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Vibroacoustic interface conditions between prestressed structures and moving fluids

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Many applications involve coupling between prestressed solids and fluids (possibly flowing). Typical problems might be given by vibroacoustics of fluid-filled pressurized cavities, wave propagation, dynamics and stability of pipes conveying fluids... The goal of this work is to investigate jump conditions that hold for small linear perturbations at any impermeable interfaces, slip or bonded, plane or not, between fluids and structures in the presence of initial flow and prestress. First, the concept of generalized functions in distribution theory and its applications to interface conditions is briefly recalled. One also gives a review about the mixed Eulerian-Lagrangian description, that yields an interesting unification between existing formulations for inviscid fluids (Galbrun's equation) and solids (updated Lagrangian formulation). Based on conservative equations obtained from this description, interface conditions are then derived in an elegant manner thanks to the concept of generalized functions. Interface conditions for displacement and stress are obtained and agree with results found in the literature. As an example, these conditions are applied to the analysis of vibroacoustic wave propagation inside an inviscid fluid-filled pressurized duct. The combined effects of prestress, initial flow and acoustic coupling are briefly discussed.

1 Introduction

Many applications involve small wave motion through an interface between prestressed solids and fluids, initially moving or not: aeroengine ducts, human arteries, acoustic measurements in geophysics, ultrasonic stress characterization, heat exchangers, nuclear reactors, storage tanks...

The motion of particles at the interface between both media in contact can occur with slip (when the fluid is considered as inviscid) or without (for a viscous fluid). In the absence of flow and prestress, interface conditions are well-established: the kinematic and dynamic jump conditions respectively correspond to the continuity of the normal components of the acoustic velocity or displacement (and their tangential components in the no slip case) and of the acoustic stress tensor. However, the derivation of the appropriate interface conditions is somewhat complicated when an initial flowing or prestressed state exists. The possibility of slip further complicates this derivation.

For the acoustic kinematic jump condition, considerable discussion appeared years ago in the literature as to whether continuity of particle normal displacement or normal velocity is the appropriate boundary condition for an inviscid flowing fluid [1]. Myers [2] derived a kinematic condition based on the Eulerian acoustic velocity. Godin [3] proved the equivalence between the latter and the normal Lagrangian displacement continuity.

Few papers deal with the dynamic jump condition for a slip interface in the presence of prestress. Poirée [4] used distribution theory in order to derive a stress jump condition for plane interfaces and Goy [5] later extended his work to arbitrary non-plane interfaces. When studying incompressible hydroelastic vibrations, Schotté and Ohayon [6] obtained linearized boundary conditions on the interface between a prestressed structure and an inviscid fluid in the absence of flow. Norris et al. [7] made a thorough analysis of non-moving fluid/solid systems and derived some stress jump conditions valid for both slip and bonded interfaces. Most of these studies point out the benefits of an intermediate formulation compared to a full Lagrangian or Eulerian approach. This intermediate formulation is often referred to as “updated Lagrangian” in non-linear mechanics and sometimes “mixed Eulerian-Lagrangian” in flow acoustics.

The goal of this paper is to investigate and clarify jump conditions that hold for linear perturbations at fluid-structure interfaces (slip or bonded, plane or not) in the presence of initial flow and prestress. This work is based on the concept of generalized functions in distribution theory and its application to a mixed Eulerian-Lagrangian formulation. It is restricted to impermeable interfaces (immiscible media always in contact with no void).

2 Theoretical background

2.1 Generalized functions and interface conditions

One elegant way to derive jump conditions is to interpret conservative equations of continuum mechanics in terms of distribution theory through the concept of generalized derivatives and to postulate that these equations hold in the sense of distributions [8, 4]. This procedure is now briefly recalled.

For instance, let us start with the momentum conservative equation:

$$\frac{\partial(\rho\mathbf{v})}{\partial t}+\nabla\cdot(\rho\,\mathbf{v}\otimes\mathbf{v}-\boldsymbol{\sigma})=\rho\,\mathbf{f} \tag{1}$$

where $\rho, \mathbf{v}, \boldsymbol{\sigma}, \mathbf{f}$ respectively denote density, velocity, Cauchy stress tensor and force per unit mass.

