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Experimental verification of range-dependent inverse method for geoacoustic parameters from modal dispersion data

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The procedure for determining the sediment compressional wave speed profile from modal dispersion data is extended to cover range-dependent environment. In this approach, the range-dependent environment is divided in to a number of range-independent sections and the sediment characteristics of each section estimated. Results of inversions performed using synthetic data show that the range-dependent properties can be obtained for an experiment conducted using multiple source receiver combinations. During the Shallow Water Experiment 2006, data were collected using a distribution of receivers and sources. The source transmitted broad band pulses over a frequency band of 50 – 250 Hz. Here we present results obtained by analysis of this data, including the sediment compressional wave speed profiles estimated for regions of the experimental area.

1. Introduction

In shallow water, the ocean bottom plays a significant role in the propagation of acoustic energy. Rajan, *et. al*.[1] have developed a method for estimating the ocean bottom acoustic properties using modal eigenvalues. An alternative method is to use the group velocity dispersion relation to determine the sediment acoustic properties. The acoustic signal in this case is generated by a broadband source. The group velocity dispersion curve is obtained by suitably processing the data acquired at the receiver. The extraction of sediment properties from group velocity data in he case of a range-independent environment has been outlined in Rajan *et. al*[1]. In this paper we present an extension to this method for estimating the sediment characteristics in a range-dependent environment.

2. Formulation of the inverse problem

In shallow water, the acoustic field can be represented as a sum of contributions from a set of propagating modes[2]. These propagating modes are identified by their eigenvalues i.e. horizontal wavenumber. The number of modes that exist in a given waveguide is dependent on the frequency of the acoustic source and the ocean environment, i.e. the geometric and geo-acoustic properties of the water column and the sediments. The group velocity of a particular mode is the energy transport velocity of the mode. The arrival time of a particular mode at the receiver will therefore depend on the group velocity. The dependence of the group velocities of the modes on the frequency is the group velocity dispersion. The group velocities themselves are dependent on the characteristics of the waveguide, including the acoustic characteristics of the sediment layers. This dependence is exploited to extract sediment acoustic properties from group velocity dispersion data. In the following paragraphs the formulations of the inverse problem for the range-dependent case are briefly presented.

The acoustic field $p(r,z_s,z_r;\omega)$ by a point harmonic source of frequency ω in a range-dependent environment is given by the expression[2]

$$p(r,z_s,z_r;\omega) = i\sqrt{\frac{2}{\pi}}\, exp(i\pi/4)\sum_{n=1}^{N}\phi_n(z_s,r=0)\phi_n(z_r,r) \times \frac{e^{i\int_0^r k_n(s)ds}}{\sqrt{k_n(r)r}} \quad (1)$$

where r is the range to the receiver, z_s is the source depth, z_r is the receiver depth, ϕ_n is the eigenfunction of mode n, k_n is the eigenvalue of mode n and N is the total number of propagating modes in the waveguide. In obtaining the above expression we have assumed that the adiabatic approximation is valid. The modal phase is therefore given by the expression

$$\theta_n = \int_0^r k_n(s)ds \quad (2)$$

Taking the difference on both sides we have

$$\Delta\theta_n = \int_0^r \Delta k_n(s)ds \quad (3)$$

Next, consider the situation where the compressional wave speed of the environment is perturbed by a small amount. Then at any given range location *s,* a perturbation in compressional wave speed is related to a change in the eigenvalue by the relation[1]

$$\Delta k_n(s) = \int_0^\infty \frac{-1}{k_n(s)} \frac{\omega^2 \Delta c(s,z)}{c_b^3(s,z)\rho_b(s,z)} \times \left|\phi_{nb}(s,z)\right|^2 dz \quad (4)$$

where $k_n(s)$ is the eigenvalue of the n[th] mode for an assumed initial unperturbed model at range s, $c_b(s,z)$ and $\rho_b(s,z)$ are the initial unperturbed sediment compressional wave speed and density profiles at range s , ϕ_{nb} is the eigenfunction corresponding to the n[th] mode at that location for the unperturbed model, and $\Delta c(s,z)$ is the perturbation in the compressional wave speed.

