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Experimental feasibility of in-duct sound source reconstruction

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Due to the expected air transport growth and stringent environmental regulations, there is a strong need to develop noise reduction techniques at acceptable cost in the aeronautical sector. A sound characterisation of the aero-acoustic sources in the nacelle acoustic duct problem plays a crucial role. In this study, it is first shown how the liners optimized impedances strongly depend on the noise source characteristics under both tonal and broadband excitation conditions, and in the latter case, for varying degree of correlations between the random source strengths. It is found under which conditions source identification methods, such as pointwise model-based inverse techniques, are able to provide reliable models of equivalent sound sources from a limited number of in-duct measurements, but which require *a priori* knowledge of the source location. The performance of such methods are compared with two different approaches, namely the use of focussed in-duct beamforming techniques to locate the unknown sources, and a decomposition of the assumed source strength into angular Fourier series for both the location and the determination of the source amplitudes. Experimental results are presented for the location and the reconstruction of the particle velocity spectrum of wall-mounted compression drivers from in-duct measurements.

1 Introduction

The growth of the air traffic density contributes to the definition of more and more restrictive legislations against the impact of noise pollution on the airport community. To meet these demanding requirements, greater efforts need to be done towards the detailed characterisation of aircraft noise sources. The development of modern turbofan engines with higher bypass ratio, and so a reduced exhaust jet velocity, has led to a relative increase of the contribution of fan noise sources to the overall radiated acoustic power, especially under take-off and approach conditions.

Direct identification of the different fan noise sources under real flight conditions is a rather complex, expensive and delicate task as it requires advanced measurement techniques that might itself perturb the noise source generation mechanisms. We note that a number of turbofan noise sources are readily modelled as a distribution of monopole, dipole or quadrupole-type elementary sources, which respectively correspond to fluctuations of flow rates, of forces exerted on a solid (eventually moving) surface and of shear velocity and which radiate an acoustic pressure the amplitude of which scales on a specific power of the flow velocity. It would therefore be advantageous to investigate the use of hybrid model-based inverse methods to reconstruct the strengths of ducted elementary source distributions given a measurement of the acoustic field radiated by the original source at a number of in-duct sensor positions and a consistent model of the propagation paths between the modelled sources and the sensor positions.

Many authors have worked on the field of inverse methods for the identification of acoustic sources, especially under free-field or weakly reactive environments. Three main approaches have been developed to solve NAH (Near-field Acoustical Holography) problems. A first approach concerns the Fourier-based methods which rely on the wavenumber relationship between an unknown source distribution and its radiated field measured over a complete hologram surface [1]. Inherent truncation effects due to the spatial Fourier transform are avoided when using the statistically optimized NAH (SONAH) which makes use of a surface-to-surface projection scheme based on the plane wave expansion of the sound field [2]. The SONAH might thus be viewed as part of a second NAH approach based on a convergent expansion of the radiated sound field as a series of orthogonal solutions of the homogeneous Helmholtz equation such as, for instance, spherical wave functions for the Helmholtz Equation Least-Squares (HELS) method [3]. A third NAH approach requires an inversion of a spatial model of the transmission paths between a source distribution and its radiated field. Depending on the choice of the integral representation of the radiated pressure field, model-based reconstruction techniques include the Inverse Boundary Element Method (IBEM) and the Equivalent Source Method (ESM). The IBEM assumes an interpolation of the unknown surface velocity distribution at the nodes of each discretized boundary element [4]. The ESM assumes a distribution of volume velocity monopole-type [5] or pressure dipole-type sources which respectively discretize, the single and double layer potential densities in an indirect integral representation of the radiated pressure field. Unlike Fourier-based methods, the two last approaches account for non-separable source geometries. Recently, closed-form expressions have been found for the model-based reconstruction of the velocity of baffled planar structures in terms of spheroidal wave functions [6].

