Identification and characterisation of the scholte A and circumferential S0 waves from the time-frequency analysis of an acoustic experimental signal backscattered by a tube

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The problem of the acoustic diffusion by a tube is a theoretical problem, which had known since few years a considerable interest. A good comprehension of the acoustic diffusion makes it possible the study of the opposite problem. A tube in the acoustic diffusion field made many theoretical and practical works. These works showed in particular that the resonance phenomenons are directly related to the physical and geometrical properties of the target. The interest of the study presented in this paper is the identification of the circumferential waves. Several frequental analysis techniques were applied to characterize the target. These analysis techniques currently used present the major disadvantage to not be able to highlight the temporal structure of the field diffused by the target. Reciprocally the acoustic temporal analysis of the target can’t always separate the successive arrivals of the various type waves in the signal and to have information on their frequental contents. The joint time-frequency representations provide this need. In this paper, the time-frequency of Wigner-Ville is applied to analyse the acoustic experimental signal backscattered by a steel tube with radii radio 0.97 immersed in water. Through the Wigner-Ville image we have identified the Scholte A and the symmetric S0 circumferential waves.

1. Introduction

The acoustic characterization of an elastic target of simple geometrical form (plate, tube,...) by the mono-dimensional analysis requires a good knowledge of the temporal and frequental characteristics of the different circumferential waves [1-2]. These mono-dimensional techniques can lead to the analysis of the wave dispersion by simple elastic object. In the frequency domain, the analysis of a target allows us to isolate and identify the frequency resonances, but cannot display prominently the temporal structure of the backscattered field. Reciprocally, a temporal analysis of the acoustic impulse response allows us to separate some events but can’t provide information on their frequental structure. This is why, in this work, the time-frequency representation which account of the time and frequency parameters is used allowing us to overcome the limitations of the mono-dimensional techniques [3-6]. Among several solutions proposed, in this work the time-frequency representation of the Wigner-Ville is retained for their interesting properties [3-8]. These representations are applied to analyse the experimental signal backscattered by a steel tube of radii ratio b/a=0.97 (a is the external radius and b is the internal radius). One of the objectives of this representation is the identification of the target starting from their acoustic signatures. In this study, we used the Wigner-Ville to analyse the circumferential waves backscattered by a submerged tube. The interest of this study is to visualize the frequental evolution in function of time of the Scholte A and the symmetric S0 circumferential waves. In this study, we show the evolution of the dispersion velocity of these waves. Through the time-frequency image obtained, we have identified the surface wave "Scholte wave" A as well as the symmetric S0 circumferential waves.

2. Time-frequency representation of Wigner-Ville

The majority of the non-parametric time-frequency representations are represented by the Cohen class. This class includes inter alias the Wigner-Ville distributions. Among the large number of existing time-frequency representations, some authors have proposed using the Wigner-Ville. The choice of this particular representation results from its interesting properties in terms of acoustic applications [3-8].

To avoid the covering of frequental components in the time-frequency representation, in this study we use in the place of the real signal x(t) the analytical signal \( x_a(t) \), defined by the expression:

\[
    x_a(t) = x(t) + iH[x(t)]
\]

(1)

Where \( \hat{t}^\approx=1 \) and \( x(t) \) is the signal with real values and \( H[x(t)] \) its transform of Hilbert.

The spectrum \( F_a(k) \) of the analytical signal \( x_a(t) \) is given by :

\[
    F_a(k) = \begin{cases} 
        2F_a & 0 < k < N/2 \\
        F_a & k = 0, N/2 \\
        0 & N/2 < k < N
    \end{cases}
\]

(2)

The Distribution of Wigner-Ville associated to a signal \( x(t) \), of finished energy, is the function \( W_x(t,\nu) \) depending of the temporal \( (t) \) and frequental \((\nu)\) parameters. This distribution is given by the following expression [5-8]:

\[
    W_{x_a}(t,\nu) = \int_{-\infty}^{\infty} x_a(t+\tau/2)x_a^*(t-\tau/2)e^{-j2\pi\nu\tau} d\tau
\]

(3)

Where \( x_a(t) \) indicates the complex conjugate of \( x_a(t) \).
The transform called the Smoothed Pseudo Wigner-Ville "SPWV" is implemented in this work to attenuate the interference terms presented between the inner components figured in Wigner-Ville representation, which decrease the visibility of the time-frequency image. The SPWV use two smoothing windows $h(t)$ and $g(t)$. These smoothing windows are introduced into the Wigner-Ville distribution definition in order to allow a separate control of interference either in time ($g$) or in frequency ($h$). The expression of this representation is defined by \[3-4\]:

$$SPWV_x(t, v) = \int_{-\infty}^{\infty} h\left(\frac{\tau}{2}\right) \int_{-\infty}^{\infty} g(t - u) \ x_x(u + \frac{\tau}{2}) \ \cdot x_x^*(u - \frac{\tau}{2}) \ e^{-2i\pi vu} \ d\tau$$  \hspace{1cm} (4)

Where $h(t)$ is a smoothing frequential window and $g(t)$ is a smoothing temporal window.

