Mechanical resonances in the low-frequency vibration spectrum of a cylindrically symmetric, anti-tank landmine

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Acoustic-based landmine detection methods have enjoyed success, in part, because of structural resonances in many landmines. The unburied VS 1.6, a member of a family of plastic, cylindrically symmetric anti-tank landmines, exhibits seven modes below 1.6 kHz and a large frequency shift of the first symmetric mode such that its frequency is greater than that of the first asymmetric mode, a phenomenon observed in timpani of reduced kettle volume [Christian *et al.*, J. Acoust. Soc. Am., 76(5), 1336-1345 (1984)]. An elastically supported, thin, elastic plate acceptably models the unperturbed modes of the pressure plate. Coupling of the acoustic analog of the first symmetric mode to the cavities beneath the plate shows those cavities to be the cause of the perturbation. Shallow burial in sand effectively removes the frequency shift of the first symmetric mode.

1 Introduction

The importance that the structural modes of land mines have in acoustic based methods of land mine detection has been a recent topic of interest in the literature [1]. However, resonance behavior has been reported often [2-5]. Early works by Scott *et al* [2] and Donskoy *et al* [3] experimentally demonstrate resonances in both buried and in unburied land mines.

Initial modeling efforts by Donskoy *et al* [3] considered the pressure plate of a landmine to act as a 1-D damped simple harmonic oscillator. Zagrai *et al* [1] built upon the work of Donskoy *et al* by modeling a pressure plate's structural modes by an elastically supported thin plate. This thin plate model was then used to create a spatially distributed set of damped simple harmonic oscillators to obtain a 2-D representation of a land mine. Parallel to Zagrai *et al*, Alberts *et al* [4,5] directly modeled the mechanical behavior of the pressure plate of a VS 1.6 landmine as an elastically supported thin plate.

This paper presents a summary of the work previously published by Alberts *et al* [4]. The following section describes the landmine under study. Section 3 briefly describes the experimental procedures and the modes observed. Subsequently, the experimental observations are quantitatively and qualitatively compared to models in the fourth section.

2 The VS 1.6 Land Mine

A cross sectioned example of the VS 1.6 is depicted in Fig. 1. The explosives are housed in the large cavity at the bottom of the land mine. The detonating assembly and fuse are located in the center of and above the explosives cavity. Contained within the detonating assembly is a small volume, 0.8 cm^3, which is connected via a small tube to the volume, 180 cm^3, directly beneath the pressure plate. These coupled volumes act together to activate the land mine's detonator and to create the land mine's shock-resisting feature. To detonate the VS 1.6, a steady pressure must be applied to the pressure plate in order to allow enough time for air to move from the large volume though the tube into the small volume in the detonating assembly. This air inflates a diaphragm that causes a small pin to initiate the fuse. This also acts as the shock-resisting feature of the mine because a blast wave or acoustic wave passing over the mine occurs on such a short time scale that air does not pass through the tube to inflate the diaphragm. The coupled volumes and the compliant pressure plate make this land mine readily detected via acoustic methods, hence the reason for choosing the VS 1.6 for this work.

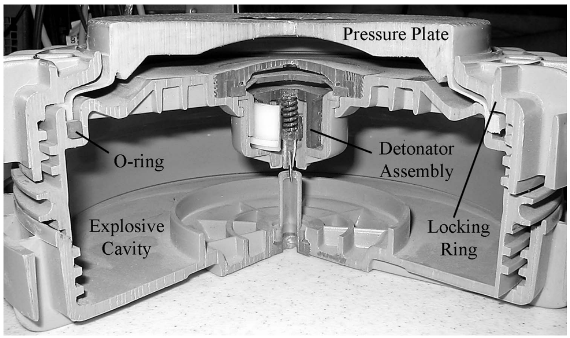

Fig. 1 Cross-sectioned example of a VS 1.6 (reproduced from Ref. 4).

3 Experimental Procedures and Results

3.1 Modal analysis of the unburied VS 1.6

The experimental procedures used to characterize the mechanical vibrations of the VS 1.6 have been well described in Ref. 4. Thus, the experimental procedures will be briefly touched upon in this and the following subsection.

