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Shape optimization of polygonal rooms for a correct modal distribution at low frequencies based on psychoacoustic criterion

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Resonances in small rooms may lead to inadequate frequency responses. In rooms, where the exigencies on the listening conditions are important, these resonances may cause non wanted coloration effects, which imply a non desirable sound quality. Choosing the correct shape and dimensions it is possible to reduce the audible effects of these resonances. The presented methodology aims to determine the shape and size of small and medium polygonal-shaped rooms based on the finite element method for modeling the physical acoustic behavior of the room; a neural network for loudness estimation and genetic algorithm for estimating the optimal dimensions. A comparison with previous techniques used to choose the dimension of rectangular room is also presented.

1 Introduction

The sound field in a room is characterized by the acoustical properties of the room and the audio system therein. The pitch response and the balance in the timbre depend on the room geometry and the position of the sound sources and of the listener. The aims of this paper is to propose a room design criterion, which diminishes the psychoacoustical effects of the low-frequency resonances, based on optimizing the room dimensions considering the one aspect of the human auditory response. The main problems at low frequencies are due to the relatively low modal density. Most of the proposed solutions to address this problem have been developed for the case of rectangular rooms, and they have been based on choosing room dimension proportions as well as by positioning the sound sources and the use of resonators. The main objective of this article is to present a different perspective to address the acoustical room design. The problem will be addressed from the field of psychoacoustic instead of the architectural and physical acoustics one.

In this aspect the method to be presented is characterized by searching for the room dimensions which produce equal loudness at the low frequency bands, i.e. the sound pressure level should agree -as far as possible- with some of the loudness curves shown in Fig.1. These curves represent the response of human the auditory system based on the sound pressure and the frequency, giving the sensation of equal sound amplitude [1,2].

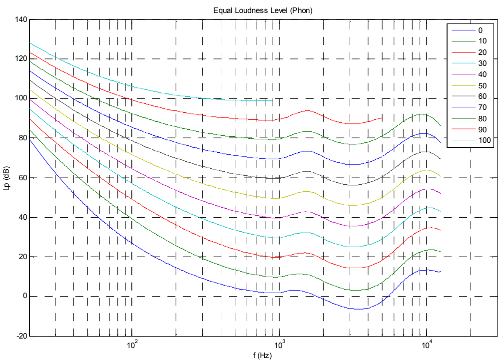

Fig.1 Equal Loudness Level Curves.

2 Bibliographical Antecedents

2.1 Previous Works

Most of the methodologies to diminish and to avoid the colorations in the design rooms are based on rectangular enclosures and mainly they consist of the choosing of the proportions adapted between width and height. Those methods try to avoid degenerated modes where multiple natural frequencies fall into a one narrow frequency band. The equation Eq. (1) determines the resonance frequencies of a rectangular enclosure; this equation is the base of diverse methods to determine the proportions an enclosure.

$$f_{n_y n_y n_z} = \frac{c}{2}\left[\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2\right]^{\frac{1}{2}} \qquad (1)$$

Where f_{n_x,n_y,n_z} are the natural frequencies, L_x, L_y, L_z are the dimensions of the rectangular room, n_x, n_y, n_z are the modal numbers and c is the speed of sound

The Bolt method [3] is based on the average distance between resonant frequencies; the most known proportions are 2: 3: 5 and 1: 21/3: 41/3. Louden [4].developed a set of more exact and preferred proportions based on the standard deviation of the space between modes and not on the distance average, producing the well-known radius 1: 1.4: 1:9. Bonello [5] developed a criterion based on the fact that the modal density must never decrease as the frequency increases. These and other methods [5,6] have their limitations, the main one is they are applicable to rectangular halls with perfectly reflecting surfaces. The absorption not only influences the amplitude of the sound pressure in the modes, it is also responsible for the resonance frequency shift. Cox, et. al. [7] developed a new methodology using optimization techniques for rectangular enclosures. They found the following optimal dimensions, 1: 2.19: 3 and 1: 1.55: 1.85.