Inserting this expression for Δk_n in equation (3), we obtain the equation

$$\Delta\theta_n = \int_0^r \int_0^\infty \frac{-1}{k_n(s)} \frac{\omega^2 \Delta c(s,z)}{c_b^3(s,z)\rho_b(s,z)} \times \left|\phi_{nb}(s,z)\right|^2 dsdz \quad (5)$$

The modal arrival time perturbation for the n[th] mode is given by

$$dt_n = \frac{\partial \Delta\theta_n}{\partial\omega} = \frac{\partial}{\partial\omega}\int_0^r \int_0^\infty \frac{-1}{k_n(s,\omega)} \times \frac{\omega^2 \Delta c(s,z)}{c_b^3(s,z)\rho_b(s,z)} \left|\phi_n(s,z,\omega)\right|^2 dsdz \quad (6)$$

The double integral can be changed into a double sum given below

$$dt_n = \sum_{p=1}^{P}\sum_{q=1}^{Q} A(s_p,z_q)\Delta c(s_p,z) \quad (7)$$

This double sum can be reduced to a matrix equation and the equation solved to determine $\Delta c(s_p,z_q), p=1,\cdots,P, q=1,\cdots,Q.$ When converting the integral to a matrix equation we assumed the region to be discretized in both range and depth. The argument s_p refers to the p[th] step in range and z_q refers to the q[th] step in depth.

In the practical implementation of this approach, the region of interest is divided in to a number of range-independent sections. The total distance R between the source and receiver is divided into M range-independent sections. Equation (6) then reduces to the form

$$dt_{n,\omega} = \frac{\partial \Delta\theta_n}{\partial\omega} = \sum_{m=1}^{M} r_m \frac{\partial}{\partial\omega}\int_0^\infty \frac{-1}{k_{nm}(\omega)} \frac{\omega^2 \Delta c_m(z)}{c_{bm}^3(z)\rho_{bm}(z)} \times \left|\phi_{nbm}(z,\omega)\right|^2 dz \quad (8)$$

In the above equation the subscript *m* to the parameters refers to the value of these parameters in the m[th] range section. The left hand side of equation is the difference in arrival time of the n[th] mode between the value determined from the experiment and the time from the initial unperturbed ocean model.

In order to estimate the sediment compressional wave speed we assume that we have some approximate estimate of the compressional wave speed profile from archival data or some other source. The true compressional wave speed of the sediment is assumed to be a small perturbation to this initial estimate of the profile. Using the formulation outlined above, the corrections to be applied to the initial estimate of the compressional wave speed can be obtained from the group velocity dispersion data for the true model as determined from a field experiment and the computed group velocity dispersion data for the assumed initial model of the compressional wave speed. It can be shown that in order to extract range-dependent sediment properties, mode arrival time data for multiple source-receiver combinations are required.

In the experimental set up for this approach, a broadband source transmits the signal. By using a multiplicity of sources and receivers we obtain sets of signals corresponding to each source/receiver combination. Time-frequency analysis of the received signals yields an estimate of the arrival time for each mode at different frequencies along the paths of each source/receiver combination. The mode arrival times for the initial model of the environment are computed. Differences between the experimentally determined arrival time data and the arrival time data for the initial model of the environment are used to extract the sediment acoustic properties for the M range intervals.

3. Shallow water experiment 2006

Shallow Water Experiments 2006 were conducted in the general area of Hudson Canyon off the New Jersey coast. Sponsored by the US Office of Navel Research, the primary goal of the experimental program was to conduct a broad range of experiments to study acoustic propagation in the littoral environment. One component of this study was a set of experiments (MIME) designed to evaluate modal inverse techniques for determining the sediment properties in a spatially varying environment. Figure 1 shows variations of the compressional wave speed in the top layers of the sediment within the experimental area. Spatial variability of the compressional wave speeds is clearly evident in the figure. The MIME experimental configuration was designed to capture this variability.

The experiment consisted of two sets of experiments one with a broad band source and the other with a CW source. The data were collected on the SHARK array, consisting of a 16 element vertical line array (VLA) and 32 element bottom mounted horizontal array. In addition, data were collected on 4 spatially distributed single hydrophone recording units (SHRU 49-53). In the case of the broad band experiment, the experimental plan was to collect data from a source whose locations were along the perimeter of a circle about 15 km from the VLA. However, during the experiment, due to time and other considerations, it was not possible to execute the experiment as planned. In the actual experiment, broad band data were collected with the source placed at locations along two circular arcs relative to the VLA.

