Thickness determination of a multilayered system of different materials by natural frequencies

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This paper is focusing on the relationship between the resonant frequencies of a multilayered system of different isotropic elastic materials and the thicknesses of each layer. A relevant ultrasonic method for thickness determination of a multilayered system by resonant frequencies has been developed. The resonant frequencies are numerically calculated from the normally incident reflection coefficient of a "steel/epoxy resin/aluminum/thin polymer" structured lamination. Some resonant frequencies are sensitive to one layer thickness in certain range while the others are mostly invariable according to the results of sensitivity analysis. This phenomenon is explained by the defined strain energy ratio, and applied to our thickness inversion. The measured resonant frequencies of a specimen show good agreement with the theoretical ones. All the resonant frequencies are taken into account, and the simulated annealing method is employed for the thickness inverse calculation of the multilayered structure. The relative errors of calculated thickness of the each layer are -0.7%, 37.5%, -1.4% and 11.8%, corresponding to the real values: 1957μm, 16μm, 1913μm and 51μm.

1 Introduction

Multilayered media are being used widely in engineering applications, such as protective coatings, lap joints, laminated and filament wound composites, and clad-metal structures[1]. And it is necessary to get some or all of the materials properties for quality controlling. Many different ultrasonic NDE techniques have been developed for the characterization of adhesive joints[2-4], a thin layer on[5, 6] or under[7, 8] a substrate. In our previous work, we focused our attention on the problem of the thickness determination of a thin layer on the inaccessible side of a single layer[8] or a multilayered structure[9] by low frequency ultrasound. Here 'thin'[6] means that the thickness of the film is far less than the wavelength of ultrasonic wave propagating through the thickness direction. In the problem of thickness determination of the thin film under a multilayered structure(steel/epoxy resin/aluminum), the thickness of the thin film and the adhesive joint varies from several to hundreds of microns. The result of the thin film layer is restricted by the accuracies of the known parameters of the other layers, especially the thickness and the longitudinal wave velocity. This problem becomes more difficult due to the complexity of the multilayered structure with different materials and the fact that the thin film is on the side opposite to that of ultrasound incidence. In this paper, all the thicknesses of the multilayered system are inversed successfully from the resonant frequencies by employing the very fast simulated annealing[10] which is efficient in multi-parameter inversion problems.

2 The Forward Problem

2.1 The reflection coefficient

Fig. 1 is a schematic description of a laminated structure consisting of parallel layers, n in number, bonded together rigidly. Each layer denoted by i ($i=1,2,\ldots,n$) and the half-spaces denoted as 0 and $n+1$ on both sides of the system can be an isotropic and homogeneous solid or fluid. The material density, thickness and longitudinal wave velocity of the ith layer are respectively ρ_i, d_i and c_i. Φ_{in} and Φ_{out} are the normal incidence wave onto the surface of the system and the reflected wave, respectively. The reflection coefficient $R_n(f)$ from the n-layer system can be written in terms of $R_{n-1}(f)$ from a ($n-1$)-layer system by the transfer matrix method[11, 12]:

$$R_n(f)=\frac{(Z_n-Z_{n+1})/(Z_n+Z_{n+1})+R_{n-1}(f)\exp[4j\pi f\bar{t}_n(1+j\alpha_n)]}{1+[(Z_n-Z_{n+1})/(Z_n+Z_{n+1})]\times R_{n-1}(f)\exp[4j\pi f\bar{t}_n(1+j\alpha_n)]}\ . \quad (1)$$

Here f is the incident wave frequency, $Z_n=\rho_n c_n$, $\bar{t}_n=d_n/c_n$ and $\alpha_n=\beta_n c_n/(2\pi)$ are the characteristic acoustic impedance, time-of-flight and normalized attenuation of layer n, respectively, β_n is the attenuation coefficient of medium n. If the semi-infinite medium under layer 1 is air, $R_0(f)\approx-1$.

