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Acoustic vector-sensor array beamforming based on fourth-order cumulants

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Abstract Some high-resolution direction-of-arrival (DOA) estimation methods, such as the MUSIC method, have been developed based on vector-sensor array. However, these high-resolution methods suffer from serious drawbacks. Indeed, they are not able to estimate coherent signals and sensitive to calibration errors. Mainly to overcome these limitations and in particular to satisfy practical engineering application, vector-sensor array beamforming based on fourth-order cumulants is adopted in this paper. The main interest in using fourth-order cumulants instead of second-order ones in vector-sensor array processing application is the aperture extension property of higher-order cumulants, which makes it possible to increase both the resolution and the number of sources to be processed from a given array, and suppress the Gaussian noise. Based on this method, smaller size of array and higher accuracy of DOA estimation can be realized, besides, this method is less sensitive to calibration errors, which makes it possible to work well in engineering application. The paper analyses the data of trial in lake, and the results of DOA estimation based on vector-sensor array are presented, knowing that experiment results are basically consistent with theory results, which can well illustrate the validity and the feasibility of the method presented in this paper.

1 Introduction

The passive direction-of arrival (DOA) estimation problem, is of great importance in many underwater applications. The technology on general acoustic pressure array had been used widely in this field. It is well known that sound wave has both scalar quantity and vector field, while traditional acoustic pressure sensor system merely makes use of its acoustic pressure information. Vector sensor, also called combined sensor, is combined by traditional and omnidirectional pressure sensor and natural dipole independent on frequency, which can co-locating and simultaneously measures pressure (scalar field) and particle velocity (vector field) of acoustic field. The technology of vector array based on technology of general acoustic pressure array brings new life to the developments of signal processing of underwater acoustics processing and sonar technology. A. Nehorai [1, 2, 3], M. Hawkes [2, 3], K. T. Wong [4, 5] and M. D. Zoltowski [4] etc. have done much work on acoustics vector-sensor arrays beamforming and direction finding.

In this paper, we use the Bartlett algorithm based on fourth-order cumulants of vector-sensor array. We are aiming to present three properties of this method: 1). the aperture extension property of higher-order cumulants [6], which can increase the aperture of a given array and, hence, improve estimation performance. So it makes it possible to use a smaller size of array to have better performance. 2). A vector-sensor linear array is used over a pressure-sensor one to improve the performance of DOA estimation. The left/ right ambiguity problem does not arise. 3). The Bartlett algorithm is not so sensitive to calibration errors as the MUSIC algorithm. Thus, it is more popular used in engineering applications. And in the end of this paper, the results of the trial are presented.

2 Hypotheses, notations, and statistics of the data

2.1 Array output assumptions

We consider n independent signals obtained by the array, with vector sensors located at $r_1,...,r_m$. The acoustic vector sensors discussed in this paper consist of a co-located pressure sensor and twosome orthogonal velocity sensors. And making the additional assumptions: the signal is narrowband plane wave travelling in an isotropic, quiescent, homogeneous fluid medium, and impinging on the array. Let u be the unit vector at the sensor pointing towards the source, which is

$$u(\Theta)=\left[\cos\varphi\sin\theta \quad \sin\varphi\sin\theta \quad \cos\theta\right]^T \quad (1)$$

Where $u(\Theta)$ is parameterized by its azimuth $\phi\in\left(0,2\pi\right]$ and elevation $\theta\in\left[-\pi/2,\pi/2\right]$.

The data vector received by the array can be written as:

$$Y(t)=A(\Theta)S(t)+N(t) \quad (2)$$

Where $A(\Theta)=\left[a_v(\Theta_1) \quad a_v(\Theta_2) \quad \cdots \quad a_v(\Theta_M)\right]$ is a $4m\times n$ matrix of the source steering vectors, which contains, in particular, the information about the direction of arrival of the sources, the steering vector $a_v(\Theta)$ is given by the Kronecker product

$$a_v(\Theta)=a_p(\Theta)\otimes h(\Theta) \quad (3)$$

$$a_p(\Theta)=\left[e^{j\omega r_1^T u(\Theta)/c} \quad e^{j\omega r_2^T u(\Theta)/c} \quad \cdots \quad e^{j\omega r_M^T u(\Theta)/c}\right]^T \quad (4)$$

$$h(\Theta)=\left[1 \quad \cos\varphi\sin\theta \quad \sin\varphi\sin\theta\right]^T \quad (5)$$

Apparently, the usual pressure-sensor array mode is a special case of this model obtained by setting $h(\Theta)=1$ (see [1] for details).

2.2 Statistical assumptions

We assume that both the signal $S(t)$ and the noise $N(t)$ are independent identically distributed, zero-mean, complex Gaussian processes.