2.2 Description of the New Method

The methodology presented in this article consists of a modification of the work by Cox et. al. using shape optimization in order to determine the room dimensions in such a way of the frequency response is isophonically flat instead of the flattest frequency response. This will be done for the frequency range 20-200 Hz. In this first stage of the work it will not be considered sound absorption of the enclosure's surfaces. The shape optimization consists in causing geometric or structural changes in order to obtain a desired frequency response of the room that is being designed. The set of modifications must be restricted in order to satisfy other requirements, such as space limitations. In this case the features of the room are modeled as a multidimensional function called objective function from now on, function of cost or fitness function, which depends on the design variables. A search region will characterize the restrictions i.e. the maximum and minimum dimensions of the room.

3 Mathematical Model of the Sound Field of a Polygonal-Shaped Room

3.1 Formulation of the Problem and Application of the Method of Separation of Variables

The enclosure is excited by a point source of flat spectrum. This can be represented using the following equation partial differential and its respective boundary conditions. In order to simplify the problem the stationary solution in the frequency domain will be only studied. When considering a harmonic solution one can obtain the Helmholtz's equation.

$$\nabla^2 P + k^2 P = 0 \qquad \nabla P \cdot \hat{n} = 0 \tag{2}$$

By using the method of variables separation the following equations and boundary conditions are obtained:

$$P(x,y,z) = P_{xy}(x,y)P_z(z) \tag{3}$$

For the dependency in z

$$\frac{\partial^2 P_z}{\partial z^2} + k_z^2 P_z = 0 \qquad \left(\frac{\partial P_z}{\partial z}\right)_{z=0} = \left(\frac{\partial P_z}{\partial z}\right)_{z=L_z} = 0 \tag{4}$$

For the dependency in x, y

$$\frac{\partial^2 P_{xy}}{\partial x^2} + \frac{\partial^2 P_{xy}}{\partial y^2} + k_{xy}^2 P_{xy} = 0 \qquad \nabla P_{xy} \cdot \hat{n} = 0 \tag{5}$$

Is due to fulfill that:

$$k^2 = k_{xy}^2 + k_z^2 \tag{6}$$

The equation and the boundary condition Eq. (4) have a well-known solution [8]. While the equation Eq. (5) and its respective boundary condition can be solved by using the Finite Elements Method [9]. Specifically it is possible to interpret these equations like a membrane with of Neumann boundary conditions. Thus the natural frequencies can be calculated as [10]:

$$\omega_{n_{xy}n_z} = c\sqrt{k_{xy}^2 + k_z^2} \tag{7}$$

Finally the sound pressure for any point $\vec{\mathbf{r}}$ inside the enclosure produced by a point source located in $\vec{\mathbf{r}}_0$ at the frequency ω, is the result of the combination of the solutions of the equations Eq. (4). and Eq. (5).

$$p(\vec{\mathbf{r}},\vec{\mathbf{r}}_0,\omega) = \sum_{n_{xy}}^{\infty}\sum_{n_z}^{\infty}\frac{A_{n_{xy}n_z}(\vec{r},\vec{r}_0,\omega)}{\omega^2 - \omega_{n_{xy}n_z}^2} \tag{8}$$

$$A_{n_{xy}n_z}(\vec{\mathbf{r}},\vec{\mathbf{r}}_0,\omega) = jU_0\rho_0 c^2\omega\left(\phi_{r,n_{xy}}\cos(k_z z)\right)\left(\phi_{r_0,n_{xy}}\cos(k_z z_0)\right) \tag{9}$$

ρ_0 is the density of the air and U_0 is the vibratory velocity of the surface of the source.

3.2 Determination of the Loudness Levels Using Neuronal Networks

The loudness may be defined as the sensation that corresponds most closely to the sound intensity of a stimulus [1]. An equal-loudness contour is a curve that ties up sound pressure levels having equal loudness as a function of frequency. In other words, it expresses a frequency characteristic of loudness sensation.

