DELTAEC is also an acoustics teaching tool

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A major revision of the Los Alamos thermoacoustics code, renamed DELTAEC, was released in 2007. It takes advantage of a user-friendly, menu-driven Windows™ environment and has indigenous plotting capabilities. DELTAEC is a differential equation solver that analyzes one-dimensional acoustical networks defined by a series of "segments" representing ducts, compliances, speakers, etc. This talk will relate experiences using this software to teach a first-year graduate core course on acoustics in fluids. Examples include illustration of effective length and quality factor of Helmholtz resonators, as well as the more challenging standing wave solutions within a resonator of variable cross-section. The plotting feature allows immediate illustration of the pressure and velocity fields as well as power flow within the resonator. DELTAEC will also adjust gas mixture concentration to match a specified resonance frequency of fixed physical dimensions. Segments representing electrodynamic loudspeakers, radiation loading, and flow resistance in porous media will be used to demonstrate the coupled-oscillatory behavior of a bass-reflex enclosure's complex electrical impedance *vs.* frequency. The "thermo-physical property" feature provides fluid and solid properties at the students' choice of pressure, temperature and frequency, making it useful as a "handbook" for other assignments.

1 Introduction and Motivation

"An acoustician is merely at timid hydrodynamacist."

A. Larraza

The education of professional acousticians in the United States since World War II has approached the subject of wave propagation in fluids by introducing the continuity equation (*i.e.,* mass conservation) and the Euler equation (momentum conservation), supplemented by an equation-of-state (typically, the adiabatic gas law), as a means of deriving the wave equation [1-11]. Once the wave equation is obtained and the general form of the solutions have been exhibited, the fundamental hydrodynamic equations are treated as subsidiary to the wave equation. Much later in the "standard curriculum", typically after systematic treatment of reflection and transmission at interfaces, radiation and scattering, absorption and attenuation, and sound in three-dimensional enclosures., the subject of lumped-parameters systems (*e. g.,* Helmholtz resonators and filters) is addressed in a way that uses analogies to mass-spring systems and ignores the fundamental role of the hydrodynamic equations in creation of the concepts such as inertance and compliance.

That traditional pedagogic sequence for the "fluids stem" in early acoustics education is contrary to the one used for the introduction of mechanical vibration, which forms the other part of an introductory student's path through an acoustics education. Invariably, instruction in the "vibration stem" begins with analysis of the single degree-of-freedom damped and driven simple harmonic oscillators which transitions logically into multiple degree-of-freedom oscillators, then on to continuous systems of successive higher dimensionality and complexity (*i.e.,* strings, bars, membranes, plates, bulk and surface waves).

The purpose of this paper is to demonstrate that a course on waves in fluids can be organized in a way that starts by applying the first-order differential equations of hydrodynamics to elements that are small compared to the wavelength of sound to introduce the concept of "acoustical compliance" as a consequence of the continuity equation and "acoustical inertance" as a consequence of the Euler equation ***before*** the equations are combined to produce the wave equation. This approach is greatly enhanced by the unlimited availability of (free) DELTAEC software [12].

This approach also provides the opportunity to introduce thermal and viscous dissipation, first by examination of the results produced by DELTAEC and then through the introduction of the fundamental transport equations (*i.e.,* the Navier-Stokes and Fourier thermal diffusion equations) [13], rather than to postpone their introduction until later in the course when attenuation is covered and the fundamental dissipative processes are frequently ignored by introduction of an artificial hysteretic "relaxation time"[10].

This new approach has been used twice by the author in the first-year graduate course[1] offered simultaneously to resident and distance education students enrolled in the Graduate Program in Acoustics at Penn State. Based on both written course evaluations and conversations with students, this approach has been successful in producing a fundamental understanding of the physical principles and mathematical techniques required to apply those principles, as well as a facility with the DELTAEC software that appears to have served the students in subsequent course work and thesis research.