Let us denote $[\![\cdot]\!]_{S=0}$ the jump $_+(\cdot)-{}_-(\cdot)$ on the interface defined by the implicit surface equation $S(\mathbf{x},t)=0$ having a unit normal $\mathbf{n}(\mathbf{x},t)$. Left subscripts $-$ and $+$ denote the media considered, respectively defined by $S<0$ and $S>0$. The unit normal is oriented from $-$ medium to $+$ so that $\mathbf{n}=\nabla S/|\nabla S|$. The interface is assumed smooth (S is differentiable). Every physical variable might be discontinuous at the interface, so that we assume the following decompositions:

$$\Psi={}_-\Psi\,H(-S)+{}_+\Psi\,H(+S) \tag{2}$$

for $\Psi=\{\rho,\mathbf{v},\boldsymbol{\sigma},\mathbf{f}\}$ and where H denotes the heavyside function.

Now, recall that time and spatial derivatives of $H(\pm S)$ are:

$$\frac{\partial}{\partial t}(H(\pm S))=\mp w_n|\nabla S|\,\delta(S)\ ,\qquad \nabla(H(\pm S))=\pm\mathbf{n}|\nabla S|\,\delta(S) \tag{3}$$

where w_n is the normal velocity of the surface $S=0$ and δ denotes the Dirac distribution.

Using Eq.(2) into (1) and using (3), it can be verified that the following equation is arrived at:

$$_{-}\mathbf{E}\,H(-S)+{}_{+}\mathbf{E}\,H(+S)+{}_{\delta}\mathbf{E}|\nabla S|\delta(S)=\mathbf{0} \qquad (4)$$

with the following notations:

$$_{\pm}\mathbf{E}=\frac{\partial({}_{\pm}\rho\,{}_{\pm}\mathbf{v})}{\partial t}+\nabla\cdot({}_{\pm}\rho\,{}_{\pm}\mathbf{v}\otimes{}_{\pm}\mathbf{v}-{}_{\pm}\boldsymbol{\sigma})-{}_{\pm}\rho\,{}_{\pm}\mathbf{f} \qquad (5)$$

and:

$$_{\delta}\mathbf{E}=[\![\rho\mathbf{v}(\mathbf{v}\cdot\mathbf{n}-w_n)-\boldsymbol{\sigma}\cdot\mathbf{n}]\!] \qquad (6)$$

The identification of each term in Eq.(4) yields $_{-}\mathbf{E}=\mathbf{0}$ for $S<0$ and $_{+}\mathbf{E}=\mathbf{0}$ for $S>0$, corresponding to the equilibrium equations (1) inside media $-$ and $+$. It also yields $_{\delta}\mathbf{E}=\mathbf{0}$ on $S=0$: this is the interface jump conditions associated with momentum conservation. Note that $w_n={}_{-}\mathbf{v}\cdot\mathbf{n}={}_{+}\mathbf{v}\cdot\mathbf{n}$ for impermeable interfaces (fundamental assumption of this paper), so that the jump condition degenerates into the well-known result $[\![\boldsymbol{\sigma}\cdot\mathbf{n}]\!]_{S=0}=\mathbf{0}$.

2.2 Eulerian-Lagrangian description

This section reviews the governing equilibrium equations associated with the mixed Eulerian-Lagrangian description.

When analyzing the dynamics of continuum media non initially at rest, one can choose to write physical fields with respect to:

- the Lagrangian variables $(\mathbf{x}_{ref},t)$ (reference configuration),
- the intermediate variables $(\mathbf{x}_0(t),t)$ (intermediate configuration corresponding the unperturbed flowing/prestress state),
- or the Eulerian variables $(\tilde{\mathbf{x}}(t),t)$ (total or current configuration including perturbations).