### 3. The acoustic experimental signal backscattered by a tube

#### 3.1 Experimental set-up

The experimental study is led in a parallelepiped tank. To obtain exploitable signals, it is necessary to avoid the reflexions on the tank side. The experimental pulse echo set-up is presented schematically in figure 1. The experimental system, show in figure 1, constitutes of a pulse generator, a transmitting transducer, a receiving transducer and a Lecroy digital oscilloscope. The broadband transducer placed opposite the tube system is used successively as emitter and receiver and is excited by a short pulse. The sample is a steel tube with a radii ratio equal to $b/a=0.97$. The tube is insonified in a water immersion tank. The water density is $\rho_{\text{water}} = 1000 \text{ kg/m}^3$ and the celerity of the acoustic waves is $c_{\text{water}} = 1470 \text{ m/s}$ measured at the ambient temperature ($20^\circ\text{C}$). The transducer is a Panmetrics transducer with a centre frequency equal to $10 \text{ MHz}$ and a diameter of $10 \text{ mm}$. The pulse generator sends an electrical pulse to the transducer, which is transformed into an acoustic wave. After propagation in water, the incident acoustic wave is partly reflected by a tube and partly propagated around a cylindrical shell. The reflected acoustic signal, composed of a series of echoes, is converted into analogue electrical signal, and is amplified and collected by a Lecroy digital oscilloscope 931OM-300 MHz. The data are sent through an IEEE 488 interface to a personal computer.

The summation of the incident wave, the reflective wave $c$, surface waves "shell waves" $d$ (whispering Gallery, Rayleigh, ...) and Scholte waves (A) $e$ connected to the geometry of the object (figure 2). The waves $d$ and $e$ are the circumferential waves. For these waves one distinguishes the Scholte A and the symmetric $S0$ circumferential waves.

![Fig. 1 Experimental set-up](image1)

![Fig. 2 Mechanisms of the formation of echoes showing the specular reflection $c$ and circumferential waves ($d$ and $e$)](image2)

### 3.2 The acoustic experimental signal

The measuring equipment, which contains a pulse generator, which excites the piezoelectric transducer electrically, is illustrated by figure 3. The electric excitation signal of the transducer can be adjusted by selection of the repetition frequency, by using a Sofranel 5052PR pulse generator.
The square wave generator, electrically exciting the transducer, which is insonifying with normal incidence, synchronizes the generator. After propagation in water, the incident acoustic wave is partly reflected by the tube and is partly propagating around the circumference of the tube. The backscattered acoustic signal, composed of a series of backscattering echoes, is converted-back into an analogue electrical signal. The received electrical signal is transmitted through the cable of the transmission line, and arrives at the T/R (Transmitter/Receiver) from which the transmission signal was sent. Because of the time delay between the transmitted and the received signal, the last-one can time-gated and visualized at the screen of a Lecroy digital oscilloscope. This last is also equipped with software of signal processing. This software makes it possible to carry out the Fast Fourier Transform of the received signal. The digital oscilloscope is connected with a personal computer by an IEEE 488.2 interface with signal processing features, not available in the Lecroy digital oscilloscope. Software, carried out in our Laboratory "LMTI", allows the dialogue between the personal computer and the numerical oscilloscope [4]. This operation makes it possible to store the experimental signals in the memory of the personnel computer, in order to analyse them thereafter using other software.

3.3 Spectral analysis

The acoustic experimental signal illustrated in the figure 4 shows a sequence of echoes related to the circumferential waves (A and S0) propagating around the circumference of steel tube. Through of the experimental point acquired the calculation of the spectrum of the whole signal gives the "backscattered signal spectrum". If we remove the first echo from the experimental signal, the spectrum of the remaining signal gives the "resonance spectrum". The observation of the acoustic experimental signal shows a succession of components more or less distinct that one then seeks to identify. The different echoes finish by overlapping and, in these conditions, the identification and measurement of the echo times (this time depends on the radii of tubes a and b) becomes difficult, perhaps impossible. This constitutes a major disadvantage of the temporal approach. The observation of the signal of figure 4 shows the presence of these echoes (① and ②) propagating around the circumference of the cylindrical shell and corresponding to the Scholte A and the symmetric S0 circumferential waves, which are recognizable on this signal. These waves are also recognizable on the resonance spectrum that is obtained by the application of the Fast Fourier Transform on the experimental signal.