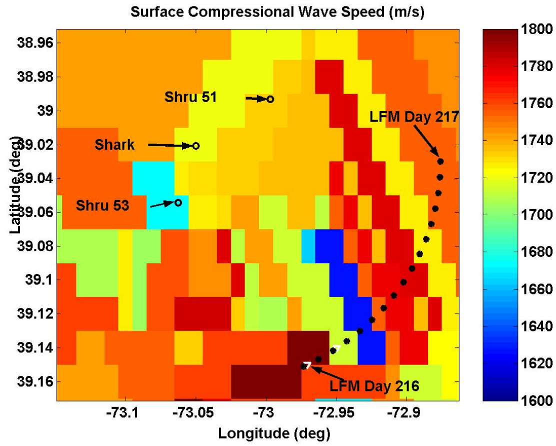

Figure 1: The figure shows the spatial variability in the compressional wave speed. The figure also shows the location of broad band shots on year days 216 and 217 and the location of the vertical line array SHARK and single element units SHRU 51 and SHRU 53.(Data on compressional wave speed from seismic survey by University of Texas, Austin)

A broad band signal (Linear frequency modulated signal) was transmitted using a J15-3 source. The bandwidth of the signal was 200 Hz i.e. the linear sweep was from 50 Hz to 250 Hz. The duration of the pulse was 0.5 sec. At each location multiple pings were transmitted so as to enable some averaging to be done to improve the SNR. However, since the source level of the signal was not high, the SNR achieved was in some cases low.

During the course of the experiment, the sound speed structure in the water column was obtained using a CTD chain. The chain was towed by the ship that towed the source and sound speed structure at the location of the source was measured. The chain had 21 sensors distributed along the length of the chain. Each element of the chain takes measurements of the temperature, conductivity and pressure every 2 seconds. The sound speed profile is computed from the values of the temperature, and conductivity at each sensor as a function of time.

4. Analysis of experimental data

As indicated earlier, broad band transmissions were made with the sources placed along an arc.

These transmissions were made on year days 216,217 and 218..

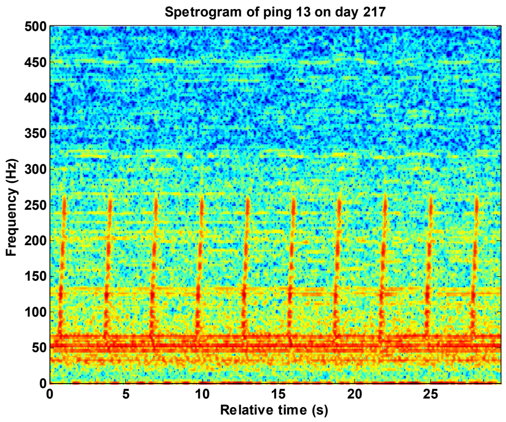

Fig. 2: Spectra of pings at location 13 on day 217

Figure 2 shows the spectra of the pings recorded at one source location on day 217. The figure shows analysis of 30 seconds of data on a single hydrophone. The sweep of the signal from 50 Hz to 250 Hz is seen in the data. Data for each transmission is processed using Short Time Fourier Transform to determine the mode dispersion data. After analyzing a number of pings, the pings with the highest signal to noise ratio are used to extract the mode arrival time data. Since the data acquisition system could not be synchronized with the transmission of the pings, the arrival time of the modes based on time of transmission of the signal could not be determined. Therefore, in order to use this data, the inverse algorithm was modified to use the time difference between the arrival of modes i.e we determine the time difference between the arrival time of say mode 1 and mode 2 at a given frequency and compare it with the time difference computed numerically for the intial model of the environment. The mode arrival structure for one of the pings is shown in Fig. 3.