Physical fields referring to these configurations will respectively be denoted with a subscript *ref*, a subscript 0 and a tilde. Their respective gradients will then be denoted by ∇_{ref}, ∇_0 and $\tilde{\nabla}$. The absence of symbol will be left for superimposed oscillatory perturbations. Denoting $\mathbf{u}$ as the particle small oscillatory displacement, $\mathbf{x}_0$ and $\tilde{\mathbf{x}}$ are then related by $\tilde{\mathbf{x}}=\mathbf{x_0}+\epsilon\,\mathbf{u}$. Let $\tilde{\Psi}(\tilde{\mathbf{x}},t)$ denote any field – scalar, vector or tensor – describing the current state in terms of Eulerian coordinates. Both the following linear perturbations can be defined:

$$\epsilon\,\Psi^E=\tilde{\Psi}(\tilde{\mathbf{x}},t)-\Psi_0(\tilde{\mathbf{x}},t),\;\; \epsilon\,\Psi^L=\tilde{\Psi}(\tilde{\mathbf{x}},t)-\Psi_0(\mathrm{x}_0,t) \qquad (7)$$

Superscripts *E* and *L* will respectively denote Eulerian and Lagrangian perturbations. One exception is for the displacement perturbation, $\mathbf{u}=\mathbf{u}^L$ (avoiding cumbersome expressions). Eulerian perturbations are naturally used in fluid acoustics. They are associated with the same geometrical point but not necessarily the same material point. Lagrangian perturbations, usually used in solid mechanics, are associated to the same particle; when they are written with respect to $(\mathbf{x}_0,t)$, as done here, the description might be called mixed Eulerian-Lagrangian description (in contrast with a full Lagrangian description, written with respect to the Lagrangian variables $(\mathbf{x}_{ref},t)$).

From definitions (7), the relation between Eulerian and Lagrangian first order perturbations is $\Psi^L=\Psi^E+(\mathbf{u}\cdot\nabla_0)\Psi_0$. The Lagrangian perturbations rules for derivatives are not straightforward and are shown to be:

$$\begin{aligned}(\tilde{\nabla}\,\tilde{\Psi})^L&=\nabla_0\Psi^L-\nabla_0\Psi_0\cdot\nabla_0\mathbf{u}\;,\\ \left(\frac{\partial\tilde{\Psi}}{\partial t}\right)^L&=\frac{\partial\Psi^L}{\partial t}-\nabla_0\Psi_0\cdot\frac{\partial\mathbf{u}}{\partial t}\end{aligned} \qquad (8)$$

Applying the perturbation rules (8) to mass conservation, one arrives at the simple result:

$$\frac{\rho^L}{\rho_0}+\nabla_0\cdot\mathbf{u}=0 \qquad (9)$$

After rearrangements, the perturbation of momentum conservation Eq.(1) yields the following conservative equilibrium equation for perturbations:

$$\begin{aligned}&\frac{\partial(\rho_0\mathbf{v}^L)}{\partial t}+\nabla_0\cdot(\rho_0\mathbf{v}^L\otimes\mathbf{v}_0-\boldsymbol{\sigma}^L\\ &-\boldsymbol{\sigma}_0\cdot((\nabla_0\cdot\mathbf{u})\mathbf{I}-\nabla_0\mathbf{u}^T))=\rho_0\mathbf{f}^L\end{aligned} \qquad (10)$$

For more details about the mixed Eulerian-Lagrangian description, the reader can refer to the work of Poirée [4].

