The sonar equations: definitions, dimensions and units of individual terms

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The passive and active sonar equations are reviewed with regard to the definitions of individual terms. It is shown on the one hand that ambiguities exist in conventional definitions that can lead to misinterpretation, and on the other that under special conditions a correction term is needed that is not usually included. Revised definitions are proposed that remove the ambiguity and hence also the likelihood of misunderstanding. The consequence of the proposed definitions are explored from the perspective of their dimensions and units. Particular emphasis is placed on the effects of variations in impedance (sound speed, or density, or both) of the medium.

1 Introduction

1.1 The sonar equation

The sonar equation is a tool for quantifying the performance of a sonar, relating the signal to noise ratio (SNR) of the sonar to the source level (SL) of the transmitted or radiated sound, the propagation loss (PL) and the background noise level (NL). In one of its most basic forms, the SNR before processing can be written (in decibels) [1]

$$\mathrm{SNR} = \mathrm{SL} - \mathrm{PL} - \mathrm{NL}\,. \qquad (1)$$

When presented with this equation we "know" intuitively how to interpret each term: they represent, in turn, the power of the source, the reduction in intensity from source to receiver due to spreading and energy loss, and the background against which the acoustic signal is to be detected. We also "know" that, should we need them, more rigorous definitions are easily available from a standard textbook [1]. Such rigorous definitions are especially important for calibration purposes. But what are these rigorous definitions, and what implications do they have for the dimensions and units of the individual terms? Similar questions are considered by Hall [2] and by Ainslie [3].

1.2 EPWI and MSP interpretations

There are two possible interpretations of sonar equation terms: one in terms of ratios of mean square pressure (MSP), relative to a reference pressure p_{ref}; the other in terms of ratios of equivalent plane wave intensity (EPWI, defined as MSP divided by the characteristic impedance of the medium), relative to a reference intensity I_{ref}. In the latter case the value of I_{ref} is chosen by convention to be the intensity of a plane wave whose RMS pressure is p_{ref}, implying a conversion of the form $I_{\mathrm{ref}} = p_{\mathrm{ref}}^2/Z_I$. This conversion gives rise to an undesirable ambiguity because the value of Z_I to be used is not specified in any internationally accepted standard. Ainslie [3] shows that if Z_I is taken to be the *local* characteristic impedance of the propagation medium, a correction factor is needed to the traditional the sonar equation, equal to the source-receiver impedance ratio.

Here an alternative assumption is explored, involving the assumption of the same *fixed* value of Z_I everywhere. The EPWI and MSP interpretations are compared with this alternative assumption in Sec. 2. An ambiguity of 1 to 5 dB is shown to occur in individual terms. While there is no net effect on SNR, the ambiguity can cause an error if the same interpretation is not used consistently throughout the entire sonar equation. It can be argued that the ambiguity is small because the impedance ratio is close to unity. This is indeed often the case, but not always. It is for exceptionally large impedance ratios, or for cases with exceptionally large precision requirements, that an unambiguous definition is needed. Such an unambiguous definition is proposed for each term in a simple sonar equation, describing the signal to noise ratio before processing.

1.3 Definitions, dimensions and units

A choice is made between the two interpretations and unambiguous definitions of individual terms presented in Sec. 3, with corresponding dimensions and reference units. Both continuous and transient forms of the passive sonar equation are presented. The active sonar equation is considered in Sec. 3.4.

This paper is a summary of the main points from Ref. [4]. Readers interested in more details can find them there. Refs. [3] and [5] are also relevant.

2 EPWI and MSP rules

2.1 The EPWI rule

2.1.1 definition

Urick emphasises that the sonar equation terms are defined in terms of intensity (EPWI) ratios. To quote directly from Urick [1][6]

*It should be emphasized that the decibel is a comparison of intensities or energy densities, rather than directly of pressures, even though "20 dB re 1 μPa" appears to refer to a pressure. What is omitted from a statement of this kind are the words re "*the intensity of a plane wave of pressure equal to*" 1 μPa.*

Two important statements are made here. The first is the EPWI rule, namely that parameters expressed in units of decibels must be ratios of EPWI (as opposed to MSP); the second is the definition of reference intensity referred to in the introduction. Earlier versions of the EPWI rule can be traced to Horton [7], Urick [8] and Camp [9]. For example [9]

… [MSP] is not proportional to the intensity and expressing [its] ratio in decibels is meaningless.

The same principle has been restated since in authoritative texts, most recently in 2007 [10][11][12].

After reading these statements, one is left in no doubt that their authors consider it incorrect to express an MSP ratio in decibels, unless the corresponding source-receiver impedance ratio happens to be unity.