However, a fewer number of studies have focussed on the identification of acoustic sources in bounded environments such as in ducts or in enclosures. A methodology based on the Green's representation of the pressure field has been proposed by Langrenne *et al.* [7] to extract the outgoing radiated component from the measured acoustic pressure generated by a machine in an enclosure. It could be well suited for the IBEM localisation of the most vibrating parts of a machine. Kim and Nelson [5] have presented experimental reconstruction results in a semi-infinite hard-wall cylindrical duct when using the ESM. They have shown that near-field measurements of the in-duct pressure field are required in order to reduce the ill-conditioning of the acoustic transfer matrix between a set of microphones and the equivalent sources and thus achieve a greater accuracy in the reconstruction results. In particular, one of the main limitations of the ESM, especially when one tries to reconstruct a set of highly-localized (or even punctual) sources, is the *a priori* choice of the locations of a limited number of equivalent sources.

The present work investigates the use of a Spectral Decomposition Method (SDM) in the ducted source reconstruction problem as an alternative to the ESM since it does not require an *a priori* knowledge of the real source location. The efficiency of the methodology is compared with respect to the ESM and to a high-resolution beamforming approach, as for the source localization. It is applied to the experimental reconstruction of the acoustic flow rate of compression chambers wall-mounted in an acoustic duct facility.

2 The ducted source reconstruction problem

The question is to reconstruct the strength distribution of ducted sources from measurement of the in-duct acoustic pressure field, given a consistent model of the transmission paths between the assumed sources and the measurement positions. In relation with the anechoic duct acoustic facility at the Université of Technology of Compiègne, one considers an infinite duct with a circular cross-section of radius R, excited by a cross-sectional distribution of sources, eventually wall-mounted on the duct, as shown in Fig. 1. The in-duct acoustic field (pressure and axial velocity) generated by the sources is measured at a number of angular and radial positions at various axial separation distances from the excitation we aim at recovering.

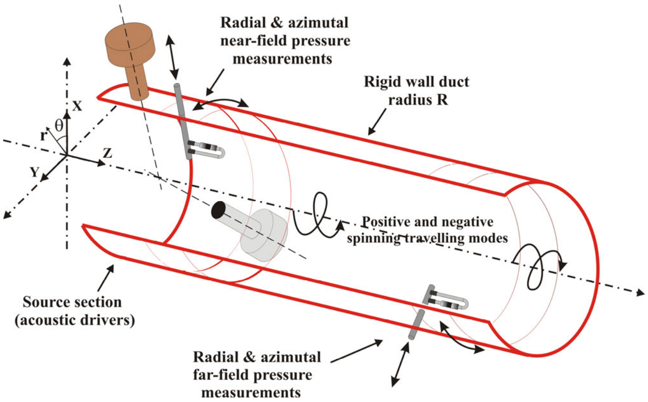

Figure 1. Cut view representation of the duct acoustic facility for inverse source strength reconstruction.

2.1 The in-duct field model

Assuming broadband stationary noise sources, the Cross-Spectral Density (CSD) matrix between the measured acoustic pressures, $\mathbf{S}_{\tilde{p}\tilde{p}}$, is expressed in terms of the unknown source strengths CSD matrix, $\mathbf{S}_{\tilde{q}\tilde{q}}$, as follows:

$$\mathbf{S}_{\tilde{p}\tilde{p}} = \mathbf{G}\mathbf{S}_{\tilde{q}\tilde{q}}\mathbf{G}^{\mathrm{H}}, \qquad (1)$$

where $\mathbf{G}$ is the acoustic transfer matrix between an assumed number of equivalent sources and the sensor positions. If one decomposes the equivalent ducted source strength distribution as a Fourier-Bessel expansion, in the form

$$\tilde{q}(r',\theta') = \sum_{mn} \tilde{\alpha}_{mn} \psi_{mn}(r',\theta') = \mathbf{\Psi}\tilde{\boldsymbol{\alpha}}, \qquad (2)$$

where $\mathbf{\Psi}$ is the matrix of the hard-walled normalized duct modes, ψ_{mn}, evaluated at the equivalent source positions and $\tilde{\boldsymbol{\alpha}}$ is the unknown vector of the source strength spectral decomposition coefficients, then Eq. (1) becomes