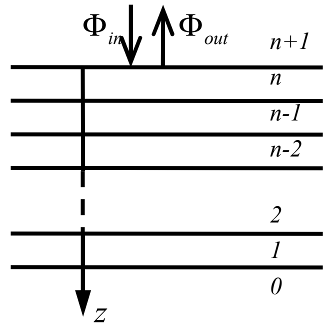

Fig. 1. Geometry of a n-layer system together in half-spaces denoted as 0 and n+1

For a multilayered system composed of a thin polymer film deposited underneath the double plates of steel and aluminium bonded by epoxy resin, n=4. The 0th medium and the 5th medium are air and a polystyrene delay layer, respectively. All the materials properties are in Table 1. The reflection coefficient is shown in Fig.2. In this figure, we marked the resonant frequencies as 'O'. Here, we call the frequency corresponding to the mth minimum of the amplitude spectra from the low frequency to the high as the mth resonant frequency, or the resonant frequency in the mth resonant mode. It is complicated and difficult to describe the resonant frequencies analytically when n is large, they are numerically calculated by searching the minima of the ultrasound amplitude spectra[7].

i	Materials	ρ_i (kg/m3)	c_i (m/s)	d_i (μm)	β_i (Np/ mmMHz)
5	Polystyrene	2060	2350		
4	Steel	7935	6049	1957	0.0017
3	Epoxy resin	1124	2350	16	0.2763
2	Aluminum	2700	6485	1913	0.0023
1	Polymer	930	1580	51	0.0531

Table 1. Physical properties of materials.

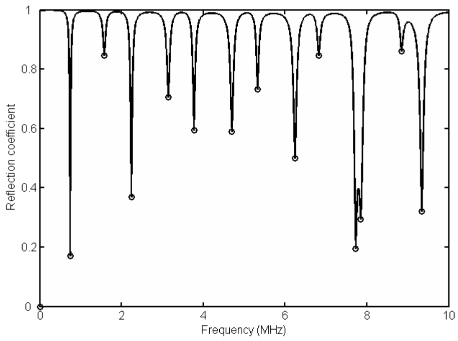

Fig.2. The amplitude spectra within the frequency range of 0-10MHz. 'O', the resonant frequencies of the system.

2.2 The resonant frequencies

The relationship between the resonant frequencies and the three acoustical parameters Z_i, $\bar{t}_i$ and α_i of layer i in the frequency range of 0-10MHz, made separately from Eq.(1). and Table 1 by numerical calculations, indicates that the resonant frequencies are very little affected by α_i and Z_i but influenced obviously by $\bar{t}_i$ .Take layer 1 for example, the resonant frequencies are nearly fixed as the normalized attenuation α_1 changes in the range of ±50%; and their changes are less than 0.51% when the characteristic acoustic impedance Z_1 varies within ±20%. However, some of the resonant frequencies shift down dramatically when the time-of-flight $\bar{t}_i$ increases. In other words, these resonant frequencies are sensitive to either the layer thickness or its longitudinal velocity. Practically, since we are concerned more with the layer thickness than its density, velocity or attenuation, it is assumed that the thickness of each layer is the variables with all the other paramters known in this problem. Fig.3. shows changes of the resonant frequency for each individual resonant mode against the variation of thin film (layer 1) thickness. For a specific mode, its resonant frequency is most sensitive to the layer thickness in a certain range. For example, the resonant frequency of the 5th mode shifts down significantly in the thickness range about 90µm to 140µm, while the resonant frequency of the 6th mode changes dramatically in the thickness range about 70µm to 100µm.

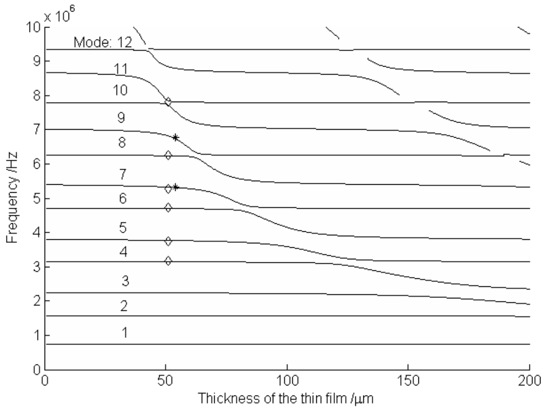

Fig.3. Resonant frequencies vs. the thin film thickness (layer 1). -, the prediction; *, the values with higher strain energy ratio in the thin film (see definition in Eq.(3).); □, the measured values.

