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Pedal notes of brass instruments, a mysterious regime of oscillation

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In reed and brass instruments, sound is produced by self-sustained mechanical oscillations driven by an airflow coming from a pressure supply such as the player's lungs. Most of the time, the fundamental frequency of a periodic oscillation is close to, and mainly controlled by, one given acoustic mode. These oscillations are the result of a complex non-linear coupling between all the acoustic modes and the mechanical valve. However, the pedal note, which is the lowest note that can be sounded on a brass instrument, is well-known as a counter-example. Eighty years ago, Bouasse did an experiment by replacing the brass mouthpiece with a reed mouthpiece on a brass instrument, and obtained a pedal note a fifth below!
An elementary model dedicated to both cane-reed and lip-reed instruments can be used in order to investigate the sound production of pedal notes, using a time-domain simulation method. Then, the periodic solutions of this model are obtained, and the mysterious oscillation regimes of the Bouasse experiment are discussed.

1 Introduction

In reed instruments the sound is the consequence of self-sustained mechanical oscillations driven by an airflow produced by a pressure supply such as the player's lungs. The oscillations are coming from the destabilization of the mechanical valve, the reed (cane-reed of woodwinds, lips of the brass player). A reed is an oscillating valve which modulates the flow driven by a pressure difference between upstream (player's mouth) and downstream (instrument's mouthpiece). Then the oscillations are induced by the coupling of a localized non-linear amplifying element (the source resulting from the valve-flow interaction) with a pipe, the instrument itself, in which acoustic energy can accumulate in resonant modes (standing waves). As a consequence, the playing frequency, i.e. the fundamental frequency of the periodic acoustic oscillation, is---most of the time---very close to one of the acoustic resonance frequencies of the instrument itself. The pedal note, which is the lowest note that can be sounded on a brass instrument (Fig.1), is well-known as a counter-example (see for example Benade [1]).

.Fig.1 Tenor trombone (left) and saxhorn (right).

When playing a Bb brass instrument, a set of notes close to the Bb harmonic series is obtained. For “conical” brass instruments such as saxhorns, the playing frequencies of the set of notes are very close to the set of the harmonically related resonance frequencies (see the measured input impedance of a saxhorn in Fig.2). For “cylindrical” brass instruments such as trombones the playing frequencies of the set of notes are close to the set of the resonance frequencies. However, the first regime, called the Bb pedal note, does not correspond to the first resonance frequency (which is something like an Eb), but is in fact a Bb, approximately a fifth higher (see the measured input impedance of a trombone in Fig.2). Eighty years ago, Bouasse [2] did an experiment illustrating these phenomena without the need of input impedance measurement set-up. A similar experiment is described here: by replacing the trombone mouthpiece by a clarinet mouthpiece and a cane reed, the lowest regime easily playable is not the Bb pedal note but lies approximately a fifth below, at the first resonance frequency.

Brass and woodwind instruments have many acoustical features in common, particularly from the valve effect point of view. Then the physics of the reed interaction with the air flow, and the coupling with the resonator, the instrument itself, can be described by an elementary model whose solutions can illustrate playing both cane-reed and lip-reed instruments. The aim of the present work is to test the elementary model by comparing its lowest periodic solutions with the pedal notes of brass instruments. In other words, is it possible to reproduce the Bouasse experiment with simulations? After this introduction (chapter 1), the theoretical context (the elementary model and the simulation method) is briefly summarised (chapter 2). Then the results of the experiments and of the simulated pedal notes are presented and discussed (chapter 3), by comparing their fundamental frequencies.

2 Modelling brass instruments

An elementary model of both cane-reed and lip-reed instruments can be summarized by the three following equations Eq.(1), (2) and (3).

The vibrating mechanical reed (cane-reed or lip reed) is represented by a one DOF system relating the opening height h(t) between the reed and the mouthpiece tip (or between the two lips), and the pressure difference between a constant mouth pressure P_m and the acoustic pressure p(t) in the mouthpiece:

$$\frac{d^2h(t)}{dt^2} + g\frac{dh(t)}{dt} + \omega_r^2\left(h(t)-h_o\right) = -\frac{P_m - p(t)}{\mu} \qquad (1)$$

where the parameters g is a damping parameter, ω_r a reed resonance angular frequency, and μ an effective mass per area parameter of the reed.