To characterize the mechanical modes of the VS 1.6, an automated modal analysis technique was used. In an attempt to isolate the modes of the pressure plate, the land mine was placed on the leveled surface of dry sand in a large container. Suspended above the mine was an electro-mechanical shaker that imparted one cycle of a 1 kHz sine wave to the edge of the pressure plate through a 3 mm diameter aluminum rod. The placement of the excitation on the pressure plate was chosen to simultaneously excite as many modes, symmetric and anti-symmetric, as possible. Triggered off the shaker was a scanning laser Doppler vibrometer (LDV) that scanned the surface of the pressure plate with a spatial resolution of 1.53 points per square centimeter. From the Fourier transformed velocity data gather by the LDV, velocity maps of the pressure plate were constructed.

The maps of the seven modes occurring below 1.6 kHz in the vibration spectrum of the VS 1.6 are shown in Fig. 2. The modes are separated by the integers "m" and "n" where "m" refers to the number of nodal diameters and "n" refers to the number of nodal circles. Also shown are the frequencies of each mode. Of note is the frequency of the (0,0) mode at 350 Hz, it occurs at a frequency higher than that of the (1,0) mode at 225 Hz. Typically modes of simpler geometrical structure are expected to occur lower in frequency.

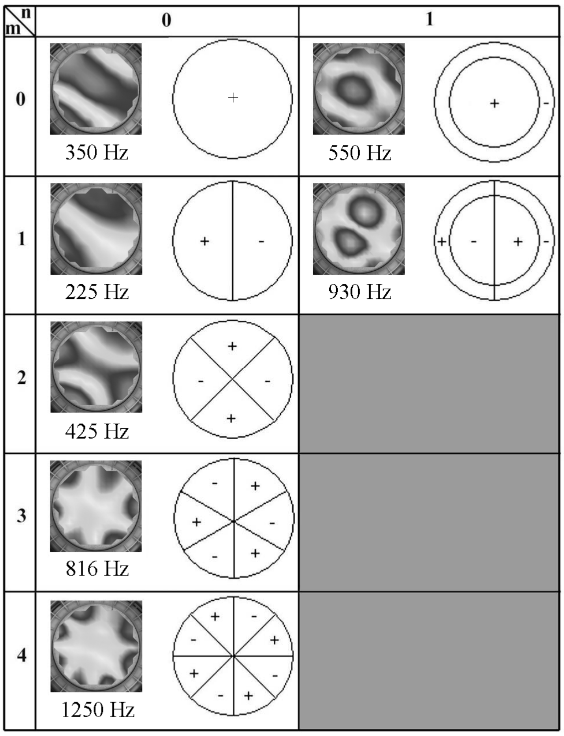

Fig. 2 Velocity maps of the modes in the pressure plate of the VS 1.6 (reproduced from Ref. 4).

3.2 Chassis loading

To determine what effects a load applied to the chassis of the land mine might have on the modes of the pressure plate, loading experiments were performed in both water and sand. The load was increased on the chassis by burying/submerging the land mine at several increments. At each increment, the previously described modal analysis procedure was performed and the mode frequencies were extracted. The results of these experiments are shown in Fig. 3 and Fig. 4, where Fig. 3 refers to the results in water and Fig. 4 refers to the results in sand. In each figure, the mode frequencies change very little until the land mine is submerged or buried to 95% of its height, which corresponds to halfway up the side of the pressure plate. In water at this depth, there is a small downward frequency shift of each mode followed by a more significant downward shift of all modes that remain visible once 100% submersion is reached. In the sand case, however, at 95% land mine height the two highest frequency modes remain at roughly the same frequency while the (1,0), (2,0), and (0,1) modes shift upward in frequency and the (0,0) modes shifts downward. These trends persist at burial to 100% land mine height with the exception that the (0,0) mode shifts to a higher frequency. Of particular interest is that the (1,0) and (0,0) modes, as measured at 100% burial, have shifted back to their expected positions.

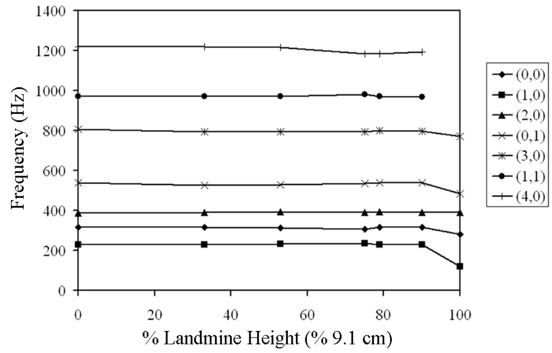

Fig 3 Chassis loading results in water (reproduced from Ref. 4)

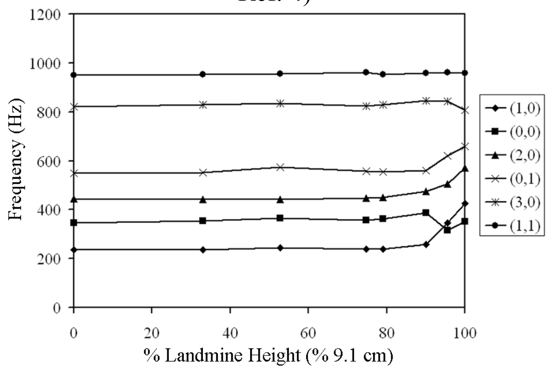

Fig. 4 Chassis loading results in sand (reproduced from Ref. 4).