2 Compliance and Inertance

The pedagogical strategy used in this exploitation of the DELTAEC software is to move continuously between hydrodynamic theory, experimental measurements, and software modelling in both lecture notes and problem sets to "spiral" into a mastery of all three. As will be explained below, disagreements between the three approaches demonstrate the subtlety that is frequently omitted from standard textbook treatments combined with the *pro forma* "find and grind/plug and chug" homework assignments.

After a brief introduction to elementary ideal gas thermodynamic concepts (*e.g.,* simple kinetic theory, heat capacity and molecular degrees-of-freedom, and the adiabatic equation-of-state), the course addresses non-dissipative lumped-element systems by introducing the concepts of compliance and inertance. This initial effort follows the development sequence, notation, and equivalent circuit element representations of Swift [14].

To emphasize the behaviour of compliance as a "gas spring", the second (weekly) problem set is dedicated to the analysis of an experiment based on Rüchardt's dynamic determination of the ratio of specific heats (also known as the polytropic coefficient, γ) [15]. The problem statement

[1] Lecture notes and problems sets for the first portion of this class in electronic form (*.pdf) are available upon request (*via* e-mail to sxg185@psu.edu) from the author.

provides the students with data obtained by measuring the oscillation frequency of a graphite piston in a glass cylinder [16]. The apparatus is shown in Fig. 1 and produces a value[2] for γ_{air} that is only 0.5% lower than the accepted value [17].[3]

Fig. 1. The Rüchardt apparatus [15] consists of a glass cylinder, with the height marked every centimetre, and a tight-fitting graphite piston attached to a brass mass. The bottom end of the glass cylinder is sealed to an aluminium platform with vacuum grease. An Endevco piezoresistive pressure sensor is threaded through the platform and is protected from contact with the graphite piston by the small PVC cylinder topped by an o-ring that acts as a shock absorber. The microphone output is connected to some signal-conditioning electronics and a digital storage oscilloscope to allow accurate measurement of the natural period of the freely-decaying oscillations.

Having developed confidence with the concept of acoustical compliance, the acoustical inertance is introduced through the application of the Euler equation to a small incompressible fluid element [14]. Combination of the inertance and compliance, along with a discussion of the continuity of pressure and volume velocity between those acoustic elements, leads directly to an expression for the Helmholtz frequency ω_o and to the derivation of the transfer function $H(f) = p_{cav}/p_1$ between the acoustical pressure outside the neck of the Helmholtz resonator p_1 and the pressure within the compliance p_{cav}.

Again, an experiment is performed (data provided in a homework problem) to measure the Helmholtz frequencies of a 500 ml boiling flask as a function of measured amounts of water added to the flask to reduce the compressible volume of the acoustical compliance element. The slope of the least-squares straight-line fit to the period squared *vs.* compliant volume produces an effective neck length. Of course, the experimental result for the pressure amplitude in the compliance p_{cav} is not infinite at resonance, nor is the neck-length corresponding to the inertance the same as the measured physical length of the flask's neck. Enter DELTAEC!

[2] By plotting the square of the oscillation period *vs.* the height of the cylinder, the slope of the best-fit line provides a value for γ, as well as its uncertainty [18].

[3] The fact that the measured value is slightly less than the accepted value is used later in the course to demonstrate that the compliance of the gas that is within a thermal penetration depth of the walls of the cylinder and piston behaves isothermally, thus introducing a small systematic error in Rüchardt's method.

3 THE DELTAEC Files

"It's not just for thermoacoustics fanatics anymore!"
Anonymous

A DELTAEC file is structured as a sequential list of "segments" such as ducts, cones, and compliances, as well as other useful segments such as electrodynamic loudspeakers, porous media, complex flow impedances, and heat exchangers. Each segment's physical properties are specified in MKS units. There are also special segments that can create side branches that are terminated or that are returned to the main system at some different location. The program behaves as a differential equation "solver" that is extraordinarily versatile in allowing the user to choose what parameters the program treats as "guesses" that the solver is empowered to modify in order to satisfy particular "target" values. In the following example, DELTAEC adjusts the Helmholtz resonator's neck length to match a specified frequency, but we could just as easily have asked DELTAEC to choose the concentration ratio of a helium/argon gas mixture to match the given frequency.