Let $R_Y=E\left\{YY^H\right\}$ denote the total covariance matrix of signal and noise received by the array, and suppose that signal is independent with the noise, then

$$R_Y=R_S+R_N \quad (6)$$

Where R_S represents the signal covariance matrix, R_N denotes the noise covariance matrix. And the Bartlett spectra is

$$P_B(\theta) = a_v^H(\theta) R_Y a_v(\theta) \quad (7)$$

2.3 Fourth-order cumualnts

Suppose Q_Y is the fourth-order cumulants of the signals, it can be defined as follows:

For $(i, j, k, l) \in \{1, \ldots, M\}$

$$\begin{aligned} Q_Y[i,j,k,l] &= cum\{Y_i(t), Y_j(t)^*, Y_k(t)^*, Y_l(t)\} \\ &= E\left[Y_i(t)Y_j(t)^* Y_k(t)^* Y_l(t)\right] - E\left[Y_i(t)Y_j(t)^*\right]E\left[Y_k(t)^* Y_l(t)\right] \\ &\quad - E\left[Y_i(t)Y_k(t)^*\right]E\left[Y_j(t)^* Y_l(t)\right] - E\left[Y_i(t)Y_l(t)\right] - E\left[Y_k(t)^* Y_j(t)^*\right] \\ &= B(\theta) Q_s B^H(\theta) \end{aligned} \quad (8)$$

Where

$$\begin{aligned} B(\theta) &= \left[b(\theta_1), \quad b(\theta_2), \quad \ldots \quad b(\theta_k)\right] \\ &= \left[a_v(\theta_1) \otimes a_v^*(\theta_1), \quad \ldots \quad a_v(\theta_k) \otimes a_v^*(\theta_k)\right] \end{aligned} \quad (9)$$

And the fourth-order Bartlett spectra is

$$P_{FOB}(\theta) = b^H(\theta) Q_Y b(\theta) \quad (10)$$

3 The aperture extension property of higher-order cumulants

Previous research shown that cumulants can be used to increase the effective aperture of an arbitrary array. In this paper, the Non-Uniform Linear Array (Non-ULA) is used instead of the Uniform Linear Array (ULA), because the Non-ULA has larger aperture extension than the sensors are located uniformly.

We consider a general pressure sensor array of three elements with locations $\{(0,0),(0,d),(0,3d)\}$ on the $x-y$ plane. The steering vector of this array is:

$$a_p(\theta) = [1, \exp(-jkd), \exp(-jk3d)]^T \quad (11)$$

If we use the definition according to (8), the new steering vector $b_p(\theta)$ is given by the Kronecker product as follows:

$$\begin{aligned} b_p(\theta) &= a_p(\theta) \otimes a_p^*(\theta) \\ &= [1, \exp(-jkd), \exp(-jk3d)]^T \otimes [1, \exp(jkd), \exp(jk3d)]^T \\ &= [1, \exp(jkd), \exp(jk3d), \exp(-jkd), 1, \\ &\qquad \exp(jk2d), \exp(-jk3d), \exp(-jk2d), 1]^T \end{aligned} \quad (12)$$

So, four virtual sensors are extended from these three actual sensors. The aperture of the array is twice larger than before. The result is shown in Fig. 1.

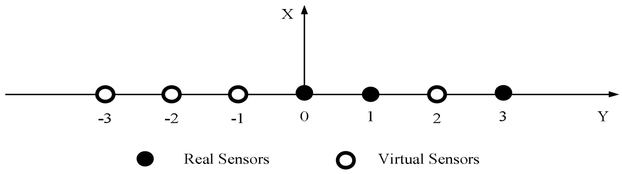
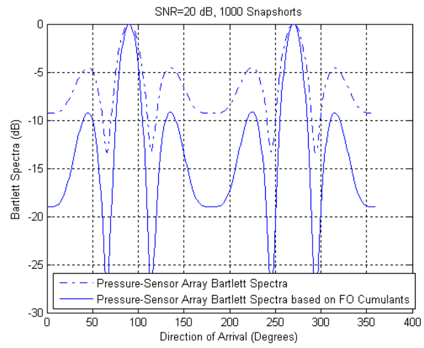


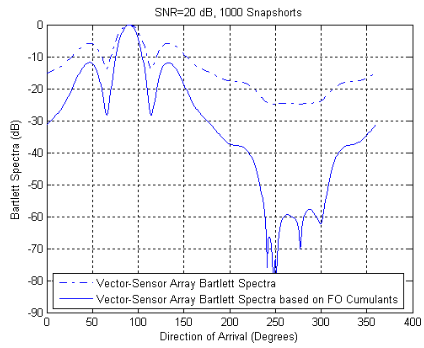
Fig. 1 Virtual aperture extension by fourth-order cumulants

Similarly, the vector-sensor arrays have the same conclusions. The larger effective aperture the narrower width of direct beam will be. Thus, bigger gain, narrower width of direct beam, and estimate more signals can be realized based on fourth-order cumulants theoretically.