4 Theoretical Models

4.1 Elastically supported plate

Based on the geometry of the VS 1.6 and the measured mode shapes of the VS 1.6, a reasonable model for this land mine's mechanical behavior is an elastically supported, thin, elastic plate. The equation of motion describing the transverse displacement, $w(r,\theta)$, of such a plate driven by an arbitrary force, $f(r,\theta)$, is:

$$\left(\nabla^4 - \frac{\rho h \omega^2}{D}\right) w(r,\theta) = \frac{f(r,\theta)}{D}. \qquad (1)$$

In Eq. 1, ρ is the mass density of the plate, h is the plate thickness, ω is the circular frequency of the plate assuming harmonic time dependence, and D is the flexural rigidity of the plate given by:

$$D = \frac{Yh^3}{12(1-\sigma^2)}. \qquad (2)$$

In Eq. 2, Y is the Young's modulus and σ is the Poisson's ratio. Fig. 5 shows the geometry of the plate and the elastic boundary conditions at the edge. Kw resists transverse motion of the plate edge and $K\psi$ resists rotation of the plate edge about the radial axis. Both springs are circumferentially distributed about the plate edge.

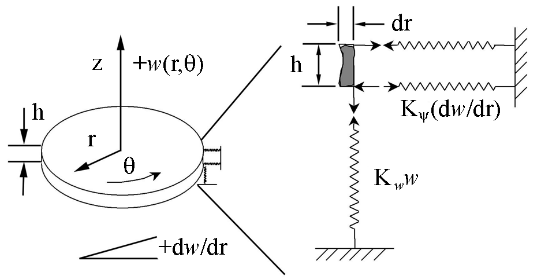

Fig. 5 Geometry of the elastically supported plate model (reproduced from Ref. 4).

The solution to Eq. 1, in terms of Bessel functions, can be found in Ref. 4. Using a plate density of 1245 kg/m^3, a vertical spring constant, *Kw*, of 0.22 MPa, and a Poisson's ratio of 0.3 and fitting to determine a Young's modulus of 2.4 GPa and a plate thickness of 4.7 mm, the elastically supported thin plate demonstrates reasonable agreement between predicted and measured mode frequencies. It was also found that the resonance frequencies showed little sensitivity to $K\psi$, so it was set to zero in all calculations.

4.2 Equivalent electrical circuit

If the dimensions and construction of the VS 1.6 and the wavelengths of interest are considered, then the large perturbation of the first symmetric mode of the VS 1.6 can readily be described by creating an electrical equivalent circuit for the first symmetric mode of the plate and for the cavities [6]. The circuit is built only for the first symmetric mode because it is expected to induce the largest net volume change when compared to other modes. The impedance of the circuit can then be evaluated to determine the effect that the coupled cavities below the pressure plate have on the first symmetric mode of the plate. This equivalent circuit is depicted in Fig. 6 where M_P and C_P are the mass and compliance of the plate, S_P is the area of the plate coupling the mechanical and acoustic portions, C_{LV} is the compliance of the large volume, M_T and R_T are the mass and resistance to motion of the air in the small tube, and C_{SV} is the compliance of the small volume in the detonating assembly.

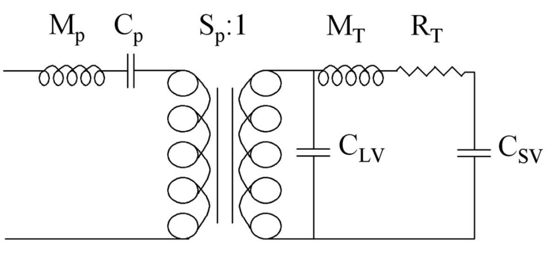

Fig. 6 Electrical equivalent circuit for the (0,0) mode of the VS 1.6 (reproduced from Ref. 4).