3.1 A 500 ml Boiling Flask

In my class, the students are provided with the first four chapters of the DELTAEC *Users Manual* and I give two lectures demonstrating how DELTAEC can be used to resolve both the divergence of p_{cav} at resonance and create an "effective length" correction to resolve the discrepancy between the theoretical and measured resonance frequency. This, of course, is supplemented with problem sets that allow the student to create their own DELTAEC files and produce their own results. The students are provided with lecture notes that provide a step-by-step process for the creation of the file and instruction on how it can be run.

Figure 2 is a screen shot of the *.out file for the 500 ml flask. The simplest method for describing the software will be to comment on 500mlFlask.OUT. After elucidating the structure, simple modifications will be introduced to illustrate the variations (*e.g.,* addition of an effective length correction, conversion to a bass-reflex loudspeaker enclosure).

Each DELTAEC file starts with a BEGIN statement (Segment #0) that can specify the initial excitation and define "global" parameters that are preserved throughout the model. In Fig. 2, "0a" specifies air at standard pressure and "0c" specifies a temperature of 22.5 °C = 295.65 K to duplicate the experimental conditions. The frequency "0b" is a "guess" (*i.e.,* a parameter that DELTAEC is allowed to vary to satisfy the "targeted" boundary conditions in "3a" and "3b"). The magnitude of the oscillatory pressure "0d" in front of the neck (Segment #1) is set to be $|p| = 1.00$ Pa and its phase "0e" is set to zero. Because this is a linear problem, the magnitude is somewhat arbitrary as is its phase. In this case, $|p|$ was chosen to be unity so that the magnitude of the pressure p_{cav} in the compliance "2A" is also numerically equal to the resonator's quality factor Q.

DELTAEC will determine the value of the volume velocity "0f" of the air in the neck that is set as another "guess". Since this file seeks the Helmholtz resonance frequency of the flask, the phase of the volume velocity "0g" is also set to zero because the driving pressure and volume velocity will be in-phase at resonance. The values of both "guesses" are shown in green because the model has run successfully. Before the program is run, those guesses are shown in red to warn the user that the values are not the result of a successful run. Input parameters that are not allowed to change are shown in blue.

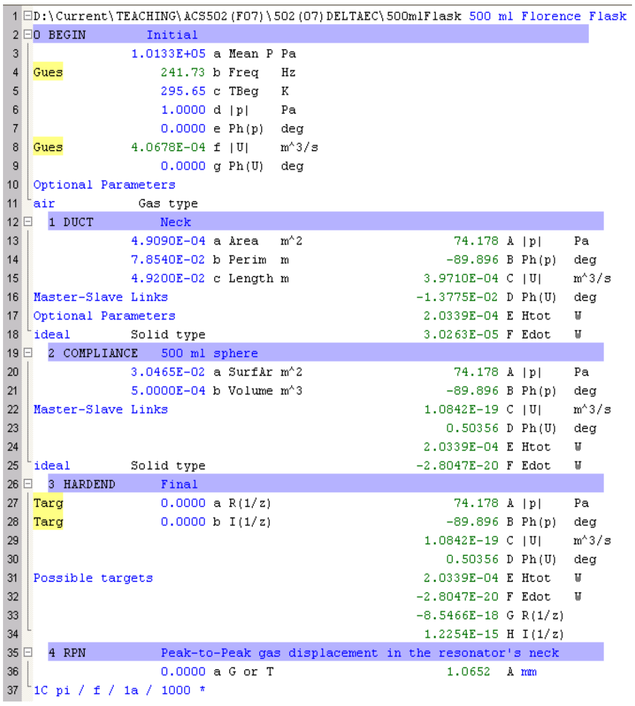

Fig. 2. DELTAEC file for 500mlFlask.OUT. Input parameters are displayed in the left-hand INPUT column in blue with the results in green. Calculated results show up in the right-hand "OUTPUT" column, also in green. In addition to the segments that describe the Helmholtz resonator geometry, an RPN segment (#4) is used to calculate the peak-to-peak motion of the gas in the neck.

