$, a field $\Phi^{(r)}$ reflected against an acoustically hard boundary of infinite extent and a scattered field $\Phi^{(s)}$. Hence:

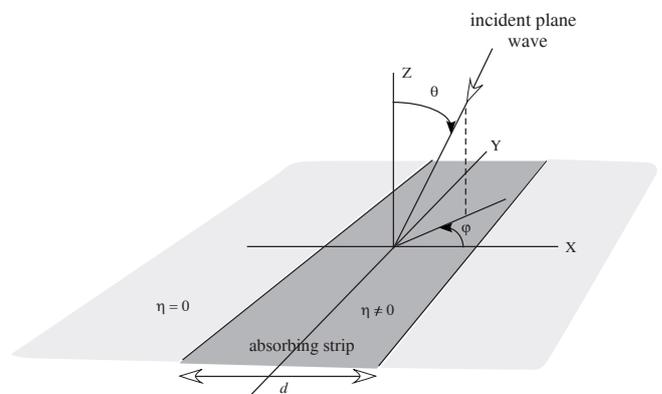


Figure 1: Configuration of the diffraction problem.

$$\Phi^{(t)} = \exp[j\alpha_o x + j\gamma_o y] + \exp[j\alpha_o x - j\gamma_o y] + \Phi^{(s)}, \quad (2)$$

in which $\alpha_o = k \sin(\theta)$ and $\gamma_o = k \cos(\theta)$; $\varphi = 0$.

Firstly a suitable representation for $\Phi^{(s)}$ is sought. An elementary solution to the Helmholtz equation is the plane wave:

$$\exp[-j\alpha x \pm j(k^2 - \alpha^2)^{\frac{1}{2}}]. \quad (3)$$

If $(k^2 - \alpha^2)^{\frac{1}{2}}$ is real, Eq. (3) represents a uniform plane wave; if on the other hand $(k^2 - \alpha^2)^{\frac{1}{2}}$ is imaginary or complex, Eq. (3) represents a non-uniform, evanescent plane wave. Now it can be shown that any solution of the Helmholtz equation can be brought into the form of an angular spectrum of plane waves [7]:

$$\frac{1}{2\pi j} \int_{\mathcal{L}} f(\alpha) \exp[\pm j\alpha x \pm j(k^2 - \alpha^2)^{\frac{1}{2}}] d\alpha, \quad (4)$$

by a suitable choice of the integration path \mathcal{L} and the function $f(\alpha)$. Such a representation is closely linked with the expression of an arbitrary function by means of a Fourier integral. The function $f(\alpha)$ is the spectrum function which specifies in terms of amplitude and phase, the weight attached to each plane wave of the

spectrum. Without loss of generality a suitable fixed path of integration can be selected so that the problem under consideration becomes a matter of determining the appropriate spectrum function $f(\alpha)$. A general solution is in the form:

$$\Phi^{(s)}(x, z) = \frac{1}{2\pi j} \int_{\mathcal{L}} f(\alpha) \exp\left[-j\alpha x - j(\kappa^2 - \alpha^2)^{\frac{1}{2}} z\right] d\alpha, \quad (5)$$

by a suitable choice of the integration path \mathcal{L} and the function $f(\alpha)$.

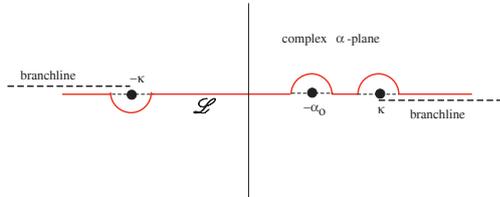


Figure 2: Path of the integration contour in the complex plane of α .