$$\mathbf{S}_{\tilde{p}\tilde{p}} = \mathbf{Z}\mathbf{S}_{\tilde{\alpha}\tilde{\alpha}}\mathbf{Z}^{\mathrm{H}}, \qquad (3)$$

with $\mathbf{Z} = \mathbf{G}\mathbf{\Psi}$, an element of which simply reads:

$$Z_{j,(m,n)} = \frac{\rho c_{mn}}{2A} \psi_{mn}(r_j,\theta_j) \mathrm{e}^{-\mathrm{i}k_{mn}\Delta z}. \qquad (4)$$

In Eq. (4), $Z_{j,(m,n)}$ relates the $(m,n)^{\text{th}}$ source spectral coefficient to the j^{th} sensor position when assuming monopole-type sources radiating in an infinite duct in the no-flow case. ρ is the fluid density, c_{mn} and k_{mn} are respectively the axial phase speed and the axial wavenumber of the $(m,n)^{\text{th}}$ duct mode, A is the duct cross-sectional area and Δz is the source-sensor separation distance. Unlike $\mathbf{S}_{\tilde{q}\tilde{q}}$, $\mathbf{S}_{\tilde{\alpha}\tilde{\alpha}}$ does not require *a priori* knowledge/assumption on the location of the elementary sources since the coefficients $\tilde{\alpha}_{mn}$ already account for the spatial distribution of the sources.

The model (1-4) has been extended to account for uniform inflow conditions and for the effect of a uniform lined section described by a locally reacting impedance [8]. In order to show the influence of both the nature of the sources and their degree of correlation on the optimal properties of a locally reacting duct liner, simulations have been carried out in a duct of radius 0.4 m with a lined section of length 0.26 m located 0.13 m apart from the source cross-section which comprises a set of nine incoherent or coherent monopoles and axial dipoles uniformly distributed over the duct diameter. The inflow velocity is 41 m/s and the liner has maximal efficiency at 2760 Hz.

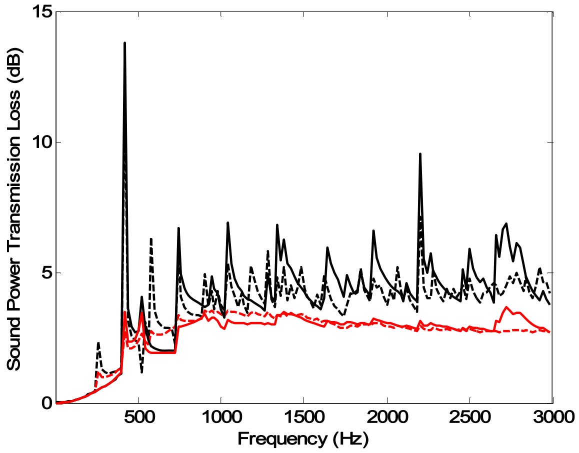

Figure 2. Sound power Transmission Loss predicted from an assumed model of 9 incoherent (dashed) or fully (solid) correlated monopoles (black) and axial dipoles (red) uniformly distributed over the duct diameter.

Fig. 2 clearly shows how the sound power Transmission Loss (TL), which scales on the ratio between the incident power before the liner and the transmitted power after the liner, significantly varies for either monopole- or dipole-type sources and, to a lesser extent, under different degrees of correlation between the sources. It can be seen that the TL is more efficient to attenuate in-duct pressure fields excited by monopole-type sources with respect to those due to dipole-type sources. In particular, the TL is more influenced by the duct cut-on frequencies under a monopole-type excitation with respect to an axial dipole-type excitation. Indeed, the former excitation induces a narrower filtering of the modes just above cut-on with respect to the latter, so that the low-order modes which are hardly dissipated by the liner are mostly filtered out when assuming a monopole-type excitation [8]. Therefore, it is important to develop suitable identification methods in order to retrieve the true source characteristics since they have a significant influence on the optimisation of the duct treatments.