Segment #1 specifies the physical properties of the neck. Since the neck is a DUCT, it is specified by a length "1c" and a cross-sectional area "1a". To accommodate different shaped ducts, the perimeter of the duct "1b" is specified independently. Segment #2 is a COMPLIANCE that is specified by its volume "2b" and the surface area "2a" of the sphere. The next segment, HARDEND (Segment #3) is one of the two possible segments (the other being SOFTEND) that is used to end a file. Since no gas flows out of the Segment #2, it is followed by the HARDEND segment which sets the real and imaginary components of its impedance to infinity by forcing the real and imaginary components of the admittance (reciprocal impedance) to both be zero.

The final segment illustrates an extraordinarily useful and very versatile feature of DELTAEC, although its function here is mundane. Segment #4 is an RPN target. These RPN segments allow the user to define mathematical functions that can use parameters of the model to calculate other parameters of potential interest. In this case, it is only used to calculate the peak-to-peak displacement of the gas in the resonator's neck by taking twice the volume velocity "1C" that exits the neck on the compliance side and divides it by the neck's cross-sectional area "1a" and the frequency obtained at "0b" times π, then multiplies by one-thousand to put the result in millimeters. As shown by the "result" in "4A", the gas's peak-to-peak displacement is just over one millimeter.

Because the thermophysical parameters of the gas and the solids are required for the solution, DELTAEC has an internal "handbook" that is also accessible to the user, both within the program and externally as the "ThermoPhys" tool. The thermophysical properties of the gas in the Helmholtz resonator are provided in Fig. 3.

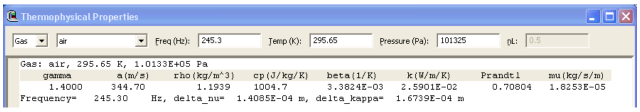

Fig. 3. Thermophysical values for air used in the Helmholtz resonator described by the file in Fig. 2.

3.2 Effective Length and *Q*

The course builds on the content of this DELTAEC model of a 500 ml flask along several directions. The first is to place an additional DUCT segment in front of the original neck (after Segment #0) which has the same cross-sectional area as the physical neck, but a miniscule perimeter to suppress thermoviscous losses. The length of this neck extension (effective length correction) is then made a "guess", while the frequency "0b" is set to the experimentally determined frequency. DELTAEC creates the required "effective length correction". The origin of this correction is not derived mathematically until much later in the semester when the complex impedance of a baffled piston is examined. ***But*** when that time comes, the students have had first-hand experience, both with measurement and with modeling, that prepares them to appreciate the significance of the reactive impedance concept (fluid inertia) that otherwise could appear vague or contrived.

The same is true for the DELTAEC result that shows the resonator's $Q = H(f) = p_{cav}/p_1$ = "2A" ÷ "1d" ≅ 74.2. Since DELTAEC correctly calculates the viscous dissipation in the neck and the thermal relaxation dissipation of the bounding (isothermal) surfaces of the compliance, it correctly determines the value of the resonator's quality factor. Further examination of the file in Fig. 2 shows in "1F" that about 30.3 μW leaves the neck and is dissipated in the compliance. ("2F" shows only a truncation error of -2.8 x 10^{-20} watts "leaves" the compliance.) The power delivered to the neck is half the product of the peak pressure and peak volume velocity: $\Pi_{in} = |p||U|/2$ = 203.4 μW, since pressure and volume velocity enter the neck in-phase. The viscous losses in the neck must therefore be the difference of 173.1 μW. These results are calculated from boundary-layer theory [14] later in the course, but again, the students' awareness has been "pre-disposed" to these concepts and their utility in a quantitative way with a concrete (actually glass 'n' gas) physical example.