The Cauchy stress increment $\boldsymbol{\sigma}^L$ (Lagrangian perturbation of Cauchy stress) is barely used in solid mechanics. Another kind of stress increment, denoted $\boldsymbol{\sigma}$, naturally appears from a transformation from current to intermediate coordinates: $\boldsymbol{\sigma}=\det(\tilde{\mathbf{X}}_0)\,\tilde{\mathbf{X}}_0^{-1}\tilde{\boldsymbol{\sigma}}\,\tilde{\mathbf{X}}_0^{-T}-\boldsymbol{\sigma}_0$ (with $\tilde{\mathbf{X}}_0=\nabla_0\tilde{\mathbf{x}}=\mathbf{I}+\nabla_0\mathbf{u}$). $\boldsymbol{\sigma}$ may be referred to as the updated Kirchhoff stress increment tensor in the so-called updated Lagrangian formulation [9]. The linearized updated Lagrangian formulation and the mixed Eulerian-Lagrangian formulation are indeed the same. A formal proof can be obtained by linearizing the above definition of $\boldsymbol{\sigma}$, yielding:

$$\boldsymbol{\sigma}^L+\boldsymbol{\sigma}_0\cdot((\nabla_0\mathbf{u})\mathbf{I}-\nabla_0\mathbf{u}^T)=\boldsymbol{\sigma}+\nabla_0\mathbf{u}\cdot\boldsymbol{\sigma}_0 \qquad (11)$$

Using Eq.(11) into (10) gives after simplifications: $\rho_0 d_0^2\mathbf{u}/dt^2-\nabla_0\cdot(\boldsymbol{\sigma}+\nabla_0\mathbf{u}\cdot\boldsymbol{\sigma}_0)=\rho_0\mathbf{f}^L$. This is the linearized updated Lagrangian formulation. For a linearly elastic solid, the stress-strain relationship is given by $\boldsymbol{\sigma}=\mathbf{C}:\boldsymbol{\epsilon}$, where $\boldsymbol{\epsilon}=1/2(\nabla_0\mathbf{u}+\nabla_0\mathbf{u}^T)$ is the linearized incremental strain tensor and $\mathbf{C}$ is the constitutive tensor.

Now for inviscid fluids, $\boldsymbol{\sigma}^L=-p^L\mathbf{I}$. p^L is barely used, in acoustics also, Eulerian perturbations being usually preferred. Nevertheless, the Eulerian-Lagrangian description yields an interesting wave equation for arbitrary inviscid flowing fluids. This equation is sometimes called Galbrun's equation and is written in terms of $\mathbf{u}$ only (further details in [4, 10, 11]).

Hence, the use of Lagrangian perturbations written in terms of the intermediate coordinates gives a straightforward unification between existing fluid and solid formulations. Another interest lies in the fact that Eqs.(9) and (10) can be interpreted in the sense of distributions, as shown in the next section.

3 Interface conditions for perturbations

3.1 Kinematic condition

One postulates that Eq.(9) is valid in the sense of distribution theory. This equation has a conservative form. Following the same procedure as in Sec. 2.1, it can be verified that its associated jump condition gives the continuity of normal Lagrangian perturbation of displacement:

$$[\![\mathbf{u}\cdot\mathbf{n}_0]\!]_{S_0=0}=0 \qquad (12)$$

The validity of this condition must be discussed. As stated earlier, some ambiguity may arise when an inviscid mean flow is present. The reference kinematic condition is the well known condition of Myers [2], valid for any stationary interface. Myers condition is written in terms of $\mathbf{v}^E$, the Eulerian acoustic velocity, and has a more complex form than condition (12). Although one can doubt about the validity of Eq.(12) at first sight, both conditions are indeed equivalent: this has been already proved by Godin [3].

3.2 Dynamic condition

The same method is now applied to Eq.(10), which is also a conservative equation. Calculations are somewhat tedious and will not be detailed here for conciseness. One arrives at an equation of the following form:

$$_{-}\mathbf{E}\,H(-S)+{}_{+}\mathbf{E}\,H(+S)+{}_{\delta}\mathbf{E}|\nabla S|\delta(S)+{}_{\delta'}\mathbf{E}\,\delta'(S)=\mathbf{0} \qquad (13)$$