2.2 The source characterization methods

Three methodologies are assessed for solving the ducted source reconstruction problem. Two model-based inverse methods are first considered: the ESM and the proposed SDM. In either case, the unknown CSD matrices, $\mathbf{S}_{\tilde{q}\tilde{q}}$ and $\mathbf{S}_{\tilde{\alpha}\tilde{\alpha}}$, for the source strength amplitudes and for their spectral coefficients, are sought in order to minimize the sum of the mean-square error between the measured pressures, $\tilde{\mathbf{p}}$, and those generated with the reconstruction sources, $\mathbf{p}$, which can be written as

$$J = \mathrm{Tr}\left(\mathbf{S}_{\tilde{e}\tilde{e}}\right), \tag{5}$$

with Tr the trace operator and $\mathbf{S}_{\tilde{e}\tilde{e}} = \mathrm{E}\left[\tilde{\mathbf{e}}\tilde{\mathbf{e}}^{\mathrm{H}}\right]$ the CSD matrix for the error vector which reads $\tilde{\mathbf{e}} = \tilde{\mathbf{p}} - \mathbf{p}$. When there are more measurement points than unknowns (either assumed sources or spectral coefficients), then Eq. (5) has a unique global minimum which corresponds to the following optimal CSD matrices for the ESM and the SDM, respectively

$$\begin{cases} \mathbf{S}_{\tilde{q}\tilde{q},\mathrm{opt.}} = \mathbf{G}^{\dagger}\mathbf{S}_{\tilde{p}\tilde{p}}\mathbf{G}^{\dagger\mathrm{H}}, \\ \mathbf{S}_{\tilde{\alpha}\tilde{\alpha},\mathrm{opt.}} = \mathbf{Z}^{\dagger}\mathbf{S}_{\tilde{p}\tilde{p}}\mathbf{Z}^{\dagger\mathrm{H}}, \end{cases} \tag{6}$$

where $\mathbf{G}^{\dagger}$ is the Moore-Penrose pseudo-inverse of the transfer matrix $\mathbf{G}$. At low frequencies, typically at frequencies for which the number of singular values of the transfer matrix (or the number of degrees of freedom of the in-duct field) falls below the assumed number of sources (resp. the number of spectral coefficients accounted for in Eq. (2)), regularization techniques are required to obtain meaningful reconstruction results. The L-curve parameter choice technique has been used due to its robustness to the presence of white noise in the measured field data.

A third method has been used which only allows the localisation of the unknown ducted sources, but not the reconstruction of their amplitudes. It is part of the beamforming approach which makes use of the phase information present in the array sensor signals to enhance, thorugh appropriate filtering of the microphones data, signals from a selected direction (or from a selected point source for focussed beamforming) while rejecting noise and interference from other directions (resp. sources).

The MUltiple SIgnal Classification (MUSIC) technique is found to be rather efficient in terms of accuracy and robustness. It belongs to the group of high-resolution subspaces methods. It is based on an eigen-decomposition of the CSD matrix, $\mathbf{S}_{\tilde{p}\tilde{p}}$, for the measured field data that provides an orthogonal basis for the signal subspace and the noise subspace onto which one builds an orthogonal projector $\mathbf{\Pi}^{\perp}$. The output power of the MUSIC focussed beamformer is then calculated for each virtual source positions and exhibits a peak when the signal is emitted from the positions of the real sources, i.e. when the signal is orthogonal to the noise subspace. It reads:

$$P(\mathbf{S}') = \frac{\mathbf{g}^{\mathrm{H}}(\mathbf{S}')\mathbf{g}(\mathbf{S}')}{\mathbf{g}^{\mathrm{H}}(\mathbf{S}')\mathbf{\Pi}^{\perp}\mathbf{g}(\mathbf{S}')}, \tag{7}$$

where $\mathbf{g}(\mathbf{S}')$ is the beamformer steering vector focussed at the virtual source position $\mathbf{S}'$. It corresponds to the pressure evaluated at the in-duct microphone positions due to a source located at $\mathbf{S}'$.