To reach a solution it is needed to formulate an expansion of the field just above the strip. A Fourier series in a specific form has been chosen for $z = 0$:

$$\Phi^{(t)}(x, 0) = \sum_{m=0}^{\infty} A_m \cos\left[\pi m \left(x/d - \frac{1}{2}\right)\right], \quad |x| < d/2, \quad (6)$$

where A_m is the complex amplitude of the field expansion of order m . The field expansion function $\cos[\pi m(x/d - \frac{1}{2})]$ is orthogonal, since for the case m is even the function is symmetric across the strip width and for m uneven the function asymmetric. There are now two representations of the total field above the strip, in which either the spectrum function $f(\alpha)$ in Eq. (5) or the constants A_m in Eq.(6) can be considered as unknowns. One of these unknowns can be eliminated with the aid of the boundary conditions in the plane $z = 0$:

$$\frac{\partial \Phi^{(t)}}{\partial z} = jk\eta \Phi^{(t)}, \quad |x| < d/2; \quad (7)$$

$$\frac{\partial \Phi^{(t)}}{\partial z} = 0, \quad |x| > d/2, \quad (8)$$

where η = the reduced specific acoustic admittance of on the surface of the material. Note that Eq.(7) is the Robin boundary condition. In fact, the elimination of the spectrum function $f(\alpha)$ has been chosen. A system of linear equations in which the amplitudes A_m occur as unknowns, is obtained. After some analysis an infinite system of equation for the amplitudes A_m is obtained:

$$A_m = b_m - \sum_{n=0}^{\infty} U_{m,n} \cdot A_n, \quad (m = 0, 1, 2, \dots) \quad (9)$$

in which:

$$b_m \stackrel{\text{def}}{=} 2\epsilon_m \cdot v_m(-\alpha_0), \quad (10)$$

$$U_{m,n} \stackrel{\text{def}}{=} \frac{k\eta \cdot d \cdot \epsilon_m}{2\pi} \int_{\mathcal{L}} \frac{v_m(\alpha) \cdot v_n^*(\alpha)}{(\kappa^2 - \alpha^2)^{\frac{1}{2}}} d\alpha, \quad (11)$$

where:

$$v_m \stackrel{\text{def}}{=} \frac{1}{d} \int_{-d/2}^{d/2} \exp(-j\alpha x) \cos[m\pi(x/d - \frac{1}{2})] dx \quad (12)$$

$$= j\alpha \frac{[\exp(-j\alpha d/2) - (-1)^m \cdot \exp(+j\alpha d/2)]}{d(\alpha^2 - \pi^2 m^2/d^2)}. \quad (13)$$

By virtue of the symmetry properties of $v_m(\alpha)$ and the path of integration \mathcal{L} , it appears that the matrix elements equal zero, if m is even and n is odd or m is odd and n is even. This system can be solved by the Jacobi iteration method, because the diagonal matrix elements are significantly dominant with respect to the off-diagonal elements. The only numerical problem is the evaluation of the integral elements $U_{m,n}$. This takes some effort to arrive at the correct values in view of the branch points at $\alpha = k$ and the pole at $\alpha = \alpha_0$.

It is a simple, efficient algorithm to calculate the field in any point as a function of the admittance, width and angle of incidence.

3 Field above the strip

3.1 Pressure amplitude just above the strip

An interesting subject is the sound pressure distribution on the absorbing strip due to the reflection of a plane wave. The pressure amplitude far outside on the hard surface is naturally twice the incident sound pressure amplitude. Closer to the strip the pressure must be lower because of the soft surface of the strip. This phenomenon yields an interesting interference pattern for the pressure on the surface.

On the surface of the strip the sound pressure amplitude is given per definition by Eq. (6).

The amplitudes have been solved by the iteration procedure of the previous section, hence the absolute value of the sound pressure (as a function of x) is simply:

$$\sqrt{|\Phi^{(t)}(x, 0)|^2} = \sqrt{\left| \sum_{m=0}^{\infty} A_m \cos\left[\pi m \left(x/d - \frac{1}{2}\right)\right] \right|^2}. \quad (14)$$

The absolute value of the sound pressure (normalized to the incident amplitude) is presented in two pictures: Figures 3 and 4.

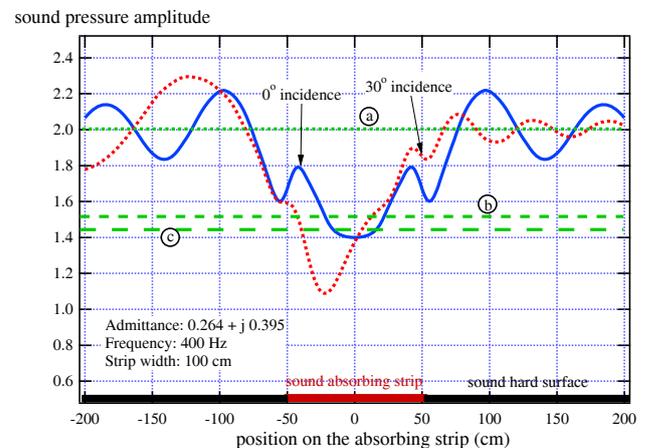


Figure 3: Absolute pressure amplitude just above the strip as a function of position just above the absorbing strip. Strip width = 1 m.

