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On some possibilities and properties of matched-field geoacoustic inversion in shallow water

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The aim of this paper is to estimate possibilities of matched-field geoacoustic inversion (MFI) in shallow water, and to recommend optimal arrangements of signal source and hydrophone array in variety of particular environments. We assumed ocean to be range independent with bottom consisting of homogeneous liquid layers. Sound fields were calculated as superposition of normal modes and continuous spectrum for tonal point source and vertical line array. MFI based on Bartlett processor was used. Possibilities of MFI were characterized by MFI penetration depth, sensitivity to various bottom parameters and non-uniqueness of inverted data. These characteristics were analysed as functions of frequency and the source depth and range for different values of sound attenuation in sediments and for various sound-speed profiles. To estimate possibilities of MFI in real ocean conditions, influence of array tilt and inadequacy of geoacoustic model were analysed. The influence of continuous spectrum was also discussed. Among the major results, optimal source ranges corresponding to the maximum penetration depth of MFI were calculated, and high influence of attenuation in sediments on possibilities of MFI was revealed.

1. Introduction

The influence of bottom structure on sound propagation is essential in shallow water, thereby it is possible to use matched field geoacoustic inversion (MFI) for remote bottom sensing. Its idea is to identify sea bed properties via inverting the acoustic field produced by a known source and measured at a receiving hydrophone array. Since 1990th MFI was studied in numerous works devoted to the inversion strategies development, computer modeling of test cases and to the application of MFI to the real ocean environment [1-8]. Nevertheless, many aspects of MFI are still rather vague. What inversion precision can be achieved, if we have particular uncertainty of the input data? What measurement precision have we provide to avoid non-uniqueness of inversion results? Is optimization of experiment geometry possible? What factors determine penetration depth of MFI? This paper represents an attempt of clarification of these questions.

2. Environmental model and investigation method

The geoacoustic model of bottom used in this paper consists of N uniform sediment layers overlying semi-infinite basement. We used up to N=50 layers to simulate real bottom structure and to describe it by inversion models with N=0-2. Each n-th layer was characterized by constant sound speed C_n, layer thickness ΔH_n, density ρ_n and attenuation coefficient α_n (fig. 1). We assumed an environment to be a range independent and no shear waves were taken in account.

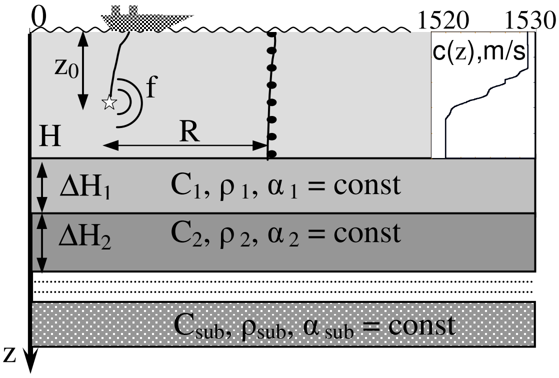

Fig. 1. Environmental model.

We used 2 types of sound speed profile in water C(z): 1) typical shallow water profile (fig. 1); 2) $C(z)=C_0$ = 1500 m/s to accelerate computations.

The acoustic field was produced by harmonic point source at depth Z_0, and measured by vertical hydrophone array at range R.

In this work the following general scheme of problem modeling was used:

1. The first stage included "real" environment parameters selection and corresponding sound field $P_e(z;\ z_0,\ R)$ computation. Thereby the input "measured" data was simulated.

2. On the next stage computation of replicas $P_{th}(z;\ z_0,\ R)$ corresponding to certain parameter set were made.

3. To determine the measure of correlation between replicas and the experimental field Bartlett processor in monochrome form was used:

$$\Re = \frac{\left|\vec{P}_{th}\vec{P}_e^*\right|^2}{\left|\vec{P}_{th}\right|^2\left|\vec{P}_e\right|^2} \qquad (1),$$

where $\vec{P}_{th}$ and $\vec{P}_e$ are vectors of measured and replica fields on array. This objective function $\Re$ has global minimum and reaches zero in case of full equality of real and replica sets of parameters. Thereby inversion can be carried out through global minimum localization while studying properties of MFI demands objective function relief analysis.