The reader is referred to Ref. 4 for the details of the calculation of the circuit impedance. The results of the impedance calculation appear in Fig. 7, which shows the impedance of the first symmetric mode of the plate with and without the backing volumes. Similar results have been observed in timpani when the kettle volume was reduced by half [7,8].

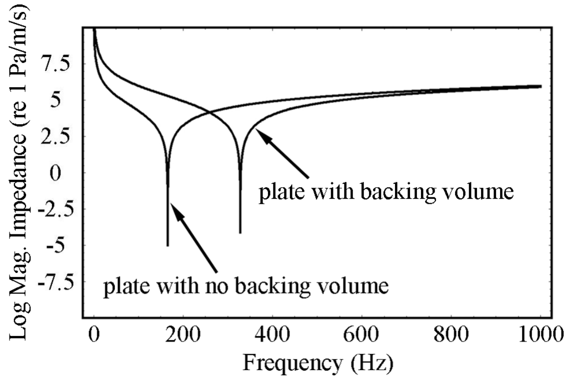

Fig. 7 Plot of the impedance of the (0,0) mode of an elastically supported plate with and without a backing volume (reproduced fro Ref. 4).

4.3 Shear contact at flush burial

The results of the chassis loading experiments demonstrated that burial to 100% land mine height in sand appears to remove the perturbation of the first symmetric mode of the plate. It is also noted that, at 95% burial, the modes began to shift. This implies that shear contact between the sand and the plate edge may have an effect on the mode frequencies. Qualitatively, this effect can be investigated by assuming welded contact between the sand and the plate edge. Under that assumption, the interest lies in the vertical component of the stress in terms of the strain. Written with the Lamé constant in terms of the compressional wave speed in the soil and the density of the soil, the stress becomes [9,10]:

$$\sigma_{zr} = \left[\frac{1-2\sigma}{2(1-\sigma)} \rho V_p^2 \right] u_{zr} . \qquad (3)$$

In Eq. 3, σ with no subscripts, ρ, V_p, and u_{zr} are the Poisson's ratio of the soil, density of the soil, compressional wave speed in the soil, and the strain, respectively. Inserting a Poisson's ratio of 0.4, a density of 1600 kg/m^3, and a compressional wave speed of 100 m/s into the term in brackets (the Lamé constant) yields a value on the order of 10^6 Pa, which, when multiplied by the strain gives units of N/m. In this qualitative example, the vertical stress can simply be added to the vertical spring constant at the plate edge, which results in a 1000% increase in the edge stiffness of the plate. Thus, the plate is forced into a more clamped condition. Fig. 8 shows the results of the circuit calculation on a clamped plate with and without a backing volume. Because flexure of the plate is the predominant vibration mechanism, the first symmetric mode changes the volume of the cavity by a smaller amount. Thus, the frequency of the first symmetric mode does not shift as far as in the elastically supported case.

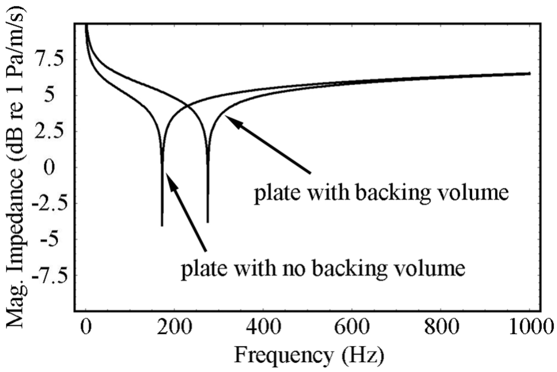

Fig. 8 Plot of the impedance for the (0,1) mode of a clamped plate with and without a backing volume (reproduced from Ref. 4).

5 Conclusion

Seven structural modes similar to those in a drumhead have been observed in the pressure plate of a VS 1.6 landmine. These modes are acceptable modeled by an elastically supported, thin, elastic plate. Mechanisms below the pressure plate cause a large perturbation of the first symmetric mode of the plate by effectively stiffening that mode. This has been confirmed by a lumped acoustic element model of the plate-volume system in the landmine. Flush burial in sand effectively removes this perturbation by forcing the plate into a more clamped condition through shear contact between the plate edge and the sand. The clamped condition lessens the effect of the volumes beneath the plate.

The study summarized here has shown interesting phenomena caused by the structure of the land mine and by its environment. The experimental observations and related explanations imply that the mechanisms encased in the landmine should be taken into consideration in modeling and that studies should consider the vibration of the land mine *in situ*.

Acknowledgments

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