3 Simulation study for the identification of ducted noise sources

In order to compare the efficiency of the above mentioned approaches, parametric studies have been carried out for the identification of two punctual volume velocity sources (located respectively at the angular positions of 0° and 150°, with respect to Fig. 1, and at the same radial position $R/2$). These are broadband stationary noise sources which are assumed to be either fully correlated or uncorrelated. The in-duct pressure data are scanned over an array of 128 sensor positions which corresponds to 8 rings of 16 angular positions, as it can be achieved in the duct acoustic facility. The measured data are simulated adding a white Gaussian noise to the noise-free field data and corresponding to a Signal-To-Noise ratio of 13 dB. In the ESM, a ring of 16 source positions is assumed to be located at $R/2$, knowing that two of them exactly coincide with the real source positions, which constitutes a strong assumption. In the SDM, the unknown source distribution is decomposed as a series of 68 duct modes so that they are all propagating at the highest frequency for the reconstruction, i.e. at 15 kHz. Although the MUSIC focussed beamformer is steered over 128 virtual positions in the source cross-section, one has plotted for convenience the results of the angular exploration over a ring of 16 virtual sources located at the duct half-radius.

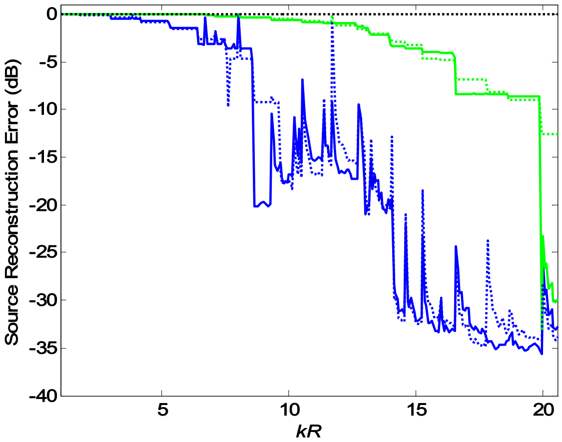

Figure 3. The mean-squared error in the reconstruction of two monopoles with random coherent (solid) and incoherent (dashed) source strengths as a function of the non-dimensional frequency kR when using the ESM (blue) and the SDM (green).

We have found useful to define two indicators. One is representative of the identification error (for the ESM and the SDM only) which accounts for both the errors on the localisation and on the reconstruction of the true source strengths. The other one only represents the localisation error (for the ESM, the SDM and the beamformer). They are plotted in Figs. 3 and 4 versus kR for a source-sensor axial separation distance, $\Delta z = 8R$.

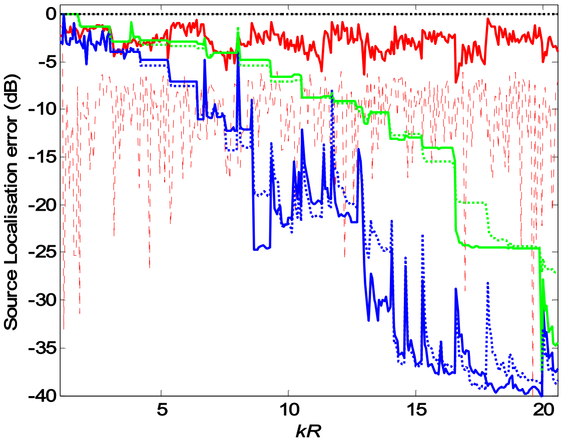

Figure 4. The error in the localisation of two monopoles with random coherent (solid) and incoherent (dashed) source strengths as a function of the non-dimensional frequency kR when using the ESM (blue), the SDM (green) and the MUSIC (red) methodologies.

