

# Suppression of Oceanic Reverberation by Subspace Methods

Xuan Li, Xiaochuan Ma and Chaohuan Hou

Institute of Acoustics, Chinese Academy of Sciences, 100080 Beijing, China hch@mail.ioa.ac.cn

Active sonar target detection in shallow water environments is a challenging problem because of the presence of reverberation noise. Based on some models of reverberation noise, data from sensors array are pre-whitened before applying a match filter. Considering reverberation noise as a sum of echoes of transmitted signal, the Principal Component Inverse (PCI) algorithm deletes the largest singulars of data matrix, which is constructed from array data. However, estimating a threshold which is needed in PCI is difficult in practice. In this paper, a new subspace method, Deleting Big Eigenvalues is proposed for suppression of broadband reverberation noise. The novel method substitutes automatic signal-number estimation for threshold estimation, and takes autocorrelation matrix eigendecomposition while PCI taking data matrix Singular Value Decomposition (SVD). According to a simulation of target detection using broadband linear frequency-modulated signal, the new method shows a better performance to estimate broadband direction-of-arrival (DOA) and smaller computing quantity compared with PCI.

## **1** Introduction

In active sonar signal processing, reverberation noise leads to complex problems for target detection and DOA estimation. Conventional method of broadband signal is taking space-time spectrum estimation by beamforming. Subspace algorithms, for instance, MUSIC and ESPRIT show excellent estimation to narrowband DOA and can be extended to broadband application. However, the application is embarrassed by reverberation noise. Reverberation, which is caused by the diffraction/ diffusion/ reflection of the transmitted signal by the ocean surface, volume, and ground, is correlated with the transmitted signal. Furthermore, the target echo's power is usually much lower and its duration is much shorter compared to the reservation noise. Low signal to reverberation ratio leads to performance degradation. How to reduce the impact of reverberation noise is a challenging problem and many models have been proposed to suppress the reverberation noise of the signal. A pre-whitening method, based on the local stationarity assumption of reverberation noise, was suggested in [1], using AR (autoregressive) filter to pre-whiten the signal in blocks in a one by one processing way. Vale'rie et al expand this method to frequency-modulated signals from monochromatic signal <sup>[2]</sup>. Besides these, one another approach, which suppresses reverberation noise in a deterministic way, was suggested by Guillaume et al in [3] as the Principal Component Inverse. The PCI method, considering reverberation noise as a sum of undesirable echoes, distinguishes reverberation noise from target echo via SVD which projects the signals into two separated subspaces, the reverberation space and the signal plus white noise space. A threshold is required in PCI. The data matrices that are constructed from the array sensor data are processed by SVD, deleting the components of larger singular values compared to the threshold. Yu Li extends two Schur-type-based PCI algorithms to adapt to parallel computing <sup>[4]</sup>.

Similar to PCI, a new method, Eigen-space Deleting Big Eigenvalues (EDBE) together with its broadband extension, is proposed in this paper. After projecting the signal separately into the echo-plus- reverberation subspace and the reverberation-plus-noise subspace, the reverberation noise is suppressed by a Minimum Variance Distortionless Response (MVDR) filter. In fact, PCI and the new methods are all subspace methods. EDBE and its extension take eigendecomposition computation of the autocorrelation matrix of spatio-temporal data, which is different from PCI that takes SVD of small-window data matrices. Furthermore, EDBE and its broadband extension automatically estimate the number of target echoes. Comparing with PCI, the new methods are more effective to suppress reverberation noise and relieve the burden in estimating the PCI threshold.

In this paper, data model is given in section 2. The new methods are proposed in section 3. Simulation about broadband DOA estimation, which shows the performance improvements, is also included in this section. Finally, we get a short conclusion in section 4.