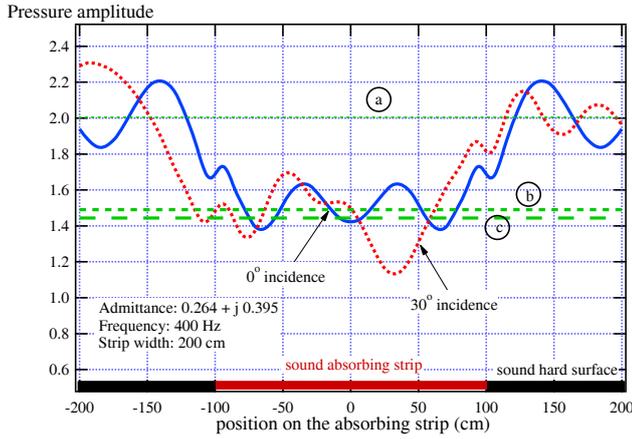


Figure 4: Absolute pressure amplitude just above the strip as a function of position just above the absorbing strip. Strip width = 2 m.

From the graphs it is evident that the pressure amplitude oscillates around the line of the absolute pressure amplitude for the infinite sample, while above the sound-hard surface the amplitude fluctuates around the value 2, being the value for sound-hard reflection. For normal incidence the pattern is symmetrical around the centre of the strip. Already on the strip near the edges the amplitude is higher than nominal value. This effect is more significant at oblique incidence. This will lead finally to an additional absorption.

3.2 Additional absorption

The absorption coefficient can be defined as the ratio of the absorbed power and the incident power, when no strip is present. The time-averaged power absorbed by a strip per unit width in the \mathcal{Y} -direction is:

$$P_a = \frac{1}{2} \frac{\text{Re}(\eta)}{\rho_a \cdot c_a} \cdot \int_{d/2}^{d/2} |\Phi^{(t)}(x, 0)|^2 dx. \quad (15)$$

The incident power is: $\frac{1}{2} \cos(\theta) / (\rho_a \cdot c_a)$. The true absorption coefficient becomes then:

$$a_{true} = \frac{\text{Re}(\eta)}{d \cdot \cos(\theta)} \cdot \int_{d/2}^{d/2} |\Phi^{(t)}(x, 0)|^2 dx. \quad (16)$$

Upon substitution of the Fourier series expansion for the total field onto the strip into Eq.(6) and evaluating the integrals we obtain:

$$a_{true} = \frac{\text{Re}(\eta)}{d \cdot \cos(\theta)} \sum_{m=0}^{\infty} |A_m|^2 / \epsilon_m, \quad (17)$$

(in which $\epsilon_m = 2$ for $m = 0$ and $\epsilon_m = 1$ for $m \neq 0$).

For very wide strips the absorption coefficient approaches the regular value for infinite surfaces:

$$a_{\theta} = \frac{4 \cdot \text{Re}(\eta) \cdot \cos(\theta)}{[\cos(\theta) + \text{Re}(\eta)]^2 + \text{Im}(\eta)^2}. \quad (18)$$

Due to the edge effect the absorbed power is increased considerably and the ratio can rise above the 100 %. From the figures below it is clear that for very narrow

strips the absorption coefficient can be significant, especially for oblique incidence. This indicates again the importance of the diffusivity of the field in a reverberation chamber.

It can be observed from Fig. 5, that for a strip width larger than 1 m, the additional absorption is marginal, certainly for normal and almost normal incidence. For smaller widths the absorption coefficient rises sharply. For very grazing incidence, it appears that the absorption coefficient is very significant, far more than the value of the infinite sample. This seems a bit strange, but it is in agreement with the results of the half-plane model. It is, however, the question whether the Robin boundary condition (Eq.(7)) is a good descriptor of the sound absorption at grazing incidence.