This form is similar to that given in Eq.(4), except for the Dirac derivative term $_{\delta'}\mathbf{E}$. Calculations show that this term indeed vanishes thanks to the kinematic condition Eq.(12). Besides, $_{-}\mathbf{E}=\mathbf{0}$ and $_{+}\mathbf{E}=\mathbf{0}$ correspond to Eq.(10) in both – and + media respectively. The dynamic jump condition that we are looking for is $_{\delta}\mathbf{E}=\mathbf{0}$, which is explicitly given as follows:

$$\begin{aligned}&[\![\left(\boldsymbol{\sigma}^L+\boldsymbol{\sigma}_0\cdot((\nabla_0\cdot\mathbf{u})\mathbf{I}-\nabla_0\mathbf{u}^T)\right)\cdot\mathbf{n}_0]\!]_{S_0=0}\\ &-\frac{1}{|\nabla_0 S_0|}\nabla_0\cdot\{(|\nabla_0 S_0|(\boldsymbol{\sigma}_0\cdot\mathbf{n}_0)\otimes[\![\mathbf{u}_\perp]\!]_{S_0=0}\}=\mathbf{0}\end{aligned} \qquad (14)$$

with $\mathbf{u}_\perp=\mathbf{u}-(\mathbf{u}\cdot\mathbf{n}_0)\,\mathbf{n}_0$ defining the displacement tangential component. Note that the second line term vanishes in the no-slip case. As a side remark, the dynamic condition remains unchanged in the presence of initial flow.

The above condition is in fact exactly the jump condition derived by Norris [7] and generalized to non-uniform prestress along the interface. This is proved from the following identity:

$$\frac{1}{|\nabla_0 S_0|}\nabla_0\cdot\{(|\nabla_0 S_0|[\![\mathbf{u}_\perp]\!]_{S_0=0}\}=[\![\nabla_0\cdot\mathbf{u}-\mathbf{n}_0\cdot(\nabla_0\mathbf{u})\cdot\mathbf{n}_0]\!]_{S_0=0} \qquad (15)$$

which allows to rewrite condition (14) as the following Norris form:

$$\begin{aligned}&[\![\mathbf{P}\cdot\mathbf{n}_0-\left(\nabla_0\cdot\mathbf{u}-\mathbf{n}_0\cdot(\nabla_0\mathbf{u})\cdot\mathbf{n}_0\right)\boldsymbol{\sigma}_0\cdot\mathbf{n}_0]\!]_{S_0=0}\\ &-[\![(\mathbf{u}_\perp\cdot\nabla_0)(\boldsymbol{\sigma}_0\cdot\mathbf{n}_0)]\!]_{S_0=0}=\mathbf{0}\end{aligned} \qquad (16)$$

with the notation: $\mathbf{P}=\boldsymbol{\sigma}^L+\boldsymbol{\sigma}_0\cdot((\nabla_0\cdot\mathbf{u})\mathbf{I}-\nabla_0\mathbf{u}^T)$.

3.3 Note for energy and entropy conditions

As far as energy and entropy conservation are concerned, the same procedure could be applied. The first step is to perform a Lagrangian perturbation (with a mixed Eulerian-Lagrangian description) of energy and entropy conservative equations. The second step is to rewrite the resulting equations under a conservative form, analogous to Eq.(10). Finally, interpreting them with the help of generalized functions will yield jump conditions for heat flux and temperature perturbations.

4 Example: vibroacoustic waves in a pressurized cylindrical shell

4.1 Equations

Wave propagation inside a pressurized cylindrical thin shell coupled with an acoustic internal fluid is studied. The prestress state is static. An initial uniform flow might be present in the fluid. For simplicity, the external fluid effect is arbitrarily neglected (vacuum). The initial fluid pressure p_0 (which should be thought as the difference between internal and external pressures) is assumed spatially constant. For the shell, the following assumptions are adopted: the radius-thickness ratio is small ($R/h>20$ in practice), the material is linearly elastic and isotropic, shear strains are neglected (Kirchhoff hypothesis) as well as normal stresses. Ones chooses to make the simplification of Reissner-Naghdi-Berry. Axial and torsional prestresses are not considered in the analysis. The only non-zero prestress is the circumferential prestress, denoted $N_{0\theta}$, due to pressurization. For an infinitely long cylinder, the approximation $N_{0\theta}=p_0 R$ holds. Wave solutions are sought of the form $e^{i(\gamma s+n\theta-\omega t)}$ ($s=x/R$ corresponds to the cylinder axis). The shell equilibrium equations are as follows (see Leissa [12] for more details):