The ESM provides a more accurate reconstruction of the source amplitudes with respect to the SDM over the whole frequency range, in a way rather independent of the degree of correlation between the sources. From Fig. 3, very accurate reconstruction results are obtained, which corresponds to levels of reconstruction error below -7 dB, obtained above about $kR = 8$ and $kR = 16$ with the ESM and the SDM, respectively. From Fig. 4, the localisation error follows the same trend although the error criterion is less stringent since an acceptable localisation of the real sources is already achieved for error levels below -5 dB.

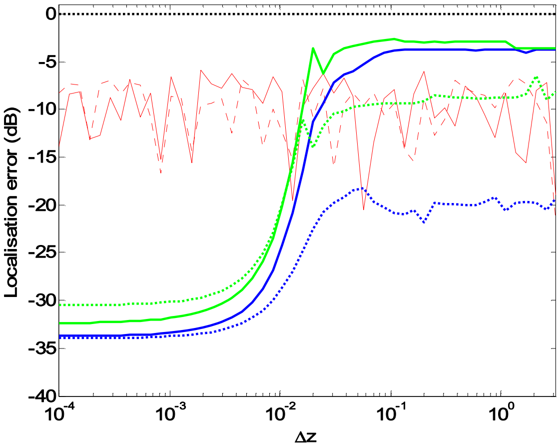

Figure 5. The error in the localisation of monopoles with random incoherent source strengths as a function of the source-sensor separation distance Δz at $kR = 3.3$ and $kR = 11.6$ when using the ESM (blue), the SDM (green) and the MUSIC (red) methodologies.

Fig. 4 shows, as expected, that the MUSIC beamformer allows a very accurate localisation of the uncorrelated sources over the whole frequency bandwidth whereas it does not perform so well in the case of correlated sources. From Fig. 5, one clearly distinguish the near-field zone ($\Delta z \leq 1\text{cm}$ at $kR = 3.3$) where both the ESM and the SDM provide very accurate localisation results due to the sensing of the duct evanescent modes which carry highly resolved information about the sources. Above this stand-off distance, the accuracy in the localisation of the sources is determined by the number of high-order propagating modes which mostly contribute to the in-duct pressure field.

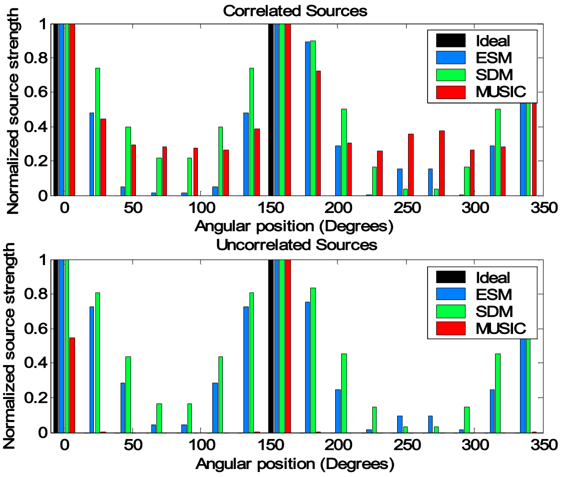

Figure 6. The normalized angular distribution of the assumed source strengths and those identified at $kR = 3.3$ when using the ESM, the SDM and the MUSIC methodologies in the coherent (top) and incoherent (bottom) cases.

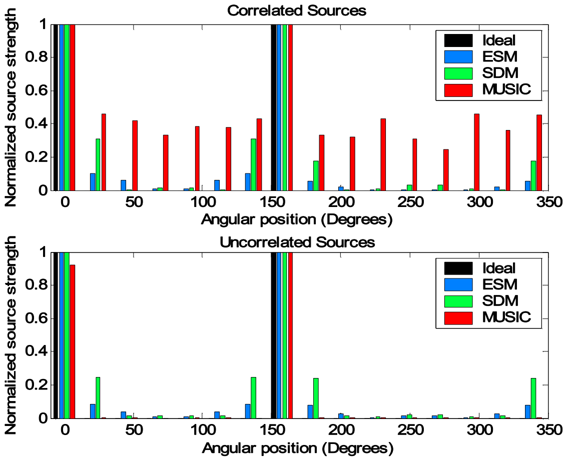

Figure 7. The normalized angular distribution of the assumed source strengths and those identified at $kR = 11.6$ when using the ESM, the SDM and the MUSIC methodologies in the coherent (top) and incoherent (bottom) cases.