DELTAEC has a native ability to plot a variety of model outputs through the creation of an "incremental plot" file. A screenshot of such a "plot window" is presented as Fig. 4.

3.3 Bass-Reflex Speaker Enclosure

Once the students have mastered the basic features of DELTAEC, it is quite easy to extend the modeling capabilities to more interesting applications such as adding an electrodynamic loudspeaker to drive the compliance of the resonator (instead of an oscillating external pressure driving the neck). In my class, I use the bass-reflex loudspeaker enclosure example provided by Beranek [4], since many of our student have access to that book, coupled to a JBL 2242 PHL 18" diameter woofer, since I happened to have measured its Small/Thiele parameters (*e.g., Bl*-product, S_D, R_m, m_o and k) using the techniques that exist in my laboratory [19].

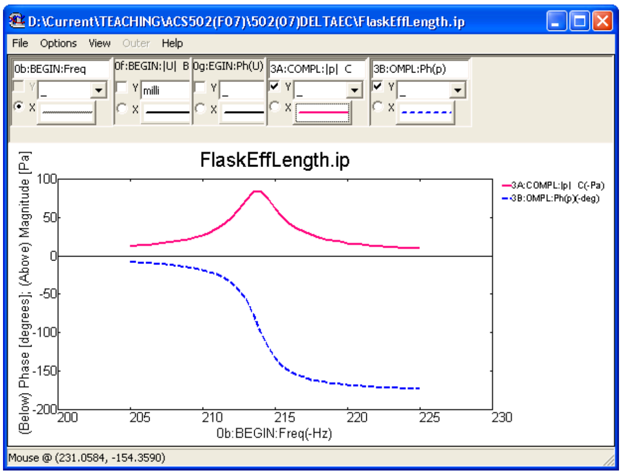

Fig. 4. DELTAEC output window for the incremental plot file FlaskEffLength.ip. I have chosen to display the pressure magnitude and phase by checking those options in the "Y" parameter boxes. By clicking on the Y-axis, I have added a label. The label on the X-axis was generated automatically, as was the legend at the right of the graph.

With the addition of an RPN target that uses the law of cosines to sum the volume velocity of the speaker cone with the volume velocity of the air exiting the "port", and placing some resistive damping material in the port to manage the system's behavior at resonance, DELTAEC provides the four response curves shown in Fig. 5.

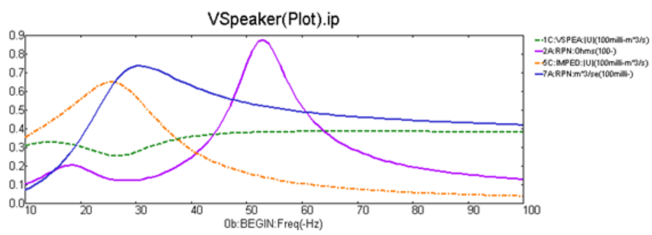

Fig. 5. Frequency response of the JBL 2242 PHL driving the Beranek bass-reflex enclosure [4]. The graph shows the magnitudes of the input electrical impedance (**purple line**), the volume velocity of the air oscillating in the port (**orange dash-dot line**), the volume velocity produced by the loudspeaker cone (**dashed green line**) and the net volume velocity produced by the vector sum of the volume velocities $|U_{net}|$ produced by the port and the loudspeaker (**blue line**).

3.4 Standing Waves

Although DELTAEC was introduced to motivate and solidify understanding of lumped-parameter systems, it is fully capable of solving for standing wave fields. This feature is particularly useful for examination of resonators with cross-section areas that vary along the resonator's axis. One such application that I use in my class is a homework assignment in which the students are tasked to model the Penn State Commemorative Beer Bottle shown in Fig. 6.