$$(\mathbf{L}+\mathbf{L_0})\mathbf{u}=\mathbf{f} \qquad (17)$$

where **L** is the Reissner-Naghdi-Berry operator:

$$\mathbf{L}=\begin{bmatrix} -\gamma^2-\frac{1-\nu}{2}n^2+\frac{1-\nu}{2}\Omega^2 & -\frac{1+\nu}{2}\gamma n & i\nu\gamma \\ -\frac{1+\nu}{2}\gamma n & -(1+k)(\frac{1-\nu}{2}\gamma^2+n^2)+\frac{1-\nu}{2}\Omega^2 & in(1+k\gamma^2+kn^2) \\ i\nu\gamma & in(1+k\gamma^2+kn^2) & 1+k(\gamma^2+n^2)^2-\frac{1-\nu}{2}\Omega^2 \end{bmatrix} \qquad (18)$$

and $\mathbf{L_0}$ is the Herrmann-Armenakas prestress operator:

$$\mathbf{L_0}=\frac{1-\nu^2}{E\,h}N_{0\theta}\begin{bmatrix}-n^2 & 0 & 0\\ 0 & -1-n^2 & 2in\\ 0 & 2in & 1+n^2\end{bmatrix} \qquad (19)$$

$\mathbf{f}$ represents the force exerted by the fluid. One has $\mathbf{u}=[u\,v\,w]^T$, where u, v w are respectively the shell displacement amplitudes with respect to the axial, azimuthal and radial directions. E, ν, ρ_s are respectively the Young modulus, Poisson coefficient, density and $k=h^2/12R^2$. Ω denotes the adimensional angular frequency, chosen as $\omega R/c_s$, where $c_s=\sqrt{E/2\rho_s(1+\nu)}$ is the shear velocity.

Note that the above expression of $\mathbf{L_0}$ also implicitly assumes that prebending stresses and predisplacements have a negligible effect upon vibrations (as usually done in prestressed dynamics).

For acoustics, the presence of internal pressure does not alter the governing equation, corresponding to the convected Helmholtz equation, so that $p^L(r)=AJ_n(\alpha r)$ with the notation: $\alpha^2R^2=(\Omega c_s/c_f-M_0\gamma)^2-\gamma^2$. ρ_f, c_f, M_0 denote fluid density, sound celerity and initial flow Mach number.

When the effect of p_0 is neglected on the shell, one simply has:

$$\mathbf{f}=\frac{1-\nu^2}{Eh}\left[0\ 0\ p^L R^2\right]^T \tag{20}$$

However, if pressurization effect is taken into account – which is barely done in the literature – it is shown that some additional terms appear in the structural equilibrium equations. A rigorous derivation of the operator $\mathbf{f}$ is now made based on the general jump conditions previously presented.

Let us start from a variational formulation of the structure. The term of interest is given by the boundary integral on the interface. The application of dynamic condition (14) yields:

$$\begin{aligned}\int_{S_0}\delta_+\mathbf{u}\cdot(\boldsymbol{\sigma}+\nabla_{0+}\mathbf{u}\cdot\boldsymbol{\sigma}_0)\cdot\mathbf{n}_0\,dS=-\int_{S_0}\delta_+\mathbf{u}\cdot(&p^L\mathbf{n}_0\\ +p_0(\nabla_0\cdot{}_-\mathbf{u})\,\mathbf{n}_0-p_0\nabla_0{}_-\mathbf{u}^T\cdot\mathbf{n}_0&\\ +\frac{1}{|\nabla_0 S_0|}\nabla_0\cdot\{(|\nabla_0 S_0|\,p_0\mathbf{n}_0\otimes[\![\mathbf{u}_\perp]\!]_{S_0=0}\})\,dS&\end{aligned} \tag{21}$$

where ${}_+\mathbf{u}, {}_-\mathbf{u}$ respectively denotes solid and fluid displacement Lagrangian perturbations. $\mathbf{n}_0$ is the outward normal of the structure.