Figs. 6 and 7 clearly show the high-resolution properties of the MUSIC beamformer when dealing with uncorrelated sources. Also, the ESM provides a better localisation of the true sources with respect to the SDM in both the correlated and uncorrelated cases.

4 Experimental reconstruction of ducted noise sources

The simulation study could benefit from experimental results obtained in the duct acoustic facility for the reconstruction of the volume velocity of a wall-mounted compression driver driven by white broadband noise and located at 45° above the horizontal duct axis (see Fig. 1). The reconstruction has been performed using the SDM from a set of in-duct pressure-based measurements acquired over a ring of 16 microphones in a cross sectional area located 0.25 m apart from the source section. Direct measurement of the driver volume velocity at the throat-duct interface has been made using a miniature Microflown particle velocity sensor. It serves as reference data with respect to which the experimental reconstruction results can be assessed.

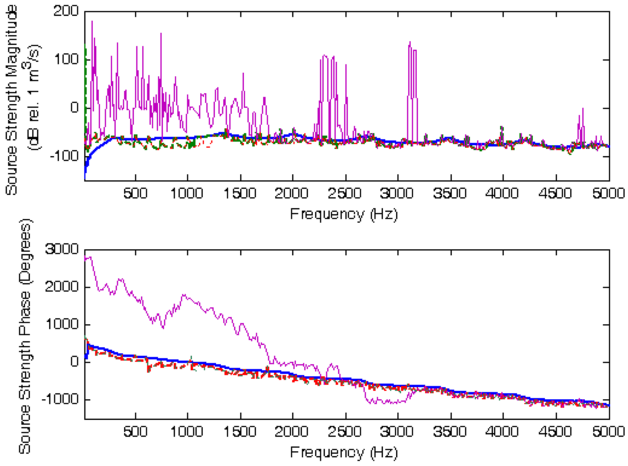

Figure 8. Source strength magnitude (top) and phase (bottom) of a compression driver wall-mounted on the duct at 45°: measured (blue) and reconstructed using the SDM with the regularized hybrid approach (GCV: magenta; L-curve: red; TSVD: green).

The reconstruction results are plotted on Fig. 8 which shows a comparison, in magnitude and phase, between the measured volume velocity and the one reconstructed with regularization. The regularised solution is obtained either with the Tikhonov method (using the L-curve and the GCV parameter-choice methods), or when using a Truncated Singular Value Decomposition (TSVD) that eliminates the low singular values responsible for the ill-conditioning of the transfer matrix. In general, the L-curve method performs better for the selection of the optimal regularisation parameter than the GCV. The TSVD based on the rank of the matrix provides almost identical values than those obtained with the L-curve. At high frequencies (above 2.3 kHz), the three identification methods converge with a good accuracy towards the measured reference volume velocity which could be reconstructed without regularization. Below 2.3 kHz, it can be seen that the L-curve and the TSVD regularized solutions provide a meaningful solution which still tracks with an acceptable accuracy the modulation frequencies of the volume velocity attributed to the driver distortion effects.

5 Conclusions

A series of simulations have shown that, when used in combination with an appropriate regularization technique, both the ESM and the SDM are able to provide accurate results for the reconstruction of the strength of ducted broadband sources, rather independently of the degree of correlation between the sources. Although the SDM provides less accurate reconstruction results with respect to the ESM, it does not require any primary assumption concerning the source distribution. Experimental reconstruction results have shown that the SDM is able to retrieve the volume velocity spectrum of a compression driver wall-mounted on a duct acoustic facility. If one deals with uncorrelated sources, an interesting alternative would be to use a high-resolution focused beamformer to provide the ESM with preliminary location of the unknown sources.

Acknowledgments

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