## 2 Signal model

We assume a linear array of M sensors, equally spaced half of the center carrier wavelength d apart. The transmitted signal,  $s_0$ , is a LFM signal. Sources are assumed to be farfield. Consider K target echoes arrive at sensors array from different directions.  $s_i$  is one of the target echoes, also signals of interest (SOI), with the DOA  $\theta_i$ .

$$s_i(t) = P_i * \exp(j * 2\pi * (f_i + \beta_i t/2)t), 0 < t < T , \quad (1)$$

where  $P_i$  is the signal amplitude,  $f_i$  is the starting frequency that has a Doppler frequency shift compared with the starting frequency of  $s_0$ ,  $\beta_i$  is the frequency modulated rate, and T is the signal time duration. At the m th sensor, the SOI is received as  $s_i(t-\tau_m)$ . Demodulating and bandpass filtering, which are needed practically in order to decrease data-storing and computing amount, are ignored here. At the output of the array, we have a sample as:

$$\boldsymbol{x}(t) = \sum_{i=1}^{K} \boldsymbol{a}_{i}(t) \boldsymbol{s}_{i}(t) + \boldsymbol{n}(t) + \boldsymbol{r}(t)$$
(2)

x(t) is the  $M \times 1$  complex vector of array observations. n(t) is Gaussian white noise with M dimension denoting the sensor noise. r(t) is reverberation noise, considered as colored noise that is nearly isotropic in space. Because reverberation noise is also created by the active sonar, it is correlated with SOI.  $a_i(t)$  is the steering vector of  $s_i$  at t, depending on the signal instantaneous frequency and direction<sup>[5]</sup>.

$$a_{i}(t) = [\exp(-j * 2\pi * \tau_{i,1} * f(t,i,1)), \cdots \\ \exp(-j * 2\pi * \tau_{i,M} * f(t,i,M))]^{\mathrm{T}}$$
(3)

f(t,i,m) is the instantaneous frequency of  $s_i$  corresponding to the *m* th sensor and time *t*.

$$f(t,i,m) = f_i + \beta t - \beta / 2 * \tau_{i,1}$$

$$\tag{4}$$

 $au_{i,m}$  is the propagation delay due to the signal direction  $heta_i$  and

$$\tau_{i,m} = (m-1)^* d / c^* \sin\left(\theta_i\right).$$
<sup>(5)</sup>

Autocorrelation (spatial) matrix is defined as  $\mathbf{R} = \mathrm{E}\left\{\mathbf{x}(t)^* \mathbf{x}(t)^{\mathrm{H}}\right\}$ . E{ } is the expectation operator. For *N* samples of the array, the autocorrelation matrix is estimated as Eq.(6).

$$\hat{\boldsymbol{R}} = \boldsymbol{X} * \boldsymbol{X}^{\mathrm{H}} / \boldsymbol{N}$$
(6)

where  $\{ \}^{H}$  denotes transpose complex conjugate. *X* is array data matrix, and

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{x}(1) & \cdots & \boldsymbol{x}(N) \end{bmatrix}$$
(7)

In order to perform broadband beamforming, frequency band of array data is partitioned to several subbands to satisfy narrow band qualification. With a suitable redundancy factor, the data is cut into blocks and transformed by FFT. For every frequency bin  $f_j$ , we can get an autocorrelation matrix  $\mathbf{R}(f_j)$  in frequency domain and estimate targets direction after beamforming. The beamformer output signal can be written as

$$Z(f_j, \theta_i) = w(f_j, \theta_i)^{\mathsf{H}} X(f_j).$$
(8)

For bearing  $\theta_i$  and frequency  $f_j$ , weight vector  $\boldsymbol{w}_{j,i}$ , short for  $\boldsymbol{w}(f_j, \theta_i)$  is calculated by Variable Loading Robust Capon Beamforming (VLRCB) in our simulation. For the interested spatial domain vector  $[\theta_1 \cdots \theta_D]$ , including the predicted DOA of SOI, beamforming weight matrices are formed as:  $\boldsymbol{W}_j = [\boldsymbol{w}_{j,1}, \cdots \boldsymbol{w}_{j,D}]$ , 0 < D < M. Multiple beam matrices are got,  $\boldsymbol{B}_j = \boldsymbol{W}_j^{\mathrm{H}} \boldsymbol{R}(f_j) \boldsymbol{W}_j$ ,  $\boldsymbol{B}_j \in C^{D \times D}$ .

At the presence of reverberation noise in power, where the beamformer doesn't provide enough SNR benefits, additional suppression of reverberation noise is required. In section 3.3, simulation will show Beamspace MUSIC's deficiency with low ratio of signal to reverberation. As a result, some methods, such as AR, PCI, etc, are taken to suppress reverberation. We propose a subspace method that work well in suppressing reverberation noise of broadband signal.