In this work we used the propagation model representing sound field as superposition of normal modes and the continuous specter field:

$$P(R,z) = 2\pi i \sum \mathrm{Res} - \int_{EJP} \tilde{P}(\xi, z) H_0^1(\xi \cdot R)\xi d\xi \qquad (2),$$

where EJP is Ewing, Jardetzky, Press brunch cut on comlex plane (ξ`,ξ``); $\xi`` = k_{sub}`k_{sub}``/\xi`$, (k is wavenumber). Special program in Matlab7 was made to compute fields.

Attenuation in n-th sediment layer was described via frequency independent non-dimensional attenuation coefficient α_n (corresponding to $\beta[\partial B/\lambda]=27.3\alpha$)

This propagation model is scale-invariant, thus using wavelength λ or water column depth H to measure ranges, we thereby determined frequency $f=C_0/\lambda$ (C_0≡1500 m/s).

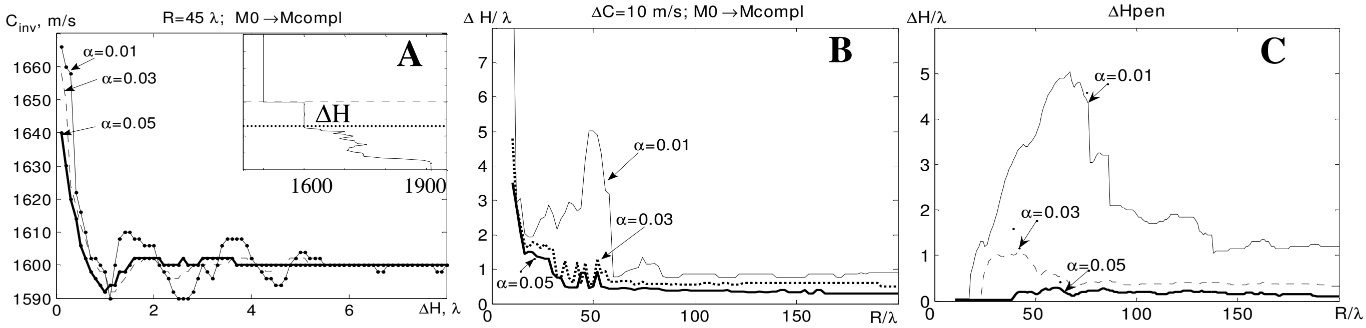

Fig. 2. Description of real model M_{compl} by inversion model M0 (water column depth H=5λ). A – sound speed inversion results $C_{inv}(\Delta H)$ at range R=45λ (true profile is shown in the top corner); B - corresponding sediment minimal thickness ΔH_{min} at ΔC=10 m/s level; C – penetration depth at mismatch level $\Delta\Re$=0.05.

3. Results and discussion

The simplest model of a sea bed is a uniform semi-infinite half space (model M0). If vertical non-uniformity in the real bottom appears deep enough it does not influence sound field in water, and M0 would be an adequate model. To estimate this required depth of uniformity the following test case was made. Real bottom was assumed to have uniform upper layer (C_1 = 1600 m/s) with a thickness ΔH and an underlying non-uniform space of complicated structure M_{compl} (described by N=50 layers – fig. 2A). Values of ΔH, the upper layer attenuation coefficient α_1 and the source-array range R were varied in the test case. Values of sound speed in the half space C_{inv}, its density and attenuation were inverted for each combination of mentioned parameters. It is well known, that the sensitivity of Bartlett processor to sound speed perturbations is much greater then one of density or attenuation, therefore we will discuss results of the test case only by the example of C_{inv}. Fig. 2A presents the results C_{inv} of inversion as a function of ΔH at α=0.01, α=0.03 and α=0.05. If the upper layer is thin (ΔH<0.1λ), then inversion gives C_{inv} = 1660 m/s, which corresponds to the average value of sound speed in upper layers (it proves an ability of MFI to average out parameters of non-uniform bottom). As thickness ΔH grows the inversion gives values converging to the true sound speed value: $C_{inv} \to C_1$ = 1600 m/s. Then it is possible to determine minimal uniform layer thickness ΔH_{min} corresponding to the following condition:

$$|C_1 - C_{inv}(\Delta H)| < \Delta C, \text{ if } \Delta H > \Delta H_{min} \qquad (3),$$

where ΔC is the allowed maximum inversion uncertainty. Fig. 2B presents minimal depth $\Delta H_{min}(R)$ of non-uniformity location as function of source-array range at different attenuations for ΔC=10 m/s. At short ranges (R~20-30 λ) field does not contain enough information about sea bed, and ΔH_{min} value is small, but growing gradually to its maximum at R~50 λ. Exact value of $\Delta H_{min}(R)$ in the maximum and its localization depend on attenuation α_1 and on H/λ. As range grows, value of ΔH_{min} decrease rapidly and at R~100λ becomes almost constant (ΔH_{min}(R>100λ) ~ λ), reaching the smaller value the bigger attenuation α_1 in uniform layer is.

Every inversion model describes real bottom approximately, that is, the mismatch $\Re$>0 between measured field and replica always exists. If we calculate the replica field in model M0 with bottom parameters similar to the parameters of the upper sediment layer in real bottom, then the mentioned mismatch will determine the measure of adequacy of M0. To analyze range dependence of this adequacy we defined a new quantity ΔH_{pen} as follows:

$$\Re(\Delta H) < \Delta\Re, \text{ if } \Delta H > \Delta H_{pen} \qquad (4)$$

where $\Delta\Re$ is the allowed mismatch. ΔH_{pen} can be associated with depth of penetration of MFI into sediments: it characterizes how deep non-uniformity should appear to provide mismatch $\Delta\Re$. Fig. 2C presents dependence $\Delta H_{pen}(R)$ for $\Delta\Re$=0.05.

The function $\Delta H_{pen}(R)$ depends mostly on attenuation α_1: low values of α_1 correspond to the big penetration depth and to the sharp peak in $\Delta H_{pen}(R)$. This function is also depends on H/λ. It can be seen, that for high values of α_1 the penetration depth is extremely small (for α_1=0.05 ΔH_{pen}<0.3λ even in the maximum). It means that a very precise field measurements and an appropriate propagation model are needed to refine inversion model and to penetrate deeper into the bottom.

At the end we can conclude, that model M0 usage is valid, if it is known a priori, that bottom can be assumed to be approximately uniform till the depth $\Delta H \geq \Delta H_{min,}$ and in this case it is also possible to refine inversion mode. If we have not such information we are forced to do it. The depth ΔH_{min} can be approximately estimated before environmental experiment.

Adding sediment layer to the inversion bottom model makes parameter classification by sound field sensitivity not universal, as it depends on sediment layer attenuation α_1, its thickness ΔH and range R. In spite of this in most cases sound field is more sensitive to sound speed C_1 in sediment layer and to its thickness ΔH than to another parameters. The examples of 2-D cross sections of the objective function $\Re(C_1, \Delta H)$ for 3 different ranges are shown on fig. 3. At short ranges (R<2H) sound field has not collected information about sea bed structure yet, and the topography of objective function is bad for inversion. As range R grows global minimum width decreases (i.e. sensitivity grows) (fig. 3B for R=5H). However at long distances absorption of bottom modes (modes, which oscillate in upper sediment layers), which embody information on bottom structure, leads to loss of sensitivity to layer thickness (fig. 3C for R = 25H). Hence, there are

