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The physics of wedge diffraction: A model in terms of elementary diffracted waves

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A top-down physical principle called virtual discontinuity principle of diffraction is applied to waves diffracted by a wedge. In the analysis diffracted waves are described by a sum of two more fundamental quantities called elementary diffracted waves and the physics of wedge diffraction is made clear in terms of the elementally diffracted waves. In addition their simple structure in the far field enables us to reproduce the far field rigorous solutions of waves diffracted by the wedge. Thus the principle is justified firmly by this result.

1 Introduction

We have proposed a new physical principle that is called virtual discontinuity principle of diffraction (abbreviated by VDPD) for analyzing waves diffracted by an obstacle [1]. Since VDPD is a top-down principle, it has been justified by the fact that the relation for diffracted waves by VDPD always satisfies the boundary condition at the surface of the obstacle. This property is not supported by other principles for analyzing diffracted waves, for examples, Kirchhoff's formula and Boundary Element Method.

In this paper VDPD is applied to waves diffracted by a wedge to make the physics of wedge diffraction clear. The main feature of VDPD lies on the assertion that waves diffracted by the apex of an obstacle are expressed by a sum of two more fundamental quantities that are called elementally diffracted waves. Since the wedge has one apex, it is suited to study the properties of elementally diffracted waves. And the potential in the wedge can be expressed by geometrical optics waves and elementally diffracted waves so that the role of elementally diffracted waves is quite understandable. Thus the physics of wedge diffraction is made clear in terms of elementally diffracted waves. In addition the elementally diffracted waves have simple structure in the far field and it enables us to reproduce the far field rigorous solutions of waves diffracted by the wedge. Thus VDPD is justified further by this result.

This paper is organized as follows. In Sec.2 a virtual space is formulated by incorporating mirror images reflected by edges of wedge into the space and the potentials in it are defined using the property of mirror reflection. In Sec.3 the Green's theorem is applied to the virtual space and the resulting relations make the physics of wedge diffraction understandable in terms of elementally diffracted waves. In Sec.4 elementally diffracted waves are derived in the far field and the rigorous solutions for diffracted waves are reproduced from them. A short summary is given in Sec.5.

2 Formulation of a virtual space

2.1 Physical description of a wedge

Let us draw a half line L_0 in the 2D space and denote the starting point as Q as shown in Fig.1 and introduce the polar coordinate system $\boldsymbol{r}=(r,\theta)$ by specifying r as a distance measured from Q and θ as an angle measured from L_0 in the anticlockwise direction. Let W_0 be a wedge of

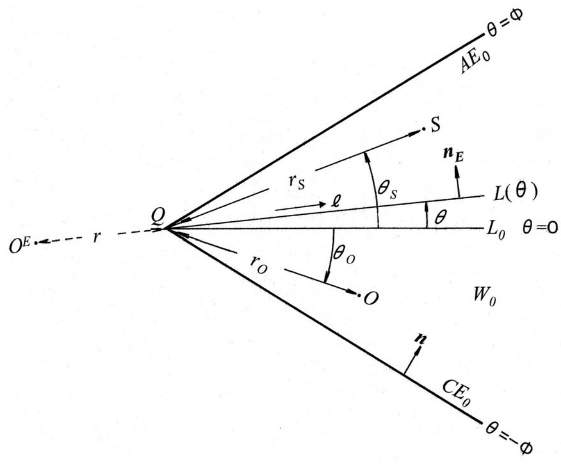

Fig.1 Configuration of a wedge.

angle 2Φ and defined by

$$W_0 = \{(r,\theta) \mid r \geq 0, -\Phi \leq \theta \leq \Phi\}\ , \qquad (1)$$

where $0<\Phi\leq\pi$ and $\Phi=\pi$ corresponds to an semi-infinite plane, $\Phi>\pi/2$ a concave wedge, $\Phi<\pi/2$ a convex wedge and $\Phi=\pi/2$ a reflecting plane. The apex of W_0 lies on Q and W_0 is bounded by two edges AE_0 and CE_0. Let us denote an half line that starts from Q and runs in the θ direction as $L(\theta)$. Then the edges can be specified as $AE_0=L(\Phi)$ and $CE_0=L(-\Phi)$ and let us denote the former as the anticlockwise edge and the latter as the clockwise edge.