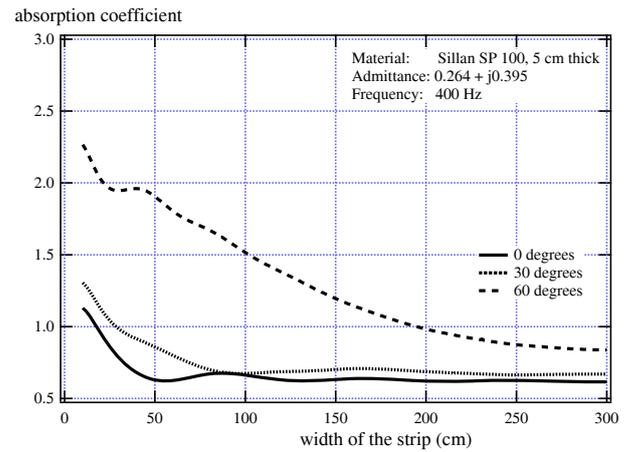


Figure 5: Absorption coefficient as a function of strip width. *Sillan* is an absorbing material.

4 Field in the space above the absorbing strip

The field is easy to calculate with a combination of the expression for the field and the amplitudes. From the two figures 6 and 7, the edge effect is clearly seen. Fig-

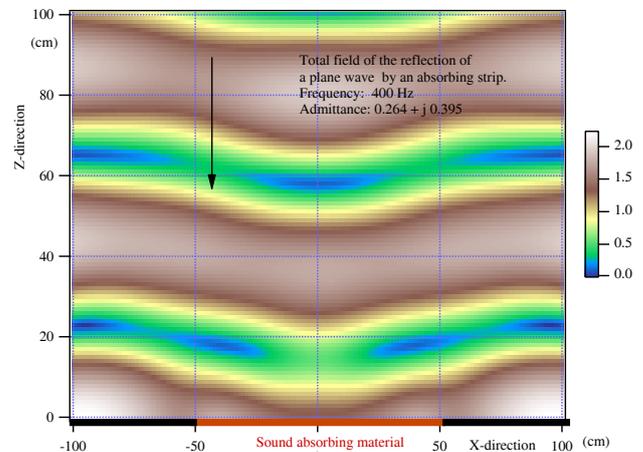


Figure 6: Total field above the strip.

ure 6 presents the interference pattern of the incident,

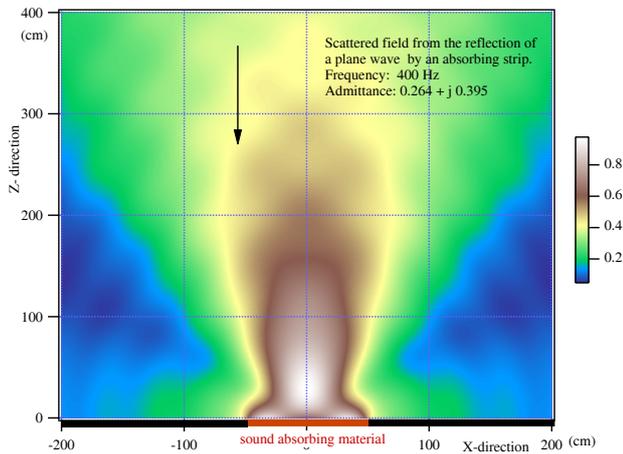


Figure 7: Scattered field above the strip.

reflected and scattered field together, a kind of standing wave. The influence of the soft strip is observed by the hollow in the interference pattern above the strip. The next figure presents the image for the scattered field $\Phi_s(x, z)$. From both edges the so-called edge wave is clearly visible.

5 FDTD approach

The *Finite-Difference Time-Domain* method is a powerful apparatus to visualize wave propagation and diffraction. It uses directly the two basic Euler equations and by taking finite steps in time and space the pressure and particle velocities in discretized points can be calculated in a straightforward manner. The method is basically simple but the practical realization is certainly not [8]. The big advantage is that diffraction can be made visible for obstacles which are not amenable for exact analysis. In the present problem the scattering of an absorbing strip can be made clearly apparent and in principle to be compared with exact data.