The **EPWI rule** (the definition of sonar equation terms in terms of EPWI ratios) is incomplete without a definition for the reference intensity (I_{ref}), as this quantity appears in the definitions of SL and NL. Urick [1] provides the following definition of I_{ref}

The unit of intensity *in underwater sound is the intensity of a plane wave having an rms pressure equal to 1 micropascal (abbreviated 1 μPa) or 10^{-5} dyne per square centimeter.*

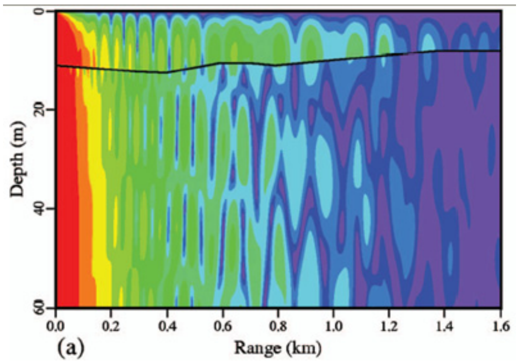
To make this definition unambiguous a value for the impedance is needed, but the precise value intended is unclear. Possible choices include:
a) the value at the measurement location
b) an internationally agreed standard reference value
c) a constant value nominally equal to the impedance of the propagation medium (usually seawater), the variations of which in time and space are considered insufficiently important to warrant explicit mention

As the impedance of the medium is in general a function of position, the intensity of a plane wave (of given RMS pressure) is also a function of position. It can therefore be argued that Urick's definition implies that I_{ref} is a function of position. This interpretation (choice a) leads to an undesirable extra term in the sonar equation [Ainslie 2004] and is not considered further here. Instead an alternative interpretation is pursued, namely that of a fixed reference intensity to be specified, independent of position and irrespective of the true impedance of the medium.

2.1.2 use in practice

The extent to which the EPWI rule is used in practice can be judged (it is rarely stated explicitly) by examining calculations of PL across a discontinuous impedance boundary. MSP is continuous across the boundary, so EPWI changes discontinuously by the impedance ratio.

The EPWI rule was the accepted definition between 1959 [7] and 1980 [13]. Recent EPWI use is confined to a handful of papers; only four instances are known to the author since DiNapoli and Deavenport [13]. Figure 1 shows a recent example, from Ref. [14]. The others are Refs. [15], [16] and [17].



,Fig.1 PL(r,z) contours for water to sediment propagation from Collis et al [14]. Notice the step change at the water-sediment boundary (ca. 10 m depth)

2.2 The MSP rule

2.2.1 definition

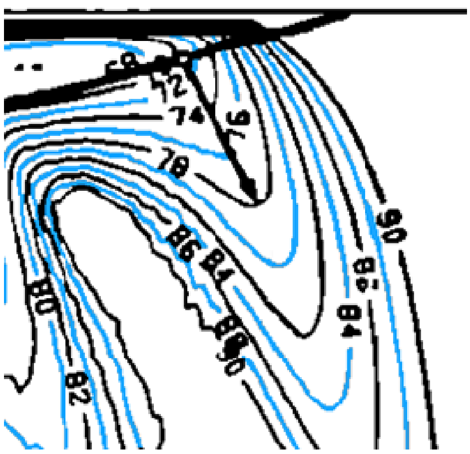
Modern standards [17][19] define sound pressure level, in decibels, as the logarithm of a ratio of MSP (not EPWI) values, so it seems reasonable to consider an alternative convention based on MSP ratios. Thus, we state the **MSP rule** as the definition of sonar equation terms in terms of MSP ratios

2.2.2 use in practice

Use of the MSP rule can be traced back to the pioneering work of Jensen and Kuperman [20], who modelled upslope sound propagation in a wedge using the parabolic equation, demonstrating how the energy associated with each mode penetrates the sediment, forming a finger-like beam as that mode reaches its cut-off depth. The last of a series of such beams is shown here in Fig. 2. The important point in the present context is how, in contrast with Fig. 1, *the propagation loss contours are continuous across the boundary*. This observation proves that the EPWI rule is not used here (a 1.2 dB step would otherwise be clearly visible).

Since 1980, nearly all cross-boundary propagation problems follow the example of Ref. [20], so that MSP has become the *de facto* standard [3][5]. The identification is in most cases by inspection, but see Ref.s [21], [22], [23] and [24] for explicit definitions. Examples of the application of MSP include at least 24 cases for the ASA wedge benchmark problems alone, early examples of which are Murphy and Chin-Bing [25] and Jensen and Ferla [26]. The MSP is also applied to many other propagation problems [4]. It is even used in an encyclopaedia article that emphasizes the importance of applying the EPWI rule with the statement [11]

The decibel … denotes a ratio of intensities (not pressures) expressed in terms of a logarithmic (base 10) scale.