After expressing the above integral in the cylindrical coordinate system and taking into account condition (12), one finally arrives at:

$$\begin{aligned}&\int_{S_0}\delta_+\mathbf{u}\cdot(\boldsymbol{\sigma}+\nabla_{0+}\mathbf{u}\cdot\boldsymbol{\sigma}_0)\cdot\mathbf{n}_0\,dS=\iint(\delta w\,p^L R^2+\\ &p_0R\left(-\delta u\frac{\partial w}{\partial s}+\delta v(v-\frac{\partial w}{\partial\theta})+\delta w(\frac{\partial u}{\partial s}+\frac{\partial v}{\partial\theta}+w)\right))ds\,d\theta\end{aligned} \tag{22}$$

where we have neglected z/R terms according to Reissner-Naghdi-Berry simplifications, as well as prebending terms. The first term of the right hand side, in terms of p^L, yields the force expression (20). The other terms (second line) are written in terms of the shell displacement only, yielding the following operator that must be added to the left-hand side of Eq.(17):

$$\mathbf{L_0'}=-\frac{1-\nu^2}{Eh}p_0R\begin{bmatrix}0 & 0 & i\gamma\\ 0 & -1 & in\\ i\gamma & in & 1\end{bmatrix} \tag{23}$$

This additional operator $\mathbf{L_0'}$ represents the effect of pressurization. It is due to the fact that a hydrostatic pressure is indeed a following force (non-constant directional). One emphasizes that this operator exactly coincides with the one found by Armenakas [13] for small h (hypothesis of our work) when studying the dynamics of in vacuo pressurized thin cylindrical shells (no fluid coupling in their analysis).

For solving the coupled vibroacoustics problem, the system must be rewritten in terms of the full eigenvector $\mathbf{u}=[u\,v\,w\,A]^T$ (now including pressure amplitude), thanks to the normal displacement continuity Eq.(12):

$$\rho_f c_f^2(\Omega c_s/c_f-M_0\gamma)^2 w=R^2\partial p^L/\partial r|_{r=R} \tag{24}$$

The obtained 4x4 eigensystem is then numerically solved by searching the roots Ω for fixed γ that make the determinant vanish with a Newton-Raphson method.

4.2 Results

For all results, solid and fluid media are assumed to have the following properties: E=2.0e+11Pa, ν=0.3, ρ_s =7800kg/m^3, c_f=1500m/s, ρ_f=1000kg/m^3. Shell radius and thickness are R=1m and h=0.01m. The initial pressure is p_0=2.2e+6Pa. Note that the effect of initial pressure might change fluid properties, but this has been neglected here. For clarity of figures, only modes with n varying from 0 to 4 are computed. Besides, the following analysis and plots are only given for $\Omega\in[0;0.5]$.

Figure 1 gives the dispersion curves Ω vs. γ of the in vacuo shell (no fluid interaction), for p_0=0 as well as for p_0=2.2e+6Pa. Pressurization effects appear negligible for n=0 and n=1 modes. However, n>1 modes are strongly affected. Their frequencies are increased for fixed k – or equivalently, their phase velocities are increased for fixed Ω. This effect is greater for small wavelengths and becomes negligible for higher wavelengths. These observations coincide with the results already found by Armenakas [13]. Figure 1 also plots the only rigid-wall acoustic mode propagating for the frequency band considered, corresponding to the n=0 plane wave mode.