In this work a loudness model, implemented by using artificial neural network, has been developed from the equal-loudness-level contours data presented in [2] and following the procedure employed by [11]. The presented model aims to perform an accurate loudness calculation at low frequencies. This objective is different to the one of the previously presented model, which is a loudness model for a wide frequency range.

The artificial neural network [12] was trained by using the Quasi Newton Backpropagation algorithm with 3000 epochs and an objective goal of 10^{-5}. The final configuration corresponds to a three layer feedforward neural network with 5 neurons in the hidden layer and 1 neuron in the output. The transfer function of the hidden layer is sigmoidal hyperbolic tangent and for the output layer is a linear function. The inputs to the neural network are the frequency and sound pressure level and the output is the respective loudness level.

4 Optimization Using Genetic Algorithm

The optimization techniques are used to determine the best possible design in engineering problems. In our case it will be used to determine the optimal shape of a small and/or medium polygonal shaped enclosure, in order to obtain the best psychoacustical response. The proposed objective function $f(\vec{\mathbf{x}})$ corresponds to the variance of the loudness level response versus the frequency.

$$f(\vec{\mathbf{x}}) = \frac{1}{N}\sum_{n=1}^{N}\left(L_L(n) - \overline{L_L}\right)^2 \tag{10}$$

Where $L_L(n)$, is the loudness level at the nth frequency, $\overline{L_L}$ is the average loudness level and $\vec{\mathbf{x}}$ is the vector of the variables of design that contains the coordinates of the points that form the room. The optimization problem that sets out is characterized by a strong nonlinear interrelation between the variables and the fitness function, also the function has many peaks and dips which makes the solution oversensitive to the dimensions of the enclosure. For this reason, the curve of frequency response has been smoothed and the method of Genetic Algorithm has been chosen [13] which have been demonstrated to be efficient in varied problems of acoustics and vibrations [14, 15]. These methods work maintaining a population of the competent designs that are combined to find improved solutions. In its basic form each member of the population is represented by a binary sequence that codifies the variables that characterize the design. The search progresses manipulating the sequences in the population to provide in the new generations of better characteristic designs on average than its processes of predecessors that are used to look for these improved designs imitate those of the natural selection.

5 Numerical Simulations

5.1 General Considerations

The optimization was done in the frequency range 20-200 Hz, although this range can be extended. The main results of the optimization process will consist in avoiding degenerated modes. The points of emission and reception are the opposite corners. In this work the optimization of the sound source position and the listener position will not be treated. The sound source is a point one with constant speed. The dimensional limits of the 21 sides polygon are, $4m \leq Lx \leq 10m$, $5m \leq Ly \leq 15m$ and $2m \leq Lz \leq 5m$. The population size is 63 and the number of generations is 1000.The comparisons with rectangular rooms are done for enclosures that have equal height and the width and length proposed in this article are obtained through the proposed relations of optimal proportionality [1, 2, 7]. The result was also compared with the one obtained from the objective function proposed by Cox et al. (see Fig.2.). The advantages of the proposed function in this work over the developed ones previously [3, 4, 5, 6, 7] are:

- To represent the response of the human auditory system and its interaction with the sound field.
- Diminish the fluctuations of the sound pressure level due the resonances.
- To increase the loudness, therefore the sound pressure level at lower frequencies.

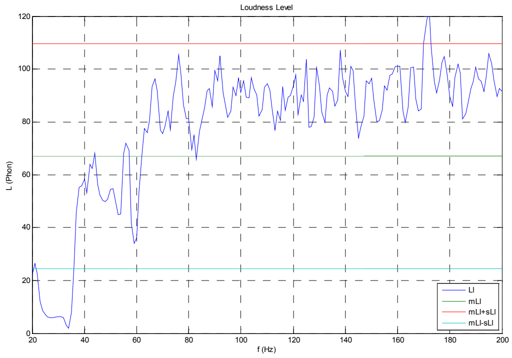

Fig.2 Proposed Fitness Function.

