individual reed characteristics due to changed damping using coupled flow-structure and time-dependent geometry changing Finite-Element calculation

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The characteristics of four saxophone reeds are investigated. Therefore, a time-dependent Finite-Element model of the mouth/reed/mouthpiece system of an alto saxophone is built. A coupling of the differential equations is used for the air flow, the reed vibration, and the change of geometry of the air flow by the reed vibrations. The four reeds are a standard reed and three deviations. All other parameters are kept the same for all reeds to make the reed movements comparable. Clear differences of the reed sounds are found showing different spectral amplitude behaviour in terms of the 3 kHz formant region, higher harmonics and the speed to reaction the reeds show with back coming impulses. Furthermore, the turbulence in the mouthpiece could be shown to be that high, that the assumption of the standard model for single reed vibrations is reasonable when setting the pressure inside the mouthpiece equal to the pressure in the tube. Also the amount of movement along the reed lengths could be shown to differ between the reeds and so the idea of the altered reeds to be more flexible in certain regions are discussed.

1 Introduction

The saxophone or the clarinet are single reed instruments. Different reeds are used by players to obtain different sounds. The reeds can differ in overall thickness or in overall geometry. This study investigates the role of different geometries of alto saxophone reeds. Four different reeds are used (see section Method), a standard reed and three derivatives which are used today by saxophone players.

To investigate this problem, the time dependent behaviour of saxophones need to be considered. The nonlinear model of the flow-reed interaction assumes a pressure difference between the mouth and the mouthpiece acting on the reed. Several analytical and experimental investigations show this behaviour (Dalmont 2007, 2003, da Silva 2007).

Reeds often show a self-sustained oscillation as with harps or accordions. Also the saxophone or clarinet mouthpiece/reed system can show such vibrations even if not attached to the resonance tube. Then the periodicity is much higher than normal tones played on these instruments and, according to the mouthpiece length. Additionally, higher blowing pressure is needed to produce this kind of oscillation. Hirschberg discusses possible reasons of the reed/mouthpiece system to be self-oscillating (Hirschberg 1995).

Another approach is to look at the system working with impulses produced by the mouth/reed/mouthpiece system. The blowing pressure of the mouth acts upon the reed on both sides, the side of the reed in the mouthpiece and the bottom reed side in the mouth cavity. The players lip touches the reed in such a way, that about 1.5 cm of the reed's bottom is free in the mouth where the mouth pressure acts upon it. As the flow of air is fast when travelling through the tip of the mouthpiece, the pressure at the upper side of the reed is much lower than that on its bottom. This pressure difference between upper side and bottom side of the reed causes the reed to move upwards and therefore closes the mouthpiece tip. Because of this closing, a pressure impulse is produced.

This impulse then travels along the tube and is partly radiated and partly reflected at the open sound whole or the saxophone bell. The reflected part of the impulse is travelling back and reaches the reed again. Here, it 'shoots' the reed open. The reed is again in an unstable condition then and the pressure difference between upper and bottom side of the reed causes it to close once more producing another pressure impulse which is again travelling down the tube. Here, a periodicity is established. Its length is determined by the time the impulse needs to travel back and forth the tube and therefore the player is able to determine the pitch of the instrument by using fingerings.

Note, that the production side of mouth/reed/mouthpiece is not a self-sustained oscillation. The pressure gradient at the reed closes the reed. The reed would never open again by itself as the pressure in the mouth is always much higher than that in the mouthpiece. The back travelling impulse from the bell is needed to open it again. Now, the precise way, the reed opens or closes is determined by many factors. As in our discussion the reed is the focus of investigation, we need to keep all parameters constant and only change the reeds geometry to find out about the basic behaviour of different reeds.

Aschhoff (Aschhoff 1936) was one of the first the discuss the acoustics of a clarinet. He asked, way it is the length of the tube determining the overall pitch of the system and not the mouth/reed/mouthpiece system. The answer is simply, that the reed is much more damped than the air column and therefore in this situation of two nonlinear coupled oscillators, the tube wins and forces the reed to vibrate with its periodicity. Only in cases where the reed is less damped like with harp or accordion reeds, a self-sustained oscillation can appear.