![Figure 3 Measuring equipment of the impulse method](image)

![Figure 4 Experimental signal backscattered by a steel tube with b/a=0.97](image)
Figure 5 illustrates the resonance spectrum of the acoustic experimental signal backscattered by a tube. On this resonance spectrum the peaks which appearing (waves A, S0) are in relation to the resonance frequencies (proper modes) of the tube [9]. In this domain, the analysis of an acoustic experimental signal backscattered by a tube allows us to isolate and identify the resonance frequencies, but cannot display prominently the temporal structure of the backscattered field. To determine the resonances, which appear on the resonance spectrum, we use the Insulation and Identification Method of the Resonances I.I.M.R developed by G.MAZE [1]. The use of the spectral analysis (figure 5) is very interesting in the field of the acoustic diffusion. Reciprocally, a temporal analysis (figure 4) of the acoustic experimental signal allows separating some events (as successive arrivals of different waves), but cannot provide any information on their frequential structure. This is why, in this paper, the time-frequency technique is used allowing us to overcome the limitations of the mono-dimensional methods. The time-frequency analysis takes into account both the time and the frequency parameters leading to synthetic images that allow us to follow the evolution of the frequential content of a wave as a function of time [3-4].

4. Wigner-Ville image of the signal backscattered by a steel tube

4.1. Wigner-Ville image

The experimental response is treated with a mixed representation called time-frequency representation of the Wigner-Ville. The acoustic experimental signal backscattered by a steel tube with b/a=0.97, is used in the following paragraph to determine the time-frequency image of the Wigner-Ville. The PWVL defined by the expression Eq.(4) is applied to analyse the acoustic experimental signal presented on the figure 4. The time-frequency image obtained is illustrated on the figure 6.

![Resonance spectrum of the experimental signal backscattered by a tube](image1)

Fig. 5 Resonance spectrum of the experimental signal backscattered by a tube, b/a=0.97

![Wigner-Ville image](image2)

Fig. 6 Wigner-Ville image of an experimental signal backscattered by a steel tube, b/a=0.97 (N=1024, smoothing temporal H= 256, smoothing frequential G= 5 windows)
4.2. Resultants and discussions

Figures 6 display time-frequency images corresponding to the Wigner-Ville of the experimental signal backscattered by a steel tube with $b/a=0.97$ presented on the figure 4. From this time-frequency image we observe two types of waves propagating around the circumference of the tube:

- The first one is the circumferential wave called “Scholte wave” $A$ (the frequencies between 0.25 MHz and 1 MHz)
- The second type is the symmetric circumferential waves $S0$ (the frequencies between 1.5 MHz and 3.75 MHz)

This figure show the synthetic time-frequency image, from which can follow the evolution of the frequential content of these circumferential waves $S0$ and $A$ in time. The dispersion (the evolution of group velocity in function of the frequency) of these waves is well localised in time-frequency plan. On this image, we observe also clearly the frequential evolution in time-frequency plan of the Scholte $A$ and the symmetric $S0$ circumferential waves. On this figure, the evolution of the symmetric $S0$ wave shows many paths of this wave. These paths are due to the impulse response of circumference of the tube. The high frequency part of the wave $S0$ arrives more belatedly than the low frequency part. This means that the group velocity of this wave reduces in function of frequency. In other respects, on time-frequency image of the figure 6, we establish that the energy of the circumferential waves $A$ and $S0$ is reduced after each round on circumference of the tube. This reduce energy is important in the high frequencies compared with the low frequencies. The time-frequency of the Wigner-Ville analysis permits to determine the group dispersion. Thus, the Wigner-Ville provides a useful tool to make measurement of the group velocity in situations where no theoretical results exist.

5. Conclusion

The methods applied in this work can be used like a new non-destructive measurement techniques to characterize a thin elastic tube. In this study, the acoustic signal backscattered by an elastic steel tube with $b/a=0.97$ is analysed using the Wigner-Ville time-frequency method. The techniques based on time-frequency analysis are actually suitable for studying the dispersion in non-stationary phenomena such as the acoustic signal backscattered by a cylindrical shell. This experimental signal is composed of several types of circumferential waves in particular symmetric $S0$ and the Scholte $A$ waves. We noted that the time-frequency image, of the Wigner-Ville make it possible qualitatively to follow the evolution of the frequential contents of the wave in function of the time and the identification of the different types of the circumferential waves ($S0$, $A$, …) propagating around the tube. The time-frequency of Wigner-Ville allowed to access to the group dispersion of the $S0$ circumferential wave. The results obtained by the Wigner-Ville analysis permits to determine the group dispersion. Thus, the Wigner-Ville provides a useful tool to make measurement of the group velocity in situations where no theoretical results exist.

References