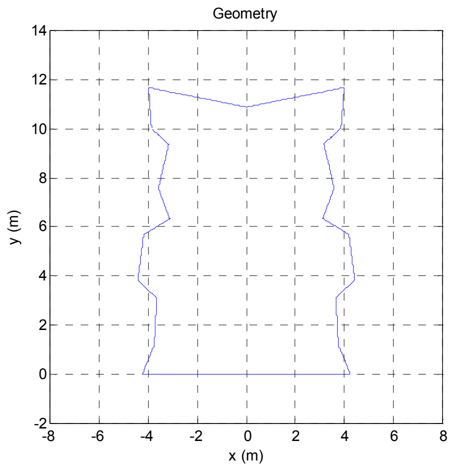

Fig.3 Optimal Geometry.

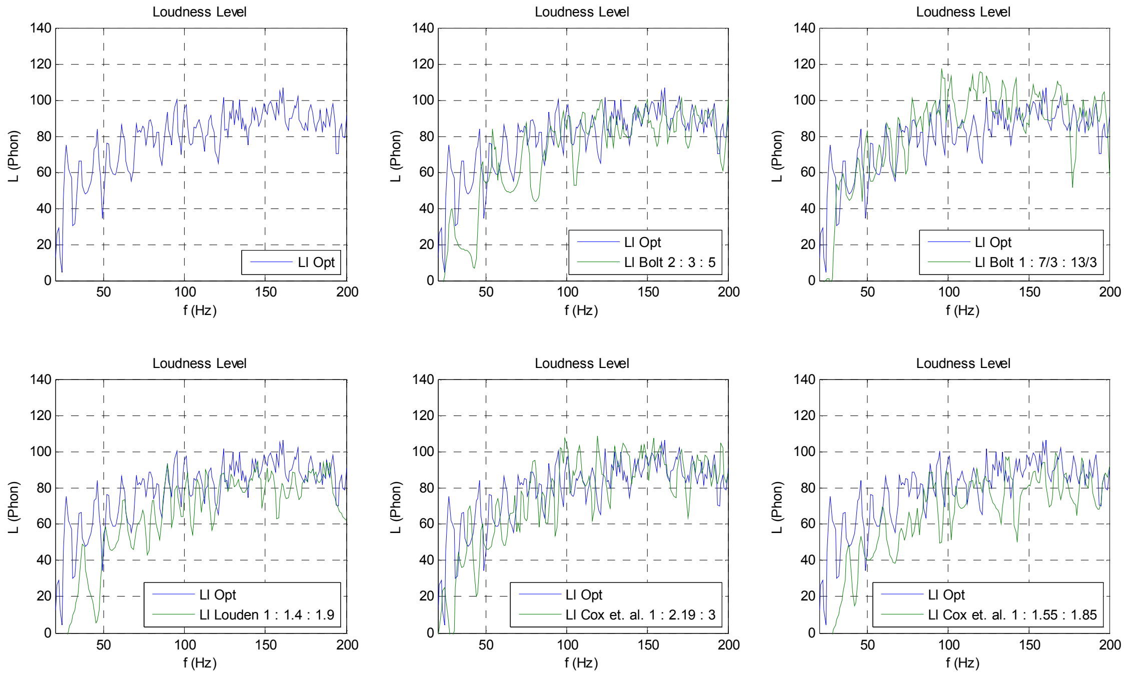

Fig.4. Loudness Levels - Differences between the presented method and the classic proportions

5.2 Results

The optimal geometry of the room is shown in the figure 3. The figure 4 shows the loudness level differences between the proposed method, i.e. the optimal polygonal shaped room, and the classic design of rectangular rooms [3, 4, 7], all having the same height (2.434 m). The polygonal shaped room has better distribution of the loudness level in frequency than the rectangular ones. In the Table 1 the values of the objective functions (Cox et. al. and present work) for the different design methodologies for rectangular rooms are presented.