Still the saxophone reed is oscillating for itself, too, where the fundamental frequency of saxophone reeds is known to be around 3 kHz. So this region has been proposed to be a kind of formant region, helping to identify saxophone tones. So what we expect to find is, that the reed reaches a stable state if no impulse is coming back the tube and, when shot open by a back impulse, is vibrating with some of its eigenvalues after the impulse is gone out again.

Here some fundamental questions can be answered:

A) How do different reeds differ in their way of vibration when shot open by an impulse?

B) Is there a formant region for the reeds and how does it look like?

C) How differs the overall amplitude behaviour for different frequency regions?

D) Are the reeds still fluctuating a bit producing the noise known from reed instruments or do they come to a rest between impulses?

E) How much turbulence is present in the mouthpiece? Is it enough to set the pressure in the mouthpiece equal to the pressure in the tube as proposed by the standard model?

F) How do different reeds react when played with different playing styles (soft, staccato, legato ect.)?

Question E) is beyond the scope of this paper and left to further investigations.

2 Method

A 2D model of the reed, the air in the mouth, and the air in the mouthpiece was built with the mouthpiece of an alto saxophone. A time dependent Finite-Element calculation was performed coupling a Navier-Stokes equation for the flow, a stress-strain equation for the reed and a moving-mesh model for the changing geometry of the flow according to the reed motion to model the different behaviour of the reeds. Fig. 1 shows the geometry of the model with the applied Finite Element mesh of triangular elements. The mouth was modelled in a way a saxophone player would do when laying the lips on the mouthpiece tip. So the air is modelled to 1.4 cm above and below the mouthpiece. The portion of the mouth air cavity which is below the mouthpiece ends at the reed and acts upon it.

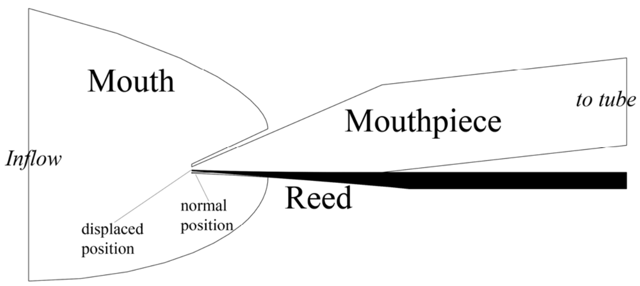

Fig. 1: Geometry of the model with mouth, mouthpiece, and reed. The inflow boundary condition is at the left of the mouth, at the right of the mouthpiece the tube would be attached. The reed is black and the undisplaced (normal) and displaced positions of the reed are shown. The reeds displacement causes a geometry change of the flow domain and therefore acts on the flow.

2.1 Reeds

Four different reeds were used according to models built by reed manufacturers. The manufacturer shall not be named here, only the basic differences between the reeds are of interest. The reeds all increase their thickness from the tip of the reed to its heel, which is 3 mm in thickness for all reeds. The reeds are named A, B, C and D here, where A is a traditional form which is relatively thin at the tip and relatively thick in the middle. B, C and D are alternative forms which are thicker at the tip and thinner in the middle.

A) Tip thickness is 0.18 mm. Although the thinnest at the tip it is the thickest in the middle.

B) Tip thickness is 0.275 mm. The thinnest of all in the middle, thicker at the tip as A and thinner as C and D.

C) Tip thickness is 0.321 mm. Still thinner in the middle than A, but thicker here than B and thinner than D.

D) Tip thickness is 0.325 mm. The thickest at the tip, still thinner than A in the middle, but thicker here than B and C.

The reeds are all 4 cm in their raising thickness part, free to vibrate here and fixed at the adjacent heels. The idea behind the B to D reeds is to have a wider flexible part of vibration at the reed tip compared to the traditional form of A. Still differences are there between B, C, and D as the overall thickness increases here. The difference between C and D is much smaller than between B and C. So here we also can test the model. The kinds of vibrations between C and D must be more similar than those of B and C.