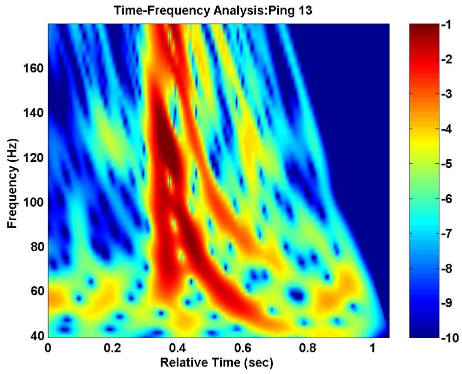

Figure 3: Time-Frequency analysis of data

The next step in the inversion of range-dependent sediment properties is to segment the region in to several range-independent sections. Based on the initial estimate of the compressional wave speed as given in Figure 1, the region of interest was divided in to sections as shown in Fig. 4. The compressional wave speed in regions marked 41, 51, 61, 42, 52 and 62 were obtained by inversion of the experimentally determined data. The sound speed profile of the water column was obtained by averaging the profiles determined from CTD chain data collected during the transmission times of the acoustic signals. The density was assumed to be a constant 1.6 gm/cc in all the sediment layers and in the half space. The compressional wave speed profiles resulting from the inversion in the different regions are shown in Fig. 5.

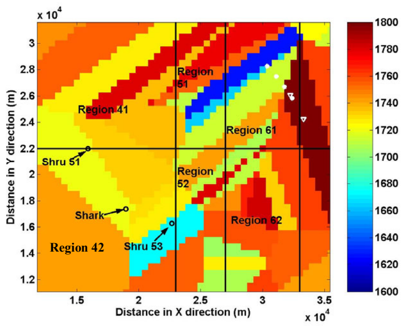

Figure 4 Range-independent sections in region between the location of the sources and receivers.

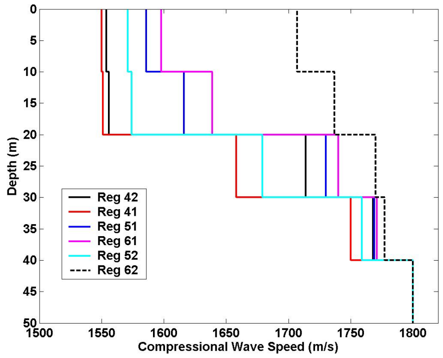

Figure 5: Compressional wave speed profiles in the five regions

We note except for region 62 which exhibit a high velocity surface layer, the other regions have similar compressional wave speed structure.

5. Conclusions

We have studied the inversion for sediment acoustic properties from mode dispersion in a range-dependent environment. The range dependent properties can be obtained by collecting data from multiple source receiver combinations. The method is based on representing the range-dependent environment as a set of range-independent sections. The data collected during the Shallow Water Experiment 2006 were analyzed to extract the mode arrival time differences. This data were used as input to the inversion algorithm and range-dependent sediment properties estimated.

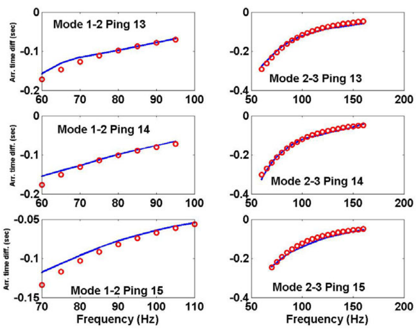

Figure 6: Comparison of mode arrival time differences from experiment and prediction by model.

As a validation of the inversion results we present in Fig. 6 a comparison of the mode arrival time differences as determined from the experiment and from model predictions. We note that the agreement between the two is good.

The variance and resolution lengths for three regions 41,51 and 61 are shown in Fig. 7. The deviation of the estimates in the top two layers is approximately 1 to 3 m/s. The resolution lengths for these two layers are 10 m. The deviation and resolution length both increase with depth

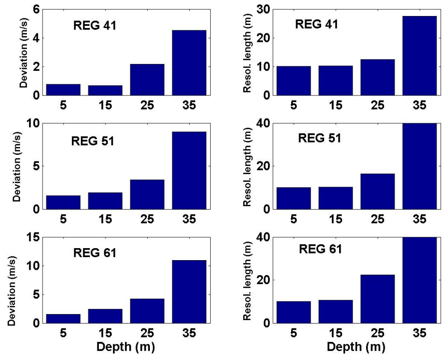

Figure 7: Deviation of estimates and resolution length for regions 41, 51 and 61.

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