2.3 Sensitivity analysis

For further analysis, the sensitivity of resonant frequency f_m to the layer thickness d_i is introduced as[5]:

$$S_{f_m,d_i} = \left| \frac{d_i}{f_m} \frac{\partial f_m}{\partial d_i} \right|, \tag{2}$$

which means that, the normalized change of f_m due to one unit normalized change of d_i. The normalized change of f_m and d_i are defined as $\varepsilon_{f_m} = \left|\partial f_m / f_m\right|$ and $\varepsilon_{d_i} = \left|\partial d_i / d_i\right|$, respectively, and connected by $\varepsilon_{d_i} = \varepsilon_{f_m} / S_{f_m,d_i}$ When S_{f_m,d_i} is small, it indicates that a small change of ε_{f_m} can lead to a evident change of d_i. Fig. 4 shows the sensitivity of the first 9 resonant frequencies to the thickness of each layer in a certain range.

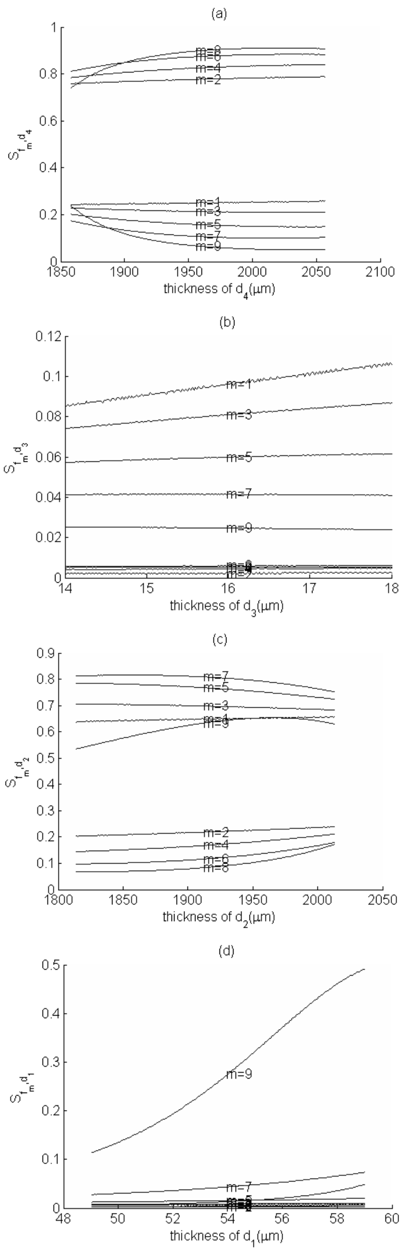

Fig.4. the sensitivity of the first 9 resonant frequencies to the thickness of each layer in a certain range. (a) steel layer; (b) epoxy resin layer; (c) aluminium layer; (d) polymer layer.

Different resonant frequencies are sensitive to each layer in certain rang. For the outer layer (layer 4), the S_{f_m,d_4} values are larger than 0.7 in all the even resonant modes while around 0.1 in other odd modes. For the inner layer (layer 2), on the contrary, the S_{f_m,d_2} values are quite larger in all the odd resonant modes. But for layer 3 and 1, most values of S_{f_m,d_3} and S_{f_m,d_1} are smaller than 0.01 while only a few of them are around 0.1. It can be concluded that all the resonant frequencies should be applied to inverse the thicknesses simultaneously. The relative errors in measurement of thicknesses of the layer 1 and 3 might be larger than the others. Here, we call the resonant frequencies with high sensitivity values to the thickness of layer *i* as the "sensitive resonant frequencies of layer *i*", while the other ones are "non-sensitive resonant frequencies of layer *i*".