,Fig.2 Propagation in a wedge from Jensen and Kuperman [20] (zoomed in version of their Fig. 2). Notice the *absence* of a step change across the water-sediment boundary. Alternate contours are coloured grey (black & white version) or cyan (colour version).

2.3 How big is the difference?

In Urick's day, the difference between levels based on the EPWI and MSP rules would usually have been small. Today two things are changing:
- we seek increasing precision in our measurements, which requires a corresponding increase in the precision of our definitions;
- we conduct measurements in increasingly complicated situations, often involving propagation between seawater and a medium of different impedance, such as the seabed, a ship wake or biological tissue.

With this in mind, and if nothing is done to prevent it, it is only a matter of time before the precision of our measurements exceeds that of our definitions. In the above examples the difference between EPWI and MSP interpretations amounts to between 1.2 and 5.5 dB. Similar differences [27] or even larger ones [28] are possible in bubbly water.

With the EPWI rule there is a further ambiguity in the value of Z_I. There is an understanding that the impedance of water is implied, but should one use freshwater or seawater? At what temperature and pressure? Urick [1] quotes a nominal value 0.67 aW/m² for the reference intensity, corresponding to an impedance (Z_I) of 1.49 MPa s/m, but there is no internationally accepted standard value. Even a small ambiguity may degrade the value of precise calibration measurements if our definitions are imprecise. (The difference in impedance between fresh- and seawater results in an ambiguity in the EPWI definition of 0.2 dB, for the same temperature and pressure.)

Such a situation is untenable in the long term. For the science of underwater acoustics to advance further it must first acknowledge that an ambiguity exists between the two interpretations, make a clear choice between them, and then apply that choice consistently. In simple terms, we must practise what we preach.

3 Definitions, dimensions and units

3.1 EPWI or MSP?

The main argument used in favour of the EPWI rule is that the decibel may only be used to relate (logarithmic) ratios of power or energy [7][9]. This reason can be questioned, however, because EPWI is not strictly equal to the acoustic intensity, nor is it proportional to either power or energy density. (In general, the sound field is not a simple plane or spherical wave, but a superposition of many such waves).

There is also a conflict between the EPWI rule and the international standard for I_{ref} because the accepted value of I_{ref} is not 1 μPa²/Z_I but 1 pW/m² [19][29][30]. Even if this conflict were resolved there remains the above-mentioned ambiguity in the value of Z_I.

By contrast, the MSP interpretation is unambiguous, is supported by international standards and is the *de facto* accepted definition (for propagation loss calculations).

Although the discrepancy (between MSP and EPWI) is most visible in the calculation of PL (see Secs. 2.1 and 2.2), the choice affects the entire sonar equation.

3.2 Passive sonar (continuous source)

In the following the MSP rule is adopted and used to define the constituent terms in the sonar equation [Eq. (1)]. The first step is to define noise level NL in terms of the mean square pressure $\langle p_N^2 \rangle$ as

$$\mathrm{NL} \equiv 10 \log_{10} \frac{\langle p_N^2 \rangle}{p_{\text{ref}}^2} . \quad (2)$$

Similarly, the source level SL is

$$\mathrm{SL} \equiv 10 \log_{10} \frac{Q_0}{p_{\text{ref}}^2 r_{\text{ref}}^2} , \quad (3)$$

where the source factor Q_0 is closely related to the radiant intensity (i.e., power per unit solid angle) J_0 (using the subscript zero to denote properties of or at the source)

$$Q_0 \equiv Z_0 J_0 \quad (4)$$

and r_{ref} is the reference distance. Equation (1) follows if propagation loss PL and SNR are defined for mean square signal pressure $\langle p_S^2 \rangle$ as

$$\mathrm{PL} \equiv 10 \log_{10} \frac{Q_0 / \langle p_S^2 \rangle}{r_{\text{ref}}^2} \quad (5)$$

and

$$\mathrm{SNR} \equiv 10 \log_{10} \frac{\langle p_S^2 \rangle / \langle p_N^2 \rangle}{1} . \quad (6)$$

For each term in the sonar equation, the reference unit must have the dimensions of the physical parameter represented by that term. For example, the physical parameter represented by the noise term NL is the mean square noise pressure, with dimensions of squared pressure and (hence) the reference unit for NL is p_{ref}^2. For the source level, the physically relevant quantity is the source factor Q_0, which [from Eq. (4)] has dimensions of squared pressure multiplied by area, hence the appearance of $p_{\text{ref}}^2\, r_{\text{ref}}^2$ in the denominator of Eq. (3). Finally the physical parameter represented by the propagation loss is an area[2], with corresponding reference unit r_{ref}^2. The signal to noise ratio is dimensionless.