The volume flow u(t) entering into the instrument is written as a function of h(t) and the pressure difference $P_m - p(t)$:

$$u(t) = w.h(t)\sqrt{\frac{2(P_m - p(t))}{\rho}} \qquad (2)$$

where w is the effective width of the reed and ρ the density of air. Note that h(t) can not be negative, if h(t) reaches the zero value then the reed is blocked against the lay of the mouthpiece and the volume flow u(t) is set to zero too.

The acoustic volume flow and the acoustic pressure in the mouthpiece are related through the acoustic resonator, the brass instrument itself, which in the linear approximation can be represented by an impedance relationship in frequency domain:

$$P(\omega) = Z_{in}(\omega).U(\omega) \tag{3}$$

where Z_{in} is the input impedance of the brass instrument.

If the above elementary model is dedicated to both cane-reed and lip-reed instruments, the values of the parameters can be significantly different. Indeed the first mechanical resonance frequency of the cane reed is much higher than the playing frequencies. The playing frequency of the cane-reed instruments is mainly controlled by one of the acoustic resonance frequencies of the instrument itself. As a consequence, a constant value of 1600 Hz is used in simulations. On the other hand, the mechanical resonance frequencies associated with the brass player's embouchure are crucial in order to get the right note associated to one of the many acoustical resonances of the brass instrument; in other words the brass player by, controlling his embouchure, is able to modify its mechanical resonance frequency to fit it around the acoustic resonance he wants to play. The great variability of the brass player's embouchure implies a great variation of the mechanical resonance frequency which follows the playing frequency in order to select a tune from low register until high register four octaves higher! As a consequence, a set of different values close to the wanted playing frequencies are used in simulations of lip reed instruments.

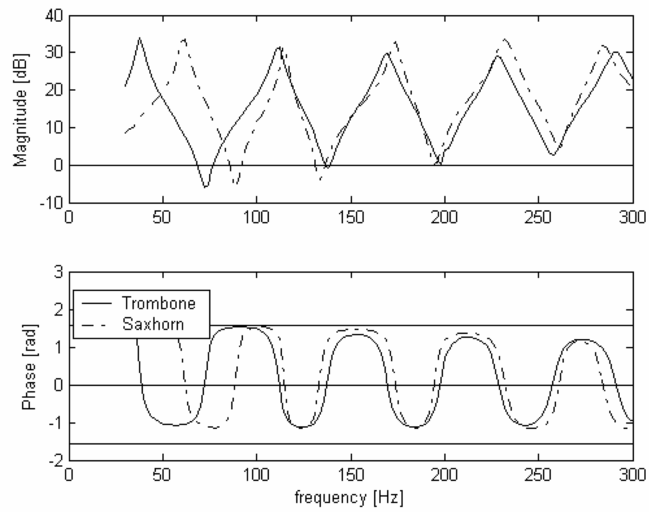
Fletcher [3] followed Helmholtz's pioneering work [4] in identifying cane-reed and lip-reed instruments as an inward or an outward striking reed, when the reed is considered as a simple one degree of freedom mechanical system as in Eq (1). These two kinds of striking reeds correspond to a positive or negative value of μ in Eq. 1 respectively. If cane-reed instruments are always considered as inward striking reed ($\mu>0$) in the literature, the lip-reed does not have such a definitive classification (see the detailed discussion in Campbell [5] for example). In our study, we assume that lip-reeds behave like outward striking reeds ($\mu<0$).

Approximate periodic solutions of the elementary model will be obtained by using the time domain simulation method described in detail in Gazengel [6], method adapted from Schumacher's work [7].