Fig. 6. The neck-length for this bottle is 17.78 mm with a radius of 8.26 mm. The volume is a cylinder that is 12.7 cm long with a radius of 24.38 mm. The length of the conical section that joins the neck to the volume is 10.0 cm.

Students are asked to use a CONE segment to join the neck to the volume and find the Helmholtz frequency (f_o = 193.1 Hz) and its Q = 58.1, as well as the frequencies f_n and Q_n's (excluding radiation losses) of the first three standing wave modes (f_1 = 1,019 Hz, Q_1 = 22.1; f_2 = 1,767 Hz, Q_2 = 15.6; and f_3 =2,466 Hz, Q_3 = 13.8 Hz). In addition to the "incremental plot" function, DELTAEC generates a "state plot" during each run that allows the physical variables to be plotted as a function of position as shown in Fig. 7

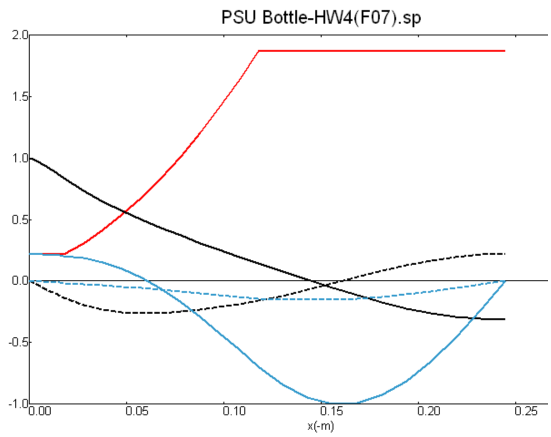

Fig. 7. Plots of the in-phase pressure (**black line**) and out-of-phase pressure times 10^2 (**dashed black**), in-phase volume velocity (**blue line**) times 10^4, out-of-phase volume velocity (**dashed blue**) times 10^5, and cross-sectional area (**red line**) times 10^3 as a function of distance from the neck.

It is useful for the students to see that the pressure is real and equal to 1.0 Pa at the open end of the neck (as specified) and that its amplitude is reduced and its phase is reversed at the closed end of the bottle. Similarly, both the in-phase and out-of-phase volume velocity vanish at the bottle's closed end.

4 El Camino Real

There are many ways to teach the fundamentals of acoustics. In more than three decades of experience since receiving my Ph.D., a knowledge of the hydrodynamic basis of sound has been both a source of psychological comfort in the unifying perspective it provides, and a reliable point-of-departure when I have had to address the behavior of new systems, complex fluids, or fluid-filled porous solids that I have encountered as a researcher in the field.

As an academic, I see it as my mission to transmit that understanding to my students in the classroom and in the laboratory in ways that are both efficient and meaningful. The approach that is outlined in this paper seems to be the most successful one that I have attempted in over thirty years in the classroom (and now on the web).

By combining theory, experiment, and the DELTAEC software, I am beginning to accumulate anecdotal evidence that my students benefit greatly from that method of instruction. Only time will tell. The purpose of an education is to provide the foundation that will allow our students to solve problems that do not exist at the time they receive their instruction and thus make humanity's collective understanding of the physical world more complete.

Since this epilog is becoming quite philosophical, I will use my remaining space for a joke about the role of education:

Question: What is the difference between the International Ladies' Garment Workers' Union and the American Psychological Association?

Answer: One generation.

Acknowledgments

I am grateful to my mentors, Izzy Rudnick and Seth Putterman, and to my colleague and good friend, Greg Swift, for teaching me to cultivate the hydrodynamic perspective on the behavior of waves in fluids. I cannot properly express my gratitude to Bill Ward and Greg Swift, the creators of DELTAEC, and to John Clark, who was responsible for creating the superb Windows™, mouse-driven user interface and native plotting features that makes this software package and its *Users Manual* so easy and satisfying for teaching the fundamentals of acoustics.

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