Room	$f(\vec{\mathbf{x}})$ Cox et. al.	$f(\vec{\mathbf{x}})$ Floody - Venegas
Bolt (2 : 3 : 5)	72.986308	720.263885
Bolt (1: 7/3 : 13/3)	98.998914	715.801163
Louden (1 : 1.4: 1.9)	61.659613	713.114722
Cox et. al.(1 : 2.19 : 3)	71.455481	681.640776
Cox et. al.(1:1.55:1.85)	59.291149	703.686551
Optimal Room (Fig. 3)	39.227699	328.153898

Table 1 Values of the Objective Functions.

The lower values of the objective function indicate a better performance of the room.

6 Conclusions

The proposed method fulfills the objectives drawn up obtaining a better performance than the recommendations of proportions of length, width and height found in literature.

The main reason is that many of these criteria were constructed on the basis of proportions, i.e. that the height of the enclosure was equaled to the unit; although the proportions of the room is maintained, when it goes to the real dimensions, not always is efficient in the frequency band of interest, because the height is considered independent variable compared to the width the length. For this reason will not recommend proportions, because each problem of optimization depends on the space search imposed by the restrictions, i.e. the maximum and minimum dimensions of the room.

On the other hand the criterion of equal loudness proposed in this work is much more demanding than the one based on flat frequency response.

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Acknowledgments

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References

[1] Zwicker, E. and Fastl, L., “Psychoacoustics-facts and models”, Berlin-Heidelberg-New York: Springer-Verlag (1990)

[2] Suzuki, Y. and Takeshima, H. “Equal-loudness-level contours for pure tones”, *Journal of Acoustics Society of America*, 116: 918–933 (2004)

[3] Bolt, R.H., “Note on the Normal Frequency Statistics in Rectangular Rooms”, *Journal of Acoustics Society of America*, 18: 130–133 (1946)

[4] Louden, M.M., “Dimension Ratios of Rectangular Rooms with Good Distribution of Eigentones”, *Acustica*, 24: 101–104 (1971)

[5] Guilford, C.L.S., “The Acoustics Design of Talk Studios and Listening Rooms”, *Journal of the Audio Engineering Society*, 27: 17–31 (1979)

[6] Walker, R., Optimun Dimension Ratios for Small Rooms, *Journal of the Audio Engineering Society*, 44: 639–645 (1996)

[7] Cox, T. J., D’Antonio, P. and Davis, M.R., “Room Sizing at Low Frequencies”, *Journal of the Audio Engineering Society*, 52: 640–651 (2004)

[8] Fahy F.J. and Walker, J.G., “Fundamentals of Noise and Vibration”, E & FN Spoon (1998)

[9] Zienkiewicz, O.C. and Taylor, R.L., “The finite element method”, McGraw Hill (1991)

[10] Floody, S and Venegas, R. “Shape Optimization in Rectangular Rooms for a Correct Modal Distribution at Low Frequencies Based on Psychoacustical Model”, *SEMACUS X Seminario Internacional de Acústica*, Santiago, Chile , 2-5 Octubre 2007.

[11] Espinoza, V., Venegas, R. and Floody, S., “Modelo de Sonoridad Utilizando Redes Neuronales”, *V Congreso Iberoamericano de Acústica*, Santiago, Chile, 25-28 Octubre 2006.

[12] M. Gupta, L. Jin, N. Homma, “Static and Dynamic Neural Network”, IEEE Press, John Wiley and Sons, 2003

[13] Goldberg, D.E., “Genetic Algorithms in Search/Optimization and Machine Learning”, Addison Wesley, 1989

[14] Keane, A. J., “Passive vibration control via unusual geometries the application of genetic algorithm optimization to structural design”, *Journal of Sound and Vibration*, 185: 441–453, 1995

[15] Ratle, A. and Berry, A., “Use of Genetic Algorithms for the Vibroacoustic Optimization of a Plate Carrying Point Masses”, *Journal of Acoustics Society of America*, 104: 3385–3397, 1998