The basic formulation of the FDTD approximation uses a Cartesian staggered grid with pressures and particle velocity as unknown quantities. The acoustical pressure is determined at the grid positions $(m\Delta x, n\Delta y)$ at time $t = i\Delta t$, with Δx and Δy the spatial discretization and Δt the time discretization step. The indices m, n mark the spatial points: the index i marks discrete time. The two components of the particle velocity (u_x, u_y) are determined at positions half way between location of the pressures:

$$u_x^{[i+\frac{1}{2}]}[(m+\frac{1}{2})x, n\Delta y]; \quad (19)$$

$$u_y^{[i+\frac{1}{2}]}[m\Delta x, (n+\frac{1}{2})\Delta y]. \quad (20)$$

and at intermediate time $t = (i + \frac{1}{2})\Delta t$. The reason for this trick is that the pressure and velocities cannot be known at the same times and positions in view of the Euler differential equations

$$-\frac{\partial u_x}{\partial t} = \frac{1}{\rho_a} \frac{\partial p}{\partial x}; \quad (21)$$

$$-\frac{\partial u_y}{\partial t} = \frac{1}{\rho_a} \frac{\partial p}{\partial y}; \quad (22)$$

$$\frac{\partial p}{\partial t} = -\rho_a c_a^2 \left[\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right]. \quad (23)$$

The difference equations read now:

$$u_x^{[i+\frac{1}{2}]}(m+\frac{1}{2}, n) = u_x^{[i-\frac{1}{2}]}(m+\frac{1}{2}, n) - \frac{\Delta t}{\rho_0 \delta x} \times [p^{[i]}(m+1, n) - p^{[i]}(m, n)], \quad (24)$$

$$u_y^{[i+\frac{1}{2}]}(m, n+\frac{1}{2}) = u_y^{[i-\frac{1}{2}]}(m, n+\frac{1}{2}) - \frac{\Delta t}{\rho_0 \delta y} \times [p^{[i]}(m, n+1) - p^{[i]}(m)], \quad (25)$$

$$p^{[i+1]}(m, n) = p^{[i]}(m, n) + \frac{\rho_0 c_a^2 \Delta t}{\delta x} [u_x^{[i+\frac{1}{2}]}(m+\frac{1}{2}, n) - u_x^{[i+\frac{1}{2}]}(m-\frac{1}{2}, n)] + \frac{\rho_0 c_a^2 \Delta t}{\delta y} [u_y^{[i+\frac{1}{2}]}(m, n+\frac{1}{2}) - u_y^{[i+\frac{1}{2}]}(m, n-\frac{1}{2})]. \quad (26)$$

The numerical implementation is not easy. A difficult point is the reflection wave on the boundaries of the working area, which can be suppressed by special algorithms, like the PML of *Berenger* [8].

5.1 Impedance condition

For the present problem the formulation of the impedance on the strip is a difficult item. There are two approaches. The first one is the direct translation of the impedance of the surface in terms of a grid. This leads to [9]:

$$p(t) = Z_{-1} \int_{-\infty}^t u_n(\tau) d\tau + Z_0 \cdot u_n + Z_1 \frac{du_n(t)}{dt}, \quad (27)$$

assuming that:

$$Z(\omega) = Z_{-1}/j\omega + Z_0 + j\omega Z_1. \quad (28)$$

According to *Botteldooren* [9]:

$$u_y^{[i+\frac{1}{2}]}(m, n+\frac{1}{2}) = \alpha_a \cdot u_y^{[i-\frac{1}{2}]}(m, n+\frac{1}{2}) - \beta_a \cdot \frac{2\Delta t}{\rho_0 \delta y} \times p^{[i]}(m, n+1), \quad (29)$$

where:

$$\alpha_a = \frac{1 - Z_0 \cdot [\Delta t / (\rho_a \delta y)] + 2Z_1 / (\rho_a \delta y)}{1 + Z_0 \cdot [\Delta t / (\rho_a \delta y)] + 2Z_1 / (\rho_a \delta y)} \quad (30)$$

$$\beta_a = \frac{1}{1 + Z_0 \cdot [\Delta t / (\rho_a \delta y)] + 2Z_1 / (\rho_a \delta y)} \quad (31)$$

The second method is the direct model of the porous material. This model supposes that the skeleton of the solid is infinitely stiff, while the air in the pores vibrates. We introduce the following quantities:

- porosity h : this is the volume fractions of the open pores of the material; $1 - h$ is the volume fraction of the solid.
- the volume velocity per area unit v : v is the average of the particle velocity with respect to the material.