2.2 Mouth and mouthpiece

The mouthpiece is the same for all reeds. It leaves a 1 mm slit to the reed at its tip, then widening to 1.8 cm in diameter over a distance of 3.5 cm only then to be prolonged over a distance of 4.5 cm with this diameter. The mouth cavity ends 1.4 cm ‘in’ the mouthpiece where the lips are placed at the top of the mouthpiece and at the bottom on the reed. The cavity itself is round and prolonged for 3 cm from the reed tip backwards. As the pressure and flow within the mouth is known - and confirmed by this calculation – to be more or less stable, the precise form of the mouth is not so much of importance here. Still it is used to change the sound of the saxophone or clarinet and need to be included in further calculations. But as we are interested in the sound of different reeds we want to keep the mouth geometry the same for all calculations.

2.2 The model

The reeds are modelled using a 2D stress-strain differential equation with four degrees of freedom, displacement and velocity in two directions.

The air model is a Navier-Stokes model. It is well accepted that the Navier-Stokes equation also covers turbulence in Finite-Element or Finite-Volume models if only enough elements are used in the turbulent region. Alternative models taking turbulence into consideration, like the k-ε model of the Reynolds-Averaged-Navier-Stokes equation (RANS), can be performed with much less elements and therefore reduce calculation cost a lot (Bader 2005). But as we are also about to discover, if the model assumption of equal pressure in the mouth piece and the tube by assuming large turbulence in the mouthpiece, it seems more appropriate to use a Navier-Stokes model with many degrees of freedom to visualize the turbulence itself.

A third system of equations need to be used here. As the vibrating reed changes the geometry of the air itself in a decisive way, a time dependent formulation of a changing geometry need to be added. Here, a Laplace smoothing algorithm was used. Because of space restrictions in this paper we do not go into mathematical details here. Still the fundamental process of the model is as follows.

A) The flow causes a pressure upon the reed.

B) This pressure changes the reed displacement.

C) The changed reed displacement means a geometry change for the flow as the inflow region between the reed tip and the mouthpiece tip changes.

D) The changed flow geometry changes the flow.

E) The changed flow is acting upon the reed again. So this E) is A) again.

Note, that all steps A) to E) are calculated simultaneously at each time step as we use an implicit time stepping algorithm. So for each time step a linear equation system is solved taking all interactions into account.

The coupling of the flow pressure upon the reed is modelled using Lagrange multipliers as additional dependent variables defined only at the flow boundaries of the reed. The same method was applied for the moving mesh equations to make the geometry changes of the flow stable and therefore add additional stability to these very complicated calculations.

The boundary conditions of the air needed to be chosen according to the very difficult calculation needs. The pressure could not be taken as high as is realistic for normal playing conditions and was set to p=150 Pa at the boundary of the mouth. Still the results are very good and therefore the method was continued with this inblow pressure.

The boundary at the tube's end of the mouthpiece was modelled time-dependent. It was assumed, that outgoing impulses were reflected at the instrument's bell and come back to the mouthpiece. There they 'shoot open' the reed and by that, another impulse is sent out to the tube again. A Gauss impulse was used as back coming impulse from the tube. The reaction of the incoming impulse by the reed is one of the aims of this investigation. The different reeds show different behaviour of reaction to an incoming impulse and therefore the impulse travelling out is different for each reed, too. This makes the differences in the sound of the reeds.

3 Results

Among the many interesting findings of the model three most important ones are discussed here briefly.

3.1 Turbulence in the mouthpiece

As shown in Fig. 3, indeed turbulence appears in the mouthpiece. This justifies the simplified model assumption of taking the pressure in the tube to be the pressure in the mouthpiece, as most of the energy is consumed in the mouthpiece and nearly no flow of air is reaching the tube.

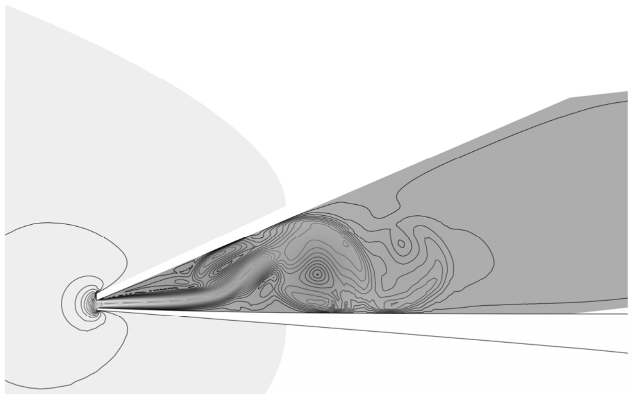

Fig. 3: Zoom into the reed region of the model for the A reed at 9 ms after model start. The contour lines show flow velocity where the background show pressure distribution (dark is low pressure, white is large pressure). The turbulent nature of the flow in the mouthpiece clearly appears. The reed is displaced here and just its boundaries are shown.