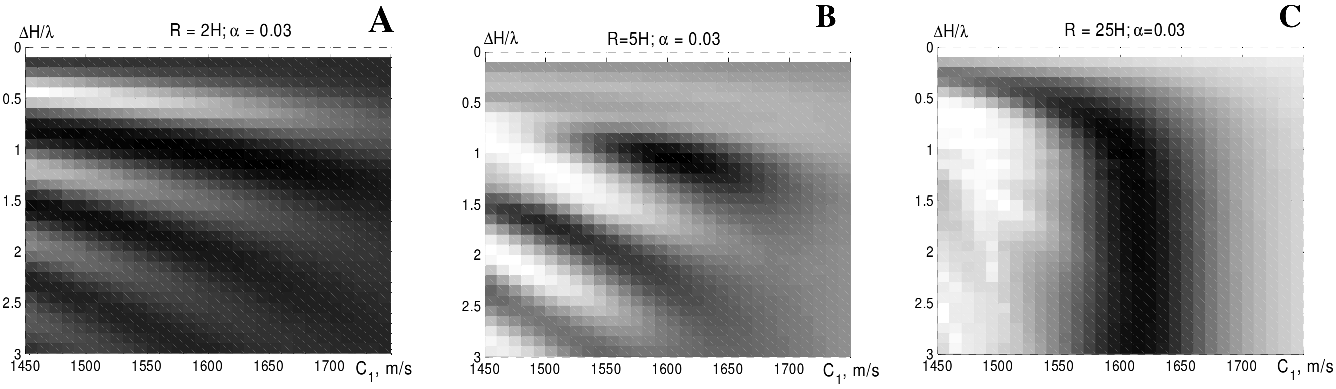

Fig. 3. 2-D cross sections of objective function $\Re(C_1, \Delta H)$ at ranges R=2H (A), R=5H (B) and R=25H (C). Minimal mismatch areas are dark.

ranges between R=2H and R=25H, which are optimal for inversion of C_1 and ΔH.

The results of inversion in real environment are always influenced by measurement errors and noise, while inversion model never describes all details of real bottom structure. That's why we assume "measured" field to be known with uncertainty, corresponding to maximum mismatch value $\Re_{er}$ between measurement result and true field. Consequently, after exact localization of the objective function global minimum, the inversion result uncertainty is governed by the width of the minimum at $\Re_{er}$ level. To reveal optimal ranges, we estimated the value of $\Re_{er}$, that is necessary for carrying out inversion with the considered result uncertainty. It was made via computation of angle θ_{min} which is defined as follows:

$$\theta_{min}=|\arg\min\{\Re(C_{01}\cos\theta, \Delta H_0\sin\theta\}| \qquad (4),$$

where $\theta\in(0,2\pi)$, C_{01} and ΔH_0 are inversion result uncertainties of C_1 and ΔH. This angle gives direction of a ravine near the global minimum. That is, if $\Re_{er}=\Re(\theta_{min})$, then sound speed in sediment layer and its thickness can be found with uncertainty $C_{01}\cos\theta$ and $\Delta H_0\sin\theta$. Fig. 4A and 4B present functions $\Re(R; \theta=\theta_{min})$ и $\theta_{min}(R)$ for different α_1. Small values of θ_{min} at short ranges (R<20H) mean essential elongation of ravine along C_1 axis. As range grows, the ravine direction rotates gradually and becomes extended along ΔH axis (i.e. sensitivity to ΔH decreases). Peaks on fig. 4B correspond to most precise inversion at given α_1, but their exact location can hardly be predicted, and, thereby, the existence of optimal ranges region R_{opt1}=20-40 H is more important. The angle θ_{min} comes to 30-60° also in the