## **3** Novel methods

Essentially, PCI algorithm is a subspace method, dividing data matrix into reverberation space and signal-plus-noise space. Nevertheless, the SVD of data matrices leads to high computational burden, and estimating threshold is inconvenient and difficult in practical application. A novel method of broadband signal is proposed in this paper.

The new methods are also elicited by subspace division of autocorrelation matrix <sup>[7]</sup>. In broadband processing, we assume reverberation noise received by the array can be modeled as broadband colored noise i.e. it is also assumed to be spatial uncorrelated. Then we construct the spatial correlation matrix in time domain and use eigendecomposition processing to construct the reverberation-noise spatial correlation matrix. By applying the reverberation-noise spatial correlation matrix in MVDR beamforming filters, the estimation of signal power and DOA is improved.

## 3.1 Eigen-space Deleting Big Eigenvalues

Firstly, Signal Number Estimator Based on Eigenvectors (SNEBE) <sup>[8]</sup>, which can be more robust and efficient in colored noise environment than some other signal number estimators, is employed to estimate the undesirable echoes number K in these methods.

After that, similar to PCI, the K largest eigenvalues are deleted and new estimation of autocorrelation matrix is got,

 $\tilde{\boldsymbol{R}} = \sum_{i=K+1}^{M} \lambda_i \boldsymbol{u}_i \boldsymbol{u}_i^{\mathrm{H}}$ .  $\boldsymbol{u}_j$  and  $\lambda_j$  are respectively eigenvector

and eigenvalue of  $\hat{R}$ . Then adaptive beamforming is taken, using  $\tilde{R} \cdot \tilde{R}$  is a narrowband autocorrelation matrix of array data.

We review the variable diagonal loading beamforming:

$$\boldsymbol{w}_{opt} = \arg\min_{\boldsymbol{w}} \left\{ \boldsymbol{w}^{H} \boldsymbol{R}_{i+n} \boldsymbol{w} \right\}, \text{ s.t. } \boldsymbol{w}^{H} \boldsymbol{a}_{0} = 1$$

$$\boldsymbol{w}^{H}\boldsymbol{R}_{i+n}^{-1}\boldsymbol{w} < \boldsymbol{\delta}$$
 (9)

Where  $w_{opt}$  is optimum weight vector,  $a_0$  is the SOI's steering vector,  $\delta$  is a small constraint factor, and  $R_{i+n}$  is interference-plus-noise correlation matrix, which taken place of sample correlation matrix  $\hat{R}$  in original algorithm. In this new method, substituting  $\tilde{R}$  for  $\hat{R}$  weakens the effect of signal-contaminant, which destroys MVDR's robustness.

#### **3.2 Broadband Application**

EDBE can be extended to broadband application in a FFT based framework. After signal number has been estimated, two subspaces are separated by autocorrelation matrix eigendecomposition, and then the array data is projected to the subspace that corresponding to small eigenvalues. The projection can also be done in frequency domain. Beamspace MUSIC is operated to estimate DOA <sup>[6]</sup>. Different from the method in [6], the frequency-domain beamspace is structured by robust adaptive beamforming using variable diagonal loading <sup>[9]</sup>; MUSIC is taken in different frequency sub-bands. The algorithm is described as follows:

- 1. Estimating signal number *K* as same as in 3.1.
- 2. Getting two subspaces:

 $\boldsymbol{U}_{R} = \begin{bmatrix} \boldsymbol{u}_{1} & \cdots & \boldsymbol{u}_{K} \end{bmatrix}, \ \boldsymbol{U}_{S} = \begin{bmatrix} \boldsymbol{u}_{K+1} & \cdots & \boldsymbol{u}_{M} \end{bmatrix}.$ 