3.2 Sound differences between the reeds

The time varying displacements of the reed tips were used to obtain a time series for the outgoing pressure impulse. Fig. 4 shows the different impulses for the four reeds.

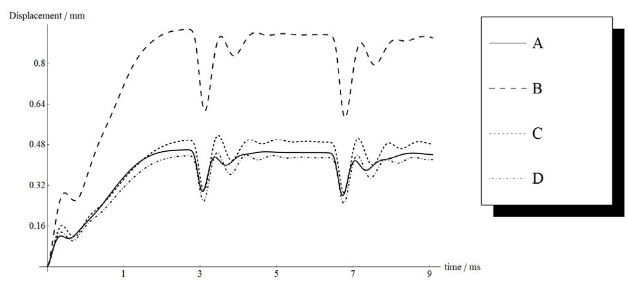

Fig. 4: Time series of the tips of the reeds A – D for 9 ms of calculation time and two back impulses from the reed. The reeds A, C, and D show about the same overall amplitude while the much more thin reed B has increased amplitude. Also the time series differ in terms of vibration caused by the different reed geometries.

The overall displacement amplitude is about the same for reeds A, B, and D and very different for reed B. This was expected as B has much less material than all others as it is thinner in the middle and only thicker than A right at the tip. Still all reeds show differences with respect to the shape of the time series which result in different spectra.

These spectra are shown in Fig. 5. Here, one period of reed displacement shown in Fig. 4 was used to construct a tone of 356 Hz by repeating the impulse shape in time. Then spectra for the time series of the four reeds were calculated by an FFT. As the pitch is arbitrary, the peaks of the spectra were connected to trajectories which are shown in Fig. 5.

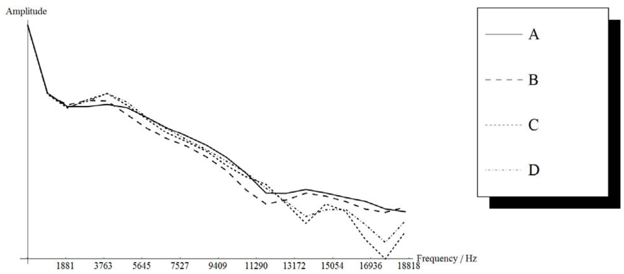

Fig. 5: Spectra of a 356 Hz tone constructed from the reed displacements, peaks connected. The reeds C and D drop amplitudes for very high frequencies. All show a peak around 3 kHz which is consistent with an eigenfrequency of the reeds.

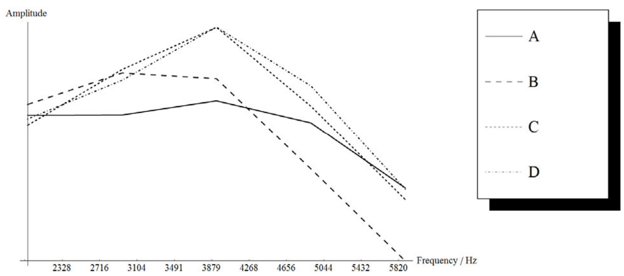

Fig. 6.: spectra of a 356 Hz tone as in Fig. 4 from 2 kHz to 6 kHz. The reeds show different resonance frequencies and resonance shapes. The traditional reed A is more flat in this region where the reeds C and D have a clear sharp peak which is also higher than the A peak. Reed B shows a kind of plateau between 3 kHz and 4 kHz.