Fig. 2 showed the Bartlett spectra for the Non-ULA which are located as Fig. 1. The space between each sensor is half-wavelength, f=6.3 KHz, fs=65.536 KHz, and a single signal is presented in the direction of 90°. Assume the vector sensors are combined by traditional pressure sensor and two dipoles located vertically at the $x-y$ plane. If we defined one direction is the x axis, then the other is the y axis. Thus, we can call them 2-D vector sensors.



(1)



(2)

Fig. 2 Bartlett spectra of the Non-ULA

(1) pressure-sensor array (2) vector-sensor array

From the simulation results, there are two points to notice. First, the spectra for the vector-sensor array have no left/right ambiguous. Second, according to Fig. 2(2) the 3dB beam-width of the main lobe is 6.5 degree narrower and the beam side lobes are 6 dB lower than before.

4 Experimental results and analyses

Experiments on vector-sensor array are carried in Sep. 2007 at Songhua Lake in China. The experiment is based on the 2-D vector sensors as mentioned above.

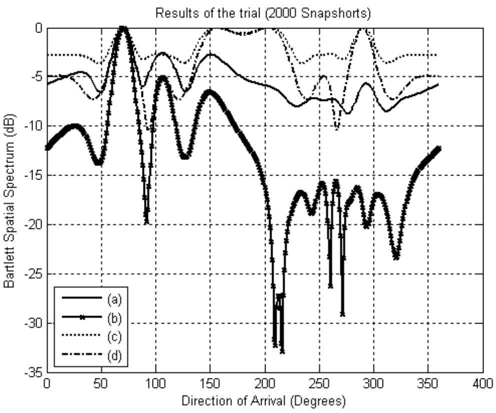
4.1 Results of the experiments

Experiment 1:

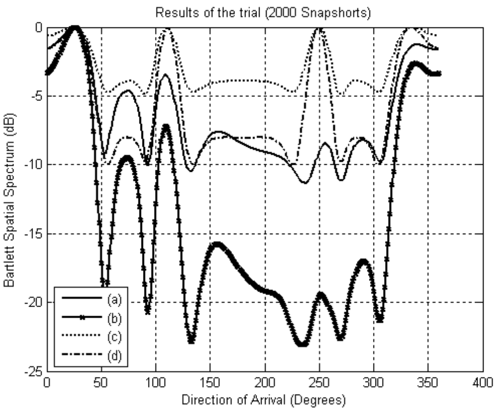
The three elements Non-ULA is fixed horizontally as Fig.1, with d=0.19m (larger than half-wavelength), the working frequency f=6.3 KHz, with ratio of signal to noise is about 12dB. The DOA estimation results are illustrated as Fig. 3(1).

Experiment 2:

The ratio of signal to noise is about 15dB. Other conditions are the same as experiment 1. The results are illustrated as Fig. 3(2).



(1)



(2)

Fig. 3 Results of the trial:

(a) vector-sensor array

(b) vector-sensor array based on cumulants

(c) pressure-sensor array

(d) pressure-sensor array based on cumulants

4.2 Discussion

The arrival angle showed in Fig. 3(1) is about 70°, and in Fig. 3(2) is about 25°, which are both coincide with reality.

From both Fig. 3(1) and Fig. 3(2), we can see that the Bartlett spectra based on fourth-order cumulants perform better than ordinary Bartlett spectra, with narrower main lobe and lower side lobes. Unlike the MUSIC algorithm, the Bartlett algorithm is less sensitive to calibration errors. So, it is popular used in engineering applications.

5 Conclusion

In this paper, fourth-order cumulants is used in vector-sensor arrays. From both simulation results and trial results, we can see this method is of advantages when working in engineering applications. Thus, a small size of array with better performance can be realized based on this method.

References

[1] Arye Nehorai, Eytan Paldi, "Acoustic vector-sensor array processing," *in Proc.26th Asilomar Conf. Signals, Syst., Comput., Pacific Grove, CA*, Oct, 192-198 (1992)

[2] Malcolm Hawkes, Arye Nehorai, "Acoustic Vector-Sensor beamforming and Capon Direction estimation," *IEEE. trans on signal processing*, 1673-1676 (1995)

[3] Malcolm Hawkes, Arye Nehorai, "Bearing estimation with acoustic vector-sensor arrays," *presented at Acoustic. Velocity-Sensor Focused Workshop , Mystic, CT.*345-358(1995)

[4] K. T. Wong, M. D. Zoltowski, "Orthogonal velocity-hydrophone ESPRIT for sonar source localization," *in Proc. MTS/IEEE Oceans Conf. Fort Lauderdale, FL,* Sept. vol. 3, 1307-1311. (1996)

[5] K. T. Wong, Hoiming Chi, "Beam patterns of an underwater acoustic vector hydrophone located away from any reflecting boundary," *IEEE, Journal of oceanic engineering,* 628-637(2002)

[6] Mithat C. Dogan and Jerry M. Mendel, "Applications of Cumulants to Array processing—Part Ⅰ: Apture Extension and Array Calibration," *IEEE trans on signal processing*, VOL. 43. NO.5, May, (1995)