2.4 The strain energy ratio

To give the physical interpretation of the sensitive resonant frequency of layer *i*, strain energy ratio is defined and calculated. In the steady static vibration model, the strain energy dominates the total energy in the lamination. For further study of the connection between the resonant frequencies of different modes and the thickness of the thin film, displacement amplitude in each layer and the strain energy[13, 14] are calculated by the transfer matrix method which we employed in the reflection coefficient deduction previously. We define strain energy ratio[9] σ_i^m in the *i*th medium for the *m*th resonant mode as:

$$\sigma_i^m = W_i^m / \sum_{i=1}^{n} W_i^m, (i = 1, 2, \cdots, n; m = 1, 2, 3, \cdots) \quad (3),$$

where the W_i^m is the strain energy in the *i*th layer for the *m*th resonant mode and $\sum_{i=1}^{n} W_i^m$ is the total strain energy in the *n*-layer system for the *m*th resonant mode. On assuming a unit displacement[13] of layer 4 at its upper surface, the strain energy ratio in each layer for the first 9 resonant modes are shown in Fig. 5.

Compared to Fig. 4, the sensitive frequencies (sensitivity values are larger than 0.1) in each layer correspond to high strain energy ratio, excepting layer 3. In layer 3, the highest strain energy ratio is $\sigma_3^2 = 1.5 \times 10^{-3}$ while the highest sensitivity value is $S_{f_1,d_3} = 0.1$. This phenomenon of 'disorder' between the strain energy ratio σ_i^m and the relevant sensitivity S_{f_m,d_i} occurs when the strain energy ratio is extra small (about 1/1000). The strain energy in layer 3 is extremely small due to its thin thickness and low density. In the high strain energy ratio mode, a disturbance of the layer thickness may cause an evident strain energy change, which dominates the composite vibration of the lamination and results in the notable resonant frequency shift; on the contrary, in the low strain energy ratio mode, the strain energy in the layer is quite negligible for any indication of the change in the layer thickness. These resonant frequencies with the high strain energy ratio are sensitive to the thickness of the layer, and they are called the *sensitive resonant frequencies* previously. The resonant frequencies with high strain energy ratios in layer 1 are marked as "*" in Fig. 3. So, compared with sensitivity parameter, the strain energy ratio is another reliable indicator for sensitive resonant frequencies when it is high. Fig.5(d) shows the capability of thickness inversion of the thin film under this multilayered media composed with different materials, which was achieved in our previous work[9].

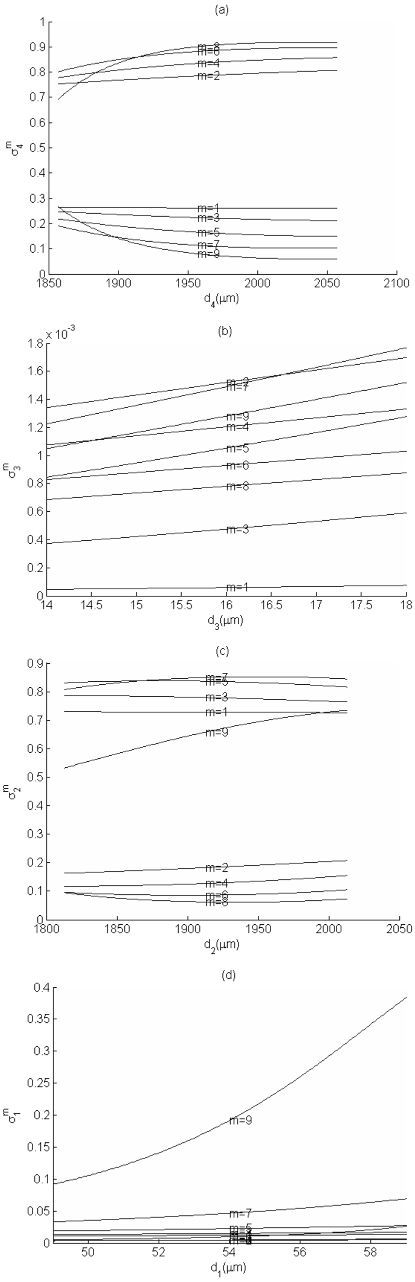

Fig.5. the strain energy ratio in each layer for the first 9 resonant modes in certain thickness ranges.
(a) steel layer; (b) epoxy resin layer;
(c) aluminium layer; (d) polymer layer.