- Structure factor k_K . This indicates the meandering of the pores, which leads to an additional density increase.
- specific flow resistance σ . This quantity describes the pressure gradient to overcome the viscous friction.

One dimensional equation of motion for waves in the material reads:

$$-\frac{\partial p}{\partial x} = j\omega\rho_a \cdot k_K \cdot \frac{v}{h} + \sigma v; \quad (32)$$

for the equation of continuity:

$$-\frac{\partial v}{\partial x} = \frac{j\omega h}{(\kappa p_a)} p. \quad (33)$$

This leads to the wavenumber:

$$k = \omega \sqrt{(\rho_a \cdot k_K + \frac{\sigma h}{j\omega}) / (\kappa p_a)} \quad (34)$$

and to the specific impedance:

$$Z_s = \frac{1}{h} \sqrt{(\rho_a \cdot k_K + \frac{\sigma h}{j\omega}) \cdot (\kappa p_a)}. \quad (35)$$

5.2 Diffracted field

A pulse either as a plane or as a cylindrical wave is sent to a sound-hard surface with a strip of porous material. This pulse has the form of cosine train with three periods with some tapering in the beginning and at the end. The wave pattern shows clearly the oscillating character

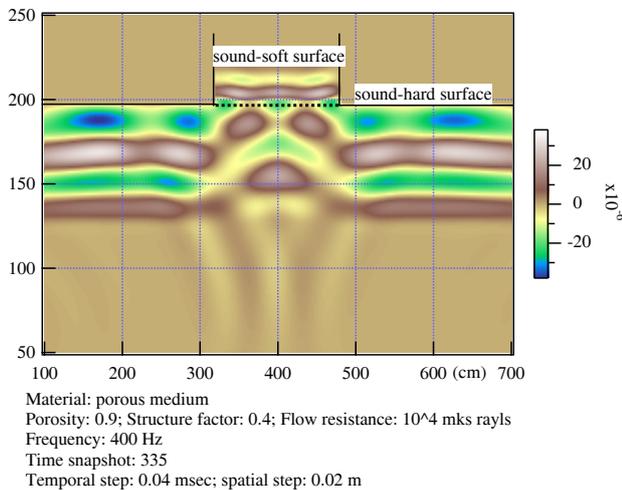


Figure 8: Diffraction of a plane-wave pulse by a porous strip. Note that part of the pulse has been reflected.

of the reflected wave on the surface. Only the magnitude is somewhat different from the exact solution. This suggests that the Robin boundary condition is just a simple approximation of the porous material.

6 Conclusions

- Diffraction of sound waves around the edge of the absorbing sample is responsible for the additional absorption, as observed in the reverberation room.

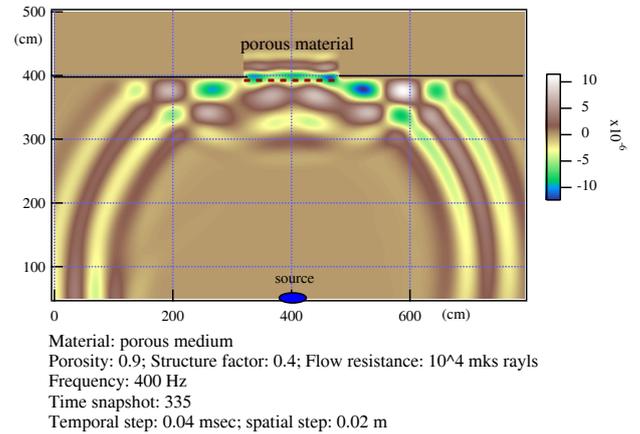


Figure 9: Diffraction of a cylindrical wave pulse by a porous strip.

- The sound pressure amplitude near the edge on the sample shows an increase towards the value of amplitude above the sound-hard source being twice the incident wave amplitude.
- It is debatable whether the Robin boundary condition describes the true diffracted sound field in an adequate way. The FDTD wave pattern using true Euler equations for the porous material suggests serious deviations.

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