2.2 Sound field

The waves propagating in W_0 are stationary in time and satisfy the following relation

$$\nabla^2 U + k^2 U = -\delta(\boldsymbol{r}-\boldsymbol{r_S}), \qquad (2)$$

where U stands for the potential of waves, k the wave number, δ the delta function, $\boldsymbol{r_S}=(r_S,\theta_S)$ the position vector of the point source S and the relations $0<r_S<\infty$ and $-\Phi<\theta_S<\Phi$ hold. In the free sound field S radiates the direct waves

$$U^F(\boldsymbol{r},\boldsymbol{r_S}) = \mathrm{H}_0^{(2)}(k\,|\boldsymbol{r}-\boldsymbol{r_S}|)/(4j), \qquad (3)$$

where U^F stands for the direct waves from S, $\mathrm{H}_0^{(2)}$ the 0-th order Hankel function of the second kind, j the imaginary unit and the stationary time function $\exp(j\omega t)$ is deleted where ω stands for the angular frequency.

The observation point O plays an important role in this paper and its position vector is denoted by $\boldsymbol{r_O}=(r_O,\theta_O)$ and it belongs to W_0. The potential is expressed as a function of $\boldsymbol{r_O}$ or $\boldsymbol{r}$, that is, $U(\boldsymbol{r_O})$ or $U(\boldsymbol{r})$. In the wedge diffraction the distance information is often unimportant and in this case the potential may be expressed as a function of θ_O or θ.

As to the boundary condition, the Dirichlet condition ($\partial U/\partial \boldsymbol{n}$=0) or the Neumann condition (U=0) is set to edges of the wedge where $\boldsymbol{n}$ stands for an inner unit vector normal to the edges. Let us denote the wedge that satisfies the Dirichet condition as the hard wedge and the Neumann condition as the soft wedge. The edge of the hard wedge can be regarded as a mirror of m=1 where m stands for the reflection coefficient of the mirror. Similarly the edge of the soft wedge can be regarded as a mirror of m=-1.

The diffracted waves can be considered as a deviation from the geometrical optics waves. Then the potential can be expressed as

$$U(\boldsymbol{r}) = U^G(\boldsymbol{r}) + U^D(\boldsymbol{r}), \tag{4}$$

where U^G stands for the potential for the geometrical optics waves and U^D that for the diffracted waves. In this paper a new field quantity that is called elementary diffracted waves is introduced by

$$U^E(r,\theta) = \int_{L(\theta)} - g(k(r+\ell))\partial U(\boldsymbol{r}_\ell)/\partial \boldsymbol{n_E}\, d\ell, \tag{5}$$

where U^E stands for the potential for the elementary diffracted waves, ℓ the coordinate taken along $L(\theta)$, r_ℓ the position vector of a point on $L(\theta)$, $\boldsymbol{n_E}$ an unit vector in the anticlockwise direction normal to $L(\theta)$, g the Green's function that is given by

$$g(x) = H_0^{(2)}(x)/(4j). \tag{6}$$

The model for wedge diffraction described in this paper is formulated in terms of U^E and it can be calculated using U in W_0 as seen in Eq.(5). Physically, however, U^E is considered as a contribution of U on $L(\theta)$ to a point located at $(r,\theta+\pi)$ as shown as O^E in Fig.1. Thus if O^E should be kept in W_0, it would be necessary to draw $L(\theta)$ in the area outside W_0.

2.3 Extension of a wedge space

Let us extend a wedge space beyond the edges by incorporating mirror images reflected by the edges in the space, that is, the edges AE_0 and CE_0 are considered as mirrors and mirrored images are assumed to be spread out beyond the edges. The wedges W_1,W_2,W_3,... are spread out beyond AE_0 and the wedges W_{-1},W_{-2},W_{-3},... are spread out beyond CE_0 as shown in Fig.2. Let us express these wedges by W_i where i stands for an integer and $|i|$ corresponds to the reflection number. The wedge W_i is bounded by $L((2i+1)\Phi)$ and $L((2i-1)\Phi)$ and let us denote the former as AE_i and the latter as CE_i. If a point in W_0 specified by (r,θ^*) is imaged to a point (r,θ) in W_i by mirror reflections, then the following relation

$$\theta = (-1)^i \theta^* + 2i\Phi, \tag{7}$$

holds. Let us denote θ^* as the original angle of θ. Then the wedge number i and the original angle θ^* for any angle θ can be assigned by the following relations