3 Experimental setup

Measurement has been carried out on a "steel/epoxy resin/aluminium/thin polymer" structured specimen with a Panametrics Ultrasonic Analyzer 5800 and a composite piezoelectric transducer with a central frequency of 6MHz and an approximate bandwidth of 6MHz(-6dB level), as Fig. 6 shows. The Panametrics 5880 was used as pulser/receiver. Longitudinal wave was transmitted normally from the upper surface of the multilayered system through a polystyrene delay layer; reflected wave from the structure is received to a Tektronix TDS3012B oscilloscope and sent to a computer. A reference signal was take when the specimen was replaced by a thick block of steel. The reflection of coefficient was calculated by an FFT algorithm from the reference and reflected signals; the resonant frequencies are acquired by searching the minima from the amplitude spectra. Here, we used the polystyrene delay layer as a buffer other than water as else always does, because it makes the resonant frequencies more obvious in the amplitude spectra. The material properties of the specimen are presented in Table 1. All these experimental data are marked as "◇" in Fig. 3. The relative errors between the experimental resonant frequencies and the predicted values are less than 2.5%.

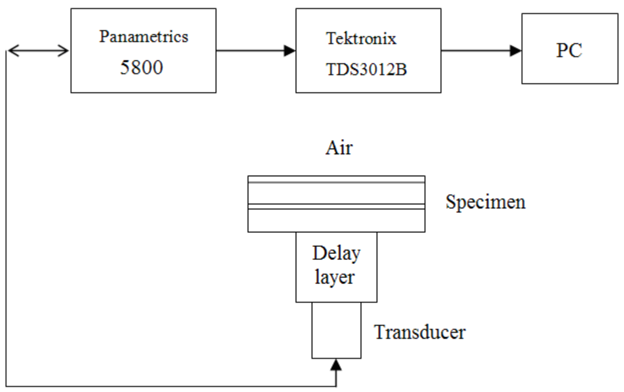

Fig. 6. Schematic of the experimental setup.

4 The Inverse Problem

4.1 The objective function

The objective function y is based on the well known least-square sense and defined as:

$$y = \sum_{i=1}^{N} (f_i - f_{0i})^2 / f_{0i}^2 , \qquad (3)$$

with $i=1,2,...,N$, f_{0i} is the measured resonant frequency value, N is the amount of measured values, and f_i is the calculated value with the assumed layer thicknesses. When y approaches to a minimum, the calculated resonant frequencies with the assumed thicknesses overlap with the measured ones, and we take these assumed values as the true thicknesses of the multilayered system.

4.2 Very fast simulated re-annealing (VFSA) and the inversion results

Simulated annealing[15] is an efficient method for optimization problems, and very fast simulated re-annealing[10] method is a robust method for multi-parameter inversion problem. Here, VFSA was employed for the inversion problem of layer thicknesses from the resonant frequencies we obtained.

All resonant frequencies including the sensitive ones obtained in our experiment (N=6) are taken into account for establishing a unique and reliable result. The results in Table 2 show good agreements with the measured thicknesses based on the VFSA method and the real values. The thickness-to-wavelength ratios (h/λ) are about 1/5 and 1/25 for the layer 1 with 51μm thick and the layer 3 with 16μm thick when the incident wave frequency is 6MHz. The relative error of layer 3 is larger than any of others because the thickness is too thin for the wavelength in it. The result of layer 3 could be better when a transducer of higher central frequency is used for the experimental measurement by the analysis of the sensitivity of some higher resonant mode to its thickness.

i.	1	2	3	4
Inverted thickness(μm)	57	1887	22	1944
Real Thickness*(μm)	51	1913	16	1957
Relative errors	11.8%	-1.4%	37.5%%	-0.7%

Table 2. Inverted thicknesses compared with the real values. (*, the corresponding measurement accuracy is 10μm)

5 Conclustion

The result shows the capability of thicknesses determination of a complex multilayered system with different materials by resonant frequencies from amplitude spectra. The most important three steps are: first, the theoretical calculation is suggested for accurate location of the sensitive resonant frequencies regions of each layer; second, a corresponding broad bandwidth transducer is indispensable for the amplitude spectra measurement and the resonant frequencies search; and the last, a reliable and rapid numerical method, like the SA method, is necessary for the thickness inversion. Actually, the precision of this thicknesses determination method of multilayered system is restricted by the accuracies of the known martial properties, especially the wave velocity of every layer. In this paper, it was assured by multiple measurements.

Acknowledgments

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