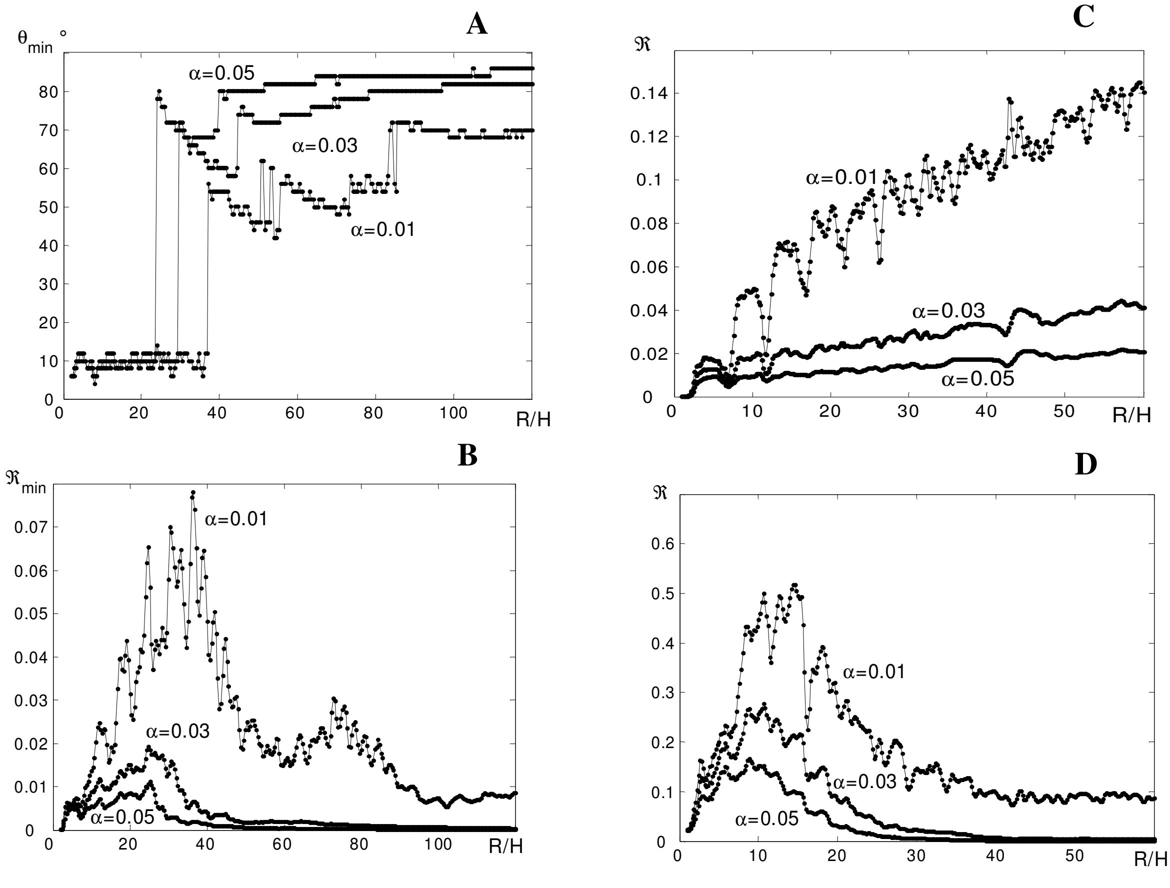

Fig. 4. Analyzing global minimum width at mismatch level C_{01}=10 m/s, $\Delta H_0=0.15\lambda$. A – $\Re_{min}$ (R; $\theta=\theta_{min}$); B – θ_{min}(R); C – $\Re_{min}$ (R; $\theta=\pi/2$); D – $\Re_{min}$ (R; $\theta=0$).

same region, allowing precise inversion of both C_1 and ΔH.

Above said means possibility to optimize the experiment geometry using source-array ranges within R_{opt1} region. Unfortunately, it is not always possible because of deep local minima existence at such ranges (this dependence on range is analyzed below). Therefore, in some cases it is reasonable to shorten range and to invert ΔH almost independently of C_1, providing much better precision for ΔH then for C_1 ("independent" inversion of ΔH is possible because of small θ_{min} values at short ranges). Fig. 4C presents function $\Re(R;\ \theta=\pi/2)$, showing that the optimal region for this approach is R_{opt2}=5-20H and depends on attenuation. If required, inverted value of C_1 can be adjusted by towing the source from the array, as C_1 "independent" inversion precision grows with range (fig. 4D).