3 Results

3.1 Measured input impedances

Input impedances of a trombone and a saxhorn have been measured using the impedance sensor described in Dalmont and Bruneau [8]. Fig.2 presents the measured input impedance curves (magnitude and phase) of a trombone and of a saxhorn. On one hand, the saxhorn curve exhibits a set of resonances whose frequencies are close to a complete harmonic series, and that is the consequence of its bore geometry close to a conical shape. On the other hand the trombone curve mainly exhibits a harmonic series too, apart from the lowest impedance peak, and that is the consequence of its bore geometry far from an ideal cone (see for example Campbell and Greated [9]). The problem is inherent in the relative proportions of cylindrical and flaring sections: the cylindrical portion of the tube tends to give intervals which are too wide, and the flaring section does not occupy a sufficient fraction of the total length to completely override this tendency!



.Fig.2 Measured input impedance (magnitude in dB and phase in radian) versus frequency of a trombone in first position (solid line) and of a saxhorn without any valves depressed (dashdot line).

Resonance frequency number	Resonance frequency in Hz			
	trombone		saxhorn	
	br.	cl.	br.	cl.
1	38	38	62	62
2	112	112	114	114
3	170	170	174	174
4	228	228	232	232
5	290	292	284	286
6	342	346	348	350
7	400	404	404	408
8	458	464	462	468

Table 1 First height resonance frequencies of a trombone and a saxhorn fitted with a brass mouthpiece (br.) or a cylindrical tube equivalent to a clarinet mouthpiece (cl.).

First the brass instruments have been measured by using their own mouthpiece (Fig.2). Secondly, they have been measured by using a cylindrical tube in place of their own brass mouthpiece. The cylindrical tube has been chosen in order to be equivalent to a clarinet mouthpiece. The resonance frequencies extracted from the two sets of input impedance measurements are shown in Table 1. Replacing the brass mouthpiece by a cylindrical tube equivalent to a clarinet mouthpiece does not influence drastically the values of the resonance frequencies.

3.2 Pedal notes and experiments

A brass player played a tenor trombone (in first position) and a saxhorn (no valve depressed) in order to get the first four periodic regimes. Their fundamental frequencies are estimated and given in Table 2 (columns “br”). The first regime, corresponding to the Bb pedal note, is played at 59Hz with the two brass instruments. If the pedal note played with the saxhorn is close to its first resonance frequency (62 Hz in Table 1, column “saxhorn – br”), the one played with the trombone is far from its resonance frequency (38 Hz in Table 1, column “trombone – br”) which is a fifth below.

Periodic regime number	Playing frequency in Hz (measurement)			
	trombone		saxhorn	
	br. (Fp/Fres)	cl.	br.	cl.
1	59 (1.55)	38	59	57
2	117 (1.04)	111	110	*0*
3	176 (1.04)	*0*	176	174
4	233 (1.02)	*0*	233	231

Table 2 Playing frequencies (in Hz) of a trombone and a saxhorn fitted with a brass mouthpiece (br.) or a clarinet mouthpiece (cl.), obtained by experienced wind instrument players. If the periodic regime is not playable, the value 0 is reported in the Table. Ratio between the playing frequency Fp and the corresponding acoustical resonance frequency Fres (in brackets).
Movies corresponding to the played notes (analyzed table 2) are available on the conference CD.

In order to reproduce Bouasse’s experiment, a second set of notes are played with the two brass instruments after replacing their own mouthpieces with a clarinet mouthpiece. Now the brass instruments are not driven by a player’s vibrating lips anymore, but by using a vibrating clarinet cane reed. If the first regime, the lowest one, is easy to play with the two brass instruments, the upper regimes are difficult to play. By using the spit valve (water key) of the brass instrument as a speaker key (register hole), the periodic regime number 2 has been obtained with the trombone (see Table 2, column “trombone – cl”), and the regimes number 3 and 4 with the saxhorn (see Table 2, column “saxhorn – cl”). Comparing all the fundamental frequencies obtained from the clarinet mouthpiece (Table 2, columns “trombone and saxhorn – cl”) with the corresponding resonance frequencies (Table 1, columns “trombone and saxhorn – cl”), it can be noticed that playing and resonance frequencies are close together. The Bouasse experiment is reproduced. There is no mysterious regime of oscillation anymore.