- 3. After FFT, transforming autocorrelation matrix R into frequency domain  $R(f_j)$ .  $f_j$  is frequency bin.
- 4. Projecting  $\boldsymbol{R}(f_i)$  into  $\boldsymbol{U}_S$ ,  $\tilde{\boldsymbol{R}}(f_i) = \boldsymbol{U}_S \boldsymbol{U}_S^{\mathrm{H}} * \boldsymbol{R}(f_i) * (\boldsymbol{U}_S \boldsymbol{U}_S^{\mathrm{H}})^{\mathrm{H}}$ .
- 5. Beamforming. For bearing  $\theta_i$  and frequency  $f_j$ , weight vector  $\boldsymbol{w}(\theta_i, f_j)$  is calculated by Variable Loading Robust Capon Beamforming (VLRCB). For the interested spatial domain vector  $[\theta_1 \cdots \theta_D]$ , including the predicted DOA of SOI, beamforming weight matrices are formed as:  $\boldsymbol{W}_j = [\boldsymbol{w}_{j,1}, \cdots \boldsymbol{w}_{j,D}]$ , 0 < D < M. Multiple

beam matrices are got,  $\boldsymbol{B}_{j} = \boldsymbol{W}_{j}^{H} \boldsymbol{R}(f_{j}) \boldsymbol{W}_{j}$ ,

 $\boldsymbol{B}_i \in C^{D \times D}$ .

6. DOA estimation by MUSIC. And then, the output is got from the average of every frequency bin.

Frequency-domain-projection is taken in the third and fourth steps. Another method of doing is time-domain-projection. Modified array data is got from this expression:  $\tilde{X} = U_S U_S^{H*} X$ . The two modes are tantamount and the computing amount of the latter is larger.

The comparison between the three methods will be shown in next subsection.

## 3.3 Simulation

Consider a 15-element ULA, spaced half of the transmitted signal's center wavelength apart (1/8 meter). Transmitted signal  $s_0$ , is LFM signal with center frequency f=6000Hz, band B=1000Hz, and duration T=0.05s. Two target echoes,  $s_1$ ,  $s_2$ , from directions 20° and 30°, impinge on the array with Doppler frequency shift 60Hz and 120Hz respectively. The targets are both far-field plot sources. The SNR (signal-to-noise ratio) are respectively 3dB and 5dB, while the RNR (reverberation-to-noise ratio) is 18dB. Reverberation is modeled as Gaussian colored noise with the same central frequency of  $s_0$  and has no directivity.

Sampling frequency fs=20000Hz. The total width of observation window is 3000 (0.15s). Each time window of FFT is 256-sample-long and the frequency bins are selected from the 69 th to the 87 th (5390.6Hz~6796.9Hz). The direction of beamforming is  $[0^{\circ} 10^{\circ} 20^{\circ} 30^{\circ} 40^{\circ}]$ .

Figure 1 presents the DOA estimation after beamforming and MUSIC, which illustrate the underlying SNR improvements. The solid, dash-dot and dash lines respectively represent the DOA estimated from original array data, data after PCI, and data after EDBE in frequency-domain.



Fig.1 DOA using BSMUSIC in the methods.

For a better understanding, we normalize the power in [0 1], and the implementation is depicted in Figure 2.



.Fig.2 Normalized DOA using BSMUSIC in the methods.

Evidently, the SOI is hardly distinguished by Beamspace MUSIC without pretreatment because too many false peaks exist; after PCI and EDBE, the SOI can be detected even with very low signal-to-reverberation ratio. EDBE shows excellent quality in that it suppresses reverberation noise power significantly. Moreover, as less SVD is needed, it has smaller computing quantity than PCI.

To analyze the estimation stationarity, we compare the algorithms by Monte Carlo simulations. 100 Monte Carlo simulations are run under the above conditions expect target echo from  $30^{\circ}$  is absent. Table 1 shows the statistical characters of the estimated direction of the target echo from  $20^{\circ}$ , including average and variance.

Method	Average	Variance
BSMUSIC	19.49	42.3736
PCI-BSMUSIC	19.77	0.2397
EDBE-BSMUSIC	19.78	0.2782

Table 1 Statistical characters of the three methods

In table 1, it's obvious that our algorithm's DOA estimation is as consistently dependable as PCI's while the first algorithm's result is with great variance.

## 4 Conclusion

This paper reviews the determined models of reverberation noise in active sonar signal processing, and then contributes an improved method, EDBE, under the highlights of PCI and Eigendecomposition. This method avoids the task of PCI threshold estimation and works better in our simulations of weak targets DOA estimation. Many more tests with real sonar data is on the way recently.

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