Analysis of different test cases showed, that distributions $\Re(R;\ \theta=\theta_{min})$ and $\Re(R;\ \theta=\pi/2)$ are rather universal: 1) their dependence on H/λ ratio is slight, thereby, they do not depend much on frequency (tests were made for frequency range, corresponding to H=3-20λ); 2) the location of optimal regions R_{opt1} and R_{opt2} does not depend on layer thickness ΔH crucially: at different ΔH this regions have common areas; 3) the variations of C_1 and C_2 do not influence R_{opt1} and R_{opt2}, though values of $\Re$ depend on them (evidently, $\Re(C_1=C_2, \Delta H)\equiv 0$). The main factor, which determines optimal ranges, is the attenuation coefficient α_1: for example, R_{opt2}=14-20H at α_1=0.01 and R_{opt2}=7-14H at α_1=0.03 are valid estimates for typical shallow water values of C_1, C_2 and for ΔH=0.5-3λ. Consequently, R_{opt1} and R_{opt2} can be estimated before environmental experiment, though a priori information about upper sediment layer attenuation coefficient can refine estimates essentially.

One of the most important questions is the non-uniqueness of inversion results. The objective function $\Re$ is multiextremal, that is, if we measure sound field with uncertainty $\Re_{er}$, inversion results will be non-unique under condition, that mismatch $\Re_{LG}$ between replica fields in the deepest local minimum and in the global one will be less then $\Re_{er}$ ($\Re_{LG} < \Re_{er} \rightarrow$ non-uniqueness). Fig. 3A-C show, that $\Re_{LG}$ depends on source-array range R: $\Re_{LG}\rightarrow 0$ at $R\rightarrow\infty$ or $R\rightarrow 0$ (in this figures $\Re_{LG}$ is close to $\Re$ value in the deepest local minimum). Hence, there are regions with maximum values of $\Re_{LG}$. To reveal these regions we made the test, resulted in distribution $\Re_{LG}(R)$ (fig. 5).

Fig. 5 shows, that the dependence $\Re_{LG}(R)$ decreases gradually from its maximum, located at R_{LG}~5H. The stability and the universality of presented dependence are rather arguable, because of the extremely time consuming procedure of $\Re_{LG}$ value computation. We found that $\Re_{LG}(R)$ is not influenced much by attenuation or by H/λ ratio. But this dependence is seemed to be influenced by layer thickness (fig. 5 corresponds to ΔH=λ). For example if ΔH=2-3λ the peak at R~5H is much more sharp and the following decreasing is much more rapid. For ΔH<0.5λ the definition $\Re_{LG}$ becomes rather vague, because of crucial loss of Bartlett processor sensitivity to C_1 perturbations. However, we consider R_{LG}~4-6H to be optimal region to provide inversion uniqueness.

To demonstrate in what way all obtained results can be used in practise we will discuss the following example. Consider, we have tonal source with λ=H/5 and we know a

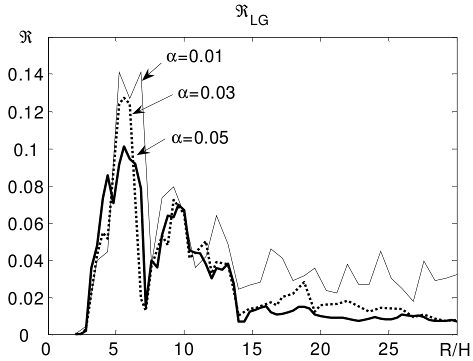

Fig.5. The mismatch $\Re_{LG}$ between replicas in the deepest local minimum and the global one as function of range at α_1=0.01, α_1=0.03 and α_1=0.05. (H=5λ, C_1=1600 m/s, ΔH=λ, C_2=1900 m/s).