3.3 Pedal notes and simulations

	cl.	br.
Effective reed width [m]	0.01	0.014
Opening height at rest (when P_m=0) [m]	0.0005	0.00058
Input radius of the mouthpiece [m]	0.0075	0.012
Damping parameter g [s^{-1}]	1.5	4
Reed effective mass per area parameter μ [$kg.m^{-2}$]	-0.28	0.0360
Reed frequency $\omega_r/2\pi$ [Hz]	1600	from 60 to 260

Table 3 Simulation parameters. The two sets of parameters correspond to a simulated brass instrument fitted with a brass mouthpiece (br.) or a clarinet mouthpiece (cl.).

Simulations of periodic trombone regimes have been carried out using the parameter’s values listed Table 3, and a wide band input impedance measurement (up to 4096 Hz). Two kinds of simulations have been done: the valve effect being controlled by a cane-reed system first (column cl. in Table 3), and by a lip-reed system (column br. in Table 3). Note that because the pedal note of the saxhorn does not behave as a “mysterious regime”, no saxhorn simulation results are reported in the present paper.

As far as the cane-reed trombone simulation is concerned, the playing frequency (fundamental frequency of the periodic regime) obtained is 36 Hz (see column cl. in Table 4) close to the first measured resonance frequency of 38 Hz (see Table 1). It corresponds to the lowest periodic regime, and can be compared with the lowest note played with the trombone fitted with the clarinet mouthpiece. Without any surprise, the simulated playing frequency is slightly lower than the resonance frequency because of the inward striking

reed hypothesis (μ <0), and does not drastically depend on the reed frequency value (1600 Hz here). The upper regimes have not been simulated. These results are qualitatively compatible with results from simulations of clarinets or saxophones.
As far as the lip-reed trombone simulation is concerned, the lip-reed resonance frequency is not set to a particular value, but varies over a wide range (from 60 to 260 Hz here) in order to reproduce the brass way of playing: the brass player is able control the periodic regime by adjusting his embouchure. By varying the lip-reed frequency from 60 to 260 Hz, four different regimes have been simulated. Frequency gaps that represent the regime transition are found. For each of the four simulated regimes, the one playing frequency given (column br. in Table 4) corresponds to the simulation where the RMS pressure is locally a maximum. They have been obtained with the following values of lip-reed frequency: 60, 100, 145 and 200 Hz respectively.

Periodic regime number	Playing frequency in Hz (simulation)	
	Trombone	
	br. (Fp/Fres)	cl.
1	86 (2.26)	36
2	148 (1.32)	
3	205 (1.41)	
4	323 (1.42)	

Table 4 Simulated playing frequencies (in Hz) of a trombone fitted with a brass mouthpiece (br.) or a clarinet mouthpiece (cl.). Ratio between the playing frequency Fp and the corresponding acoustical resonance frequency Fres (in brackets).

As far as the simulated regime number 2, 3 and 4 are concerned, the playing frequencies F_p (148, 205 and 323 Hz in Table 4) are drastically higher than the corresponding measured resonance frequency F_{res} (112, 170 and 228 Hz in Table 1). The F_p values are higher as a consequence of the outward striking reed hypothesis (μ >0). The large difference between F_p and F_{res}, illustrated by a frequency ratio F_p/F_{res} from 1.32 to 1.42 (Table 4), may be the consequence of a non optimal choice of the simulation parameters. If the first simulated regime was mainly driven by the first acoustic resonance (F_{res}=38 Hz) in the same way as the regime numbers 2, 3 and 4 are driven by the resonance frequencies 2, 3 and 4, the playing frequency should be found between 50 and 55 Hz keeping the frequency ratio F_p/F_{res} in the range [1.32-1.42]. But the simulation of the first regime, the simulated "pedal note", leads to a playing frequency of 86 Hz. One may note that by using the ratio F_p/F_{res} in the range [1.32-1.42], a fictitious resonance frequency between 60 and 65 Hz is obtained. This fictitious frequency fits quite well as the first "harmonic" of the quasi-harmonic resonance series of the trombone set of resonances (from 112 Hz, the 2nd resonance, to 458 Hz, the 8th resonance in Table 1). Then the simulation tool based on the elementary model described chapter 2 is able to generate the pedal note of a trombone, this "mysterious regime of oscillation" which is not mainly lead by the first resonance frequency of the instrument.