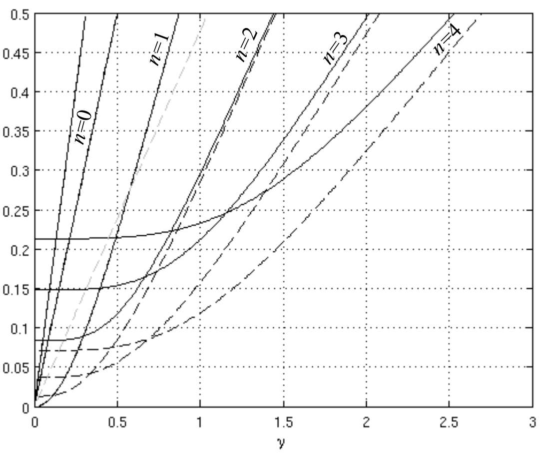

Fig.1 Ω vs. γ for in vacuo elastic modes with pressurization (solid lines), without (dashed), and for the rigid wall acoustic mode (dashed gray).

Figure 2 exhibits the dispersion behavior of the coupled fluid-solid system. Strong differences are observed when compared to the results of Fig.1. It can be seen that the fluid loading have a significant effect for the *n*=0 fluid-type mode as well as *n*=1 modes. These modes are yet only little changed by pressurization. However, frequencies of *n*>1 modes are sensitive both to pressurization and fluid loading. Taking into account fluid loading significantly reduces their frequencies compared to the in vacuo pressurized shell results.

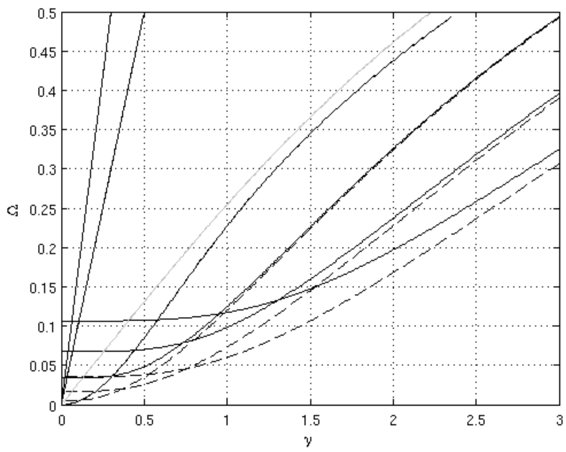

Fig.2 Ω vs. γ for structure-type modes with pressurization (solid lines), without (dashed). Gray lines: fluid-type mode.

Figure 3 shows the influence of an initial flow $M_0 = 0.05$ on dispersion curves (with pressurization). As can be observed, the presence of flow increases the frequencies of non-axisymmetric modes ($n \neq 0$) and of the fluid-type mode (*n*=0). Axisymmetric structure-type modes are left nearly unchanged.

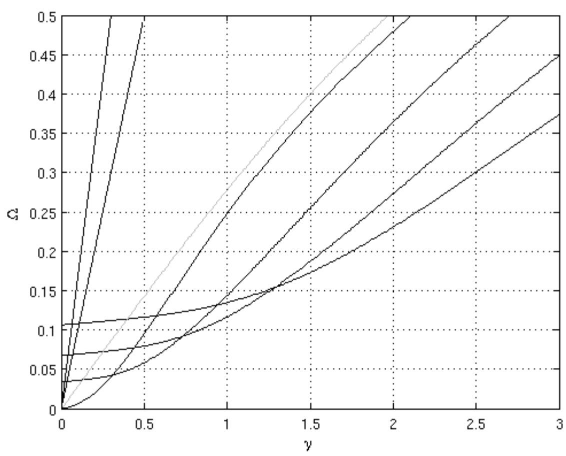

Fig.3 Ω vs. γ for $M_0 = 0.05$ (with pressurization). Same legend as Fig. 2.

5 Conclusion

In this paper, one has derived some general vibroacoustic interface conditions in the presence of an initially flowing fluid and a prestressed structure. These conditions have been obtained in an elegant manner thanks to the concept of generalized functions and the use of a mixed Eulerian-Lagrangian description. They have then been applied to the dispersion analysis of vibroacoustic wave propagation inside an inviscid fluid-filled pressurized shell. The combined effects of pressurization, fluid loading and initial flow have been briefly outlined.

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