Internationally accepted reference values for pressure and distance are [29][30] p_{ref} = 1 μPa and r_{ref} = 1 m, from which it follows, for example, that the reference unit of source level is $p_{\text{ref}}^2\, r_{\text{ref}}^2$ = 1 μPa² m².

3.3 Passive sonar (transient source)

For a transient sound of short duration, the statistics are non-stationary and the MSP is therefore not well defined.

In this situation it becomes necessary to replace MSP, wherever it occurs, with the total value of p^2 integrated over the duration of the sound [31]

$$\mathrm{NL_E} \equiv 10\log_{10}\frac{\int p_N(t)^2\,\mathrm{d}t}{p_{\mathrm{ref}}^{\;2}t_{\mathrm{ref}}}. \quad (7)$$

Correspondingly the energy source level SL is defined as

$$\mathrm{SL_E} \equiv 10\log_{10}\frac{E_0}{p_{\mathrm{ref}}^{\;2}r_{\mathrm{ref}}^{\;2}t_{\mathrm{ref}}}, \quad (8)$$

where

$$E_0 \equiv \int Q_0(t)\mathrm{d}t. \quad (9)$$

Equation (1) follows if propagation loss PL is defined as

$$\mathrm{PL} \equiv 10\log_{10}\frac{E_0\Big/\int p_S(t)^2\,\mathrm{d}t}{r_{\mathrm{ref}}^{\;2}} \quad \text{(A)}$$

and

$$\mathrm{SNR} \equiv 10\log_{10}\frac{\int p_S(t)^2\,\mathrm{d}t\Big/\int p_N(t)^2\,\mathrm{d}t}{1}. \quad \text{(B)}$$

Although ANSI [29] does not specify a reference time [needed for Eqs. (7), (8)], a natural choice is the reciprocal of the reference frequency of one hertz, i.e., t_{ref} = 1/(1 Hz) = 1 s.

If PL is known or can be estimated, the energy source level $\mathrm{SL_E}$ of a transient sound can be estimated by combining Eqs. (8) and (A) to give

$$\mathrm{SL_E} = \mathrm{PL} + 10\log_{10}\frac{\int p_S(t)^2\,\mathrm{d}t}{p_{\mathrm{ref}}^{\;2}t_{\mathrm{ref}}} \quad \text{(C)}$$

De Jong and Ainslie [32] use Eq. (C) to estimate the energy source level due to a single blow from a pile driver from the measurements of Robinson et al [33].

3.4 Active sonar

The above definitions are extended to the active sonar equation in Ref. [4]. Two remarks are made here. The first concerns the definition of the source level term SL: The source level (or source factor) is a measure of the source power (strictly the radiant intensity) radiated into the far field [2]. For this reason, Urick [1] places great emphasis on the far-field nature of the source level concept. This point is central to the correct interpretation of the term, but is missing from at least two mainstream modern definitions [18][19].

The second is about the consequences of the sonar target being in a medium of different density than the sonar. For the special case of monostatic active sonar with a buried target involving a single propagation path from sonar transmitter to target (along which the propagation loss is denoted $\mathrm{PL_1}$), the sonar equation is found to be (if the target is buried in a medium of relative density ρ)

$$\mathrm{SNR} = \mathrm{SL} - 2\mathrm{PL_1} + \mathrm{TS} - \mathrm{NL} - 10\lg\rho^2, \quad \text{(D)}$$

where NL is the total background, including both noise and reverberation, and TS is the target strength. Thus, application of the MSP rule results in a correction factor equal to the square of the relative density. If the object is buried in sand, the correction term amounts to ca. 5 dB. Equation (D) is derived in Ref. [4]. A similar correction is required also for the EPWI rule, except that the correction factor depends on the relative sound speed instead of the density [7].

4 Conclusions

- An ambiguity arises in individual terms in the sonar equation unless it is specified whether the MSP or EPWI rule is applied. A typical magnitude for the ambiguity for water-sediment propagation is 1-5 dB.
- A smaller ambiguity (order 0.2 dB) arises with the EPWI rule because of the ambiguity in the associated reference intensity.
- The mandatory nature of the EPWI rule, as described by Horton in 1959 [7] and repeated as recently as 2007 [12] is not supported by modern standards bodies.
- The overwhelming majority of papers published since February 1980 in the Journal of the Acoustical Society of America apply the MSP rule and not the EPWI rule.
- Propagation loss of an impulse converts between the time integral of sound pressure squared (a measure of the total energy), not between MSP or peak pressure.
- A correction term, not normally included in the sonar equation, of order 5 dB in magnitude, is needed in the active sonar equation for modelling detection of an object buried in sand.

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