Reeds do have a resonance frequency of 3 kHz. Below this they are passive systems which can be seen by the behaviour of all reeds below this region, they are all the same. Around 3 kHz the behaviour differs a lot. The massive reeds C and D have a sharp peak here and are more or less the same which corresponds to the fact that they do not differ very much in terms of their geometry. The traditional reed A shows a much lower resonance in this region which is more a plateau and therefore a resonance region. Still reed A has higher amplitudes than all other reeds beyond the 3 kHz region all through the highest frequencies. It therefore can be considered to be the most stable of all reeds in terms of the spectrum. Remember, that the key idea behind the reeds B to D is to make the reed tips more flexible. This seems to cause spectra which are more flexible, too. So with this resonance, the reeds reacts much more than the traditional one, in all other cases they are more calm.

It is interesting to note, that the first harmonic of these reeds used here were calculated to be around 1.4 kHz. Only the second partial was around 3 kHz. It need to be investigated further if the reeds are ‘speeded up’ by the air pressure or if the resonance found in the spectra are caused by the second partial rather than the first one.

Additionally, reeds C and D show a drop-off at higher frequencies from about 12 kHz on. This is not true for reed B. The reasons here are not clear yet, remember that reed B has least material. It could be because of higher damping because of the enhanced material. It is also interesting to note that reed B drops below all other reeds between the fundamental resonance region around 3 kHz up to about 12 kHz. As the higher flexibility of the reeds are also thought to make initial transients faster and give the player a broader range of variation of tones, further investigations with different tone beginnings need to be done here to really get into the reaction behaviour of these reeds. Here, the boundary conditions of inflow would need to be changed in terms of slow amplitude rise, sudden changes in pressure ect.

3.3 Behaviour of vibration along the reed length

The flexibility of vibration of the reeds must change according to their different thickness structures. Fig. 7 shows the time-averaged displacements for the four reeds over the vibrating length of the reed compared to reed A. Here the maximum displacement of the reed tips were normalized to make the movements comparable. Reed C and D are both more moving at their tips (left) compared to reed A. So indeed they are more flexible than A at their tips. Contrary to this, they are less moving near the heel. The enlarged mass at the tip of these reeds causes the enhanced vibration at this tip. Reed B has less mass all over. This leads to a more smooth movement of the whole reed. Comparing these averaged movements with the time series in Fig. 4, one can realize, that reed C and D have much more movements at the reed tip as reeds A and B.

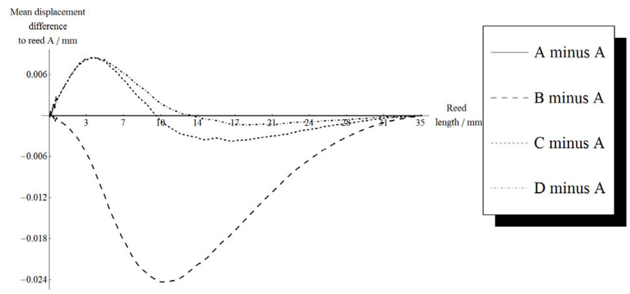

Fig. 7: Relative displacements of the reeds to the reference reed A over the reed length integrated over one impulse period in time (reed tip on the left). Note that the maximum displacements are normalized and so the much larger displacement of reed B shown in Fig. 4 is not considered. Reed B shows the largest negative difference all over where only reed C and D show a more flexible behaviour at the tip.

4 Conclusions

The different reeds clearly show different behaviour. It is interesting to see, that the altered reeds B, C, and D show more pointed spectra than the standard model which is much more consistent over all amplitudes. This may be a reason for modern saxophone players to work with these reeds which show more ‘character’ than the standard geometry reed. The spectral fluctuations at higher harmonics with the reeds C and D is surprising and need to be investigated further. Still all reeds show less amplitude in frequency regions beyond 4 kHz compared to the standard reed.

Further investigations need to consider different playing styles like staccato to get an idea about the speed of reaction of the different reeds. We may get a first idea about that by comparing the speed of the different reeds they need to reach their stable state right from the start shown in Fig. 4. Astonishingly, reed A seems to be first. Reed C and D show a similar decay in their slope at the point reed A converges but they still keep rising. Reed B has a much higher amplitude and can therefore hardly be compared with the others.

The model also shows much turbulence in the mouthpiece pointing to much damping of the flow there. It therefore seems indeed reasonable to set the pressure within the mouthpiece equal to the pressure in the tube as is proposed by the standard model of single reed behaviour.

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