3.4 Concluding remarks

Cane-reed and lip-reed instruments are musical instruments in which sound is produced by self-sustained oscillations of the mechanical valve, reed or lips, destabilized and then driven by the air flow. Useful information about the nature of the destabilization process and the near-threshold playing behavior can be obtained from study of a linearized model which takes into account only a single acoustic resonance coupled to a single mechanic resonance (see for example Cullen [10]). The linear stability analysis can be a valuable first step to get realistic values of the periodic regimes fundamental frequencies. As far the brass instruments are concerned, all the periodic regimes in the Bb harmonic series of the saxhorn (and probably of the other conical brass instruments) can be approached by the linear stability analysis. And it is the same for almost all the periodic regimes in the Bb harmonic series of the trombone (and of the other cylindrical brass instruments probably). The trombone pedal note is the exception, in the sense it is a "mysterious regime of oscillation". Bouasse's experiment told us that the trombone pedal note is not an exception anymore if the vibrating lips are replaced by a single reed!

Prediction of the large amplitude behavior of self-sustained oscillations needs more than a linear stability analysis applied to the physical model. One needs to investigate its periodic solutions. The preliminary study discussed in the present paper shows that an elementary model of reed instruments exhibits the pedal note regime among the periodic solutions. This result gives a quantitative illustration of the multiple-mode cooperation [1] leading to mysterious regimes of oscillation. This preliminary study needs further and more systematic work. For further studies of the simulation method based on modal decomposition [11] would be an attractive tool to investigate the pedal note regime. This method could be used to do a numerical morphing from a resonator having a set of harmonic resonances (saxhorn-like resonator) to a set of incomplete harmonic peaks (trombone-like resonator) by slightly moving down the first resonance frequency out of the harmonic series.

Acknowledgments

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References

[1] A.H. Benade, "Fundamentals of musical acoustics", *Oxford University Press, New York* (1976)

[2] H. Bouasse, "Instruments à vent – Tome 1", *Librairie Scientifique et Technique Albert Blanchard, Paris* (1986)

[3] N. Fletcher, "Excitation mechanisms in woodwind and brass instruments ", *Acustica* 43, 63-72 (1979)

[4] H. Helmholtz, "On the sensation of tone", *Dover, New York,*(1877) translated by A.J. Ellis (1954)

[5] M. Campbell, "Brass instruments as we know them today", *Acta Acustica united with Acustica* 4, 600-610 (2004)

[6] B. Gazengel, J. Gilbert, N. Amir, "Time domain simulation of a single reed wind instrument. From the measured input impedance to the synthesis signal. Where are the traps?", *Acustica* 3, 445-472 (1995).

[7] R.T. Schumacher, "Ab initio calculations of the oscillation of a clarinet", *Acustica* 48, 71-85 (1981)

[8] J-P. Dalmont, A-M. Bruneau, "Acoustic impedance measurement: plane-wave mode and first helical mode contributions ", *J. Acoust. Soc. Am.* 91, 3026-3033 (1991)

[9] D. Campbell, C. Greated, "The musician's guide to acoustics", *Oxford University Press, Oxford* (1987)

[10] J. Cullen, J. Gilbert, M. Campbell, "Brass instruments linear stability analysis and experiments with an artificial mouth ", *Acustica* 86, 704-724 (2000)

[11] F. Silva, V. Debut, J. Kergomard, C. Vergez, A. Deblevid, P. Guillemain, "Simulation of single reed instruments oscillations based on modal decomposition of bore and reed dynamics ", *19th International Congress on Acoustics*, Madrid (2007).

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