priory that attenuation in sediments is not too high (α_1~0.01). Also we assume the accuracy of the field measurements to be about ε=10%. If we use model M0 we can obtain unique inversion results, which will be the more accurate, the longer the range will be (the range length is bounded by environment

If we use model M1 (trying to invert ΔH, C_1 etc.) we should provide inversion uniqueness, working at ranges R=4-6H or R=9-11H (ε=10% corresponds to $\Re_{er}$~0.1). But we also should take into account, that estimate of mismatch $\Re_{LG}$

For example, if the hydrophone line array is not vertical, but has a tilt in direction of sound source, we have to determine the tilt angle with rather high precision. In the case being discussed uncerainty of this angle has to be no more then 2-3° (it corresponds to maximum hydrophone-source range uncertainty of only 0.3λ).

The usage of model M1 at all mentioned conditions will give us the value of ΔH with the uncertainty less then 0.1λ. But the uncrtainty of sound speed in upper sediment layer will be high (ΔC_1~40 m/s).

It was also analyzed if it is necessary in propagation model to take into account the field of continuous specter. Though its contribution decreases with range rapidly, it can cause additional mismatch at short ranges. The value of this mismatch can be sufficient ($\Re$~0.05) at R<50λ especially if we use model M0.

4. Conclusions

Value of the minimum uniform layer thickness ΔH_{min} in realistic bottom, which is necessary for inversion model M0 (uniform half-space) to be adequate, is influenced by a source-array range R and attenuation α_1 in the layer. At R>100λ this value is less then wavelength (ΔH_{min}~λ at α_1=0.01, ΔH_{min}~ 0.5λ at α_1=0.05). If $\Delta H<\Delta H_{min}$ inversion at short ranges averages real sea bed parameters, and corresponding penetration depth can be rather high (H_{pen}~ 10λ).

For environmental model being used, optimal value of source depth $z_0 \approx \lambda/2$ was obtained.

Model M1 (uniform layer and a half space) analysis revealed 3 types of optimal range regions:

1) in the region R_{LG} inversion uniqueness is obtained at minimal requirements to input data precision. Location of R_{LG} does not depend on attenuation α_1 and approximately R_{LG}=4-6 H.

2) in the region R_{opt1} highest inversion precision of both C1 and ΔH is obtained. Location of this region depends on α_1 attenuation essentially: R_{opt1} = 15-25 H at α_1=0.05 and R_{opt1} = 25-45 H at α_1=0.014;

3) in the region R_{opt2} highest inversion precision for ΔH is obtained, while uncertainty of C_1 is rather high. This region ranges are approximately twice shorter then for R_{opt1}: R_{opt2} =7-14H at α_1=0.05 and R_{opt1}=14-20H at α_1=0.01.

The inversion precision for C_1 grows with range (the growing is more rapid, the lower attenuation α_1 is).

Location of mentioned regions doesn't depend on H/λ ratio and depends slightly on true ΔH value. Hence, in sufficiently narrow frequency range (f_{max}/f_{min}<2-3) this location is fixed (results were obtained at f, corresponding to H=3-20λ). The location of mentioned regions can be approximately estimated before environmental experiment. A priori information about attenuation in sediments can make these estimates more precise.

Therefore, if a purpose of the environmental experiment is a rude estimation of sea bed parameters via minimal efforts (for example, only single array and monochrome source are available), then it is reasonable to work in R_{LG} region (especially if $R_{LG} \in R_{opt2}$), because of importance to provide inversion uniqueness. On the other hand, if inversion uniqueness can be provided at another ranges (for example it's possible to use several frequencies), then it is a good way to work at R_{opt1}.

In this work we revealed optimal ranges for inversion of only sound speed profile in upper bottom layer (C_1 and ΔH), using simplest inversion model. But it is seemed to be, that optimal ranges for the inversion of attenuation and density or for any parameters of more complicated models can also be revealed and estimated before environmental experiment.

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