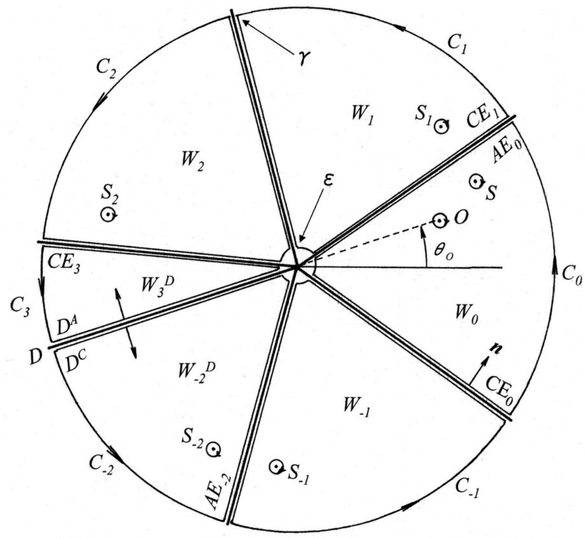

Fig.2 Formulation of a virtual space.

$$i = d(\theta) = \mathrm{int}((\theta/\Phi + \mathrm{sgn}(\theta))/2), \tag{8}$$

$$\theta^* = h(\theta) = (-1)^i(\theta - 2i\Phi), \tag{9}$$

where int(x) and sgn(x) are functions that show the integral part and sign of the real number x respectively. Then the potential at (r, θ) can be assigned by the following relation

$$U(r,\theta) = m^i U(r,\theta^*), \tag{10}$$

where i and θ^* are calculated by Eqs.(8) and (9) and m^i reflects the amplitude reversal in case of the soft wedge (m=-1). Since the original angle is symmetric with respect to the edge, $U(\theta)$ in the hard wedge is symmetric with respect to the edge and antisymmetric in the soft wedge. According to the boundary conditions, that is, $\partial U/\partial\theta$=0 for the hard wedge and U=0 for the soft wedge, the continuity of U and $\partial U/\partial\theta$ at the edge holds for hard and soft wedges. Similarly the following relation holds for the elementally diffracted waves

$$U^E(r,\theta) = (-m)^i U^E(r,\theta^*). \tag{11}$$

Consequently $U^E(\theta)$ in the hard wedge is antisymmetric with respect to the edge and symmetric in the soft wedge. The continuity of U^E and $\partial U^E/\partial\theta$ at the edge also holds for hard and soft wedges since at the edge U^E=0 is immediately obtained from Eq.(5) for the hard wedge and $\partial U^E/\partial\theta$=0 can be also derived for the soft wedge. Thus U and U^E are extended beyond the edges continuously.

For the sake of later references, let us denote the mirror image of S in W_i as S_i and its position vector as $\boldsymbol{r_{Si}}=(r_S,\theta_{Si})$, then

$$\theta_{Si} = (-1)^i \theta_S + 2i\Phi \tag{12}$$

holds and the direct waves from S_i is expressed as

$$U^F(\boldsymbol{r},\boldsymbol{r_{Si}}) = m^i \mathrm{H}_0^{(2)}(k|\boldsymbol{r}-\boldsymbol{r_{Si}}|)/(4j), \tag{13}$$

where in the case of i=0 this relation is reduced to Eq.(3) since $S_0 = S$.

2.4 Formulation of a virtual space V

A virtual space V is defined as a space that can be observed by O where mirrored images are assumed to be spread out beyond the edges. Let us introduce a half line $D=L(\theta_O+\pi)$, that is, a half line that starts from Q and runs in the direction of $\theta_O+\pi$ as shown in Fig.2. If O is rotated in the anticlockwise direction until it touches D, the contact takes place in W_p where $p=d(\theta_O+\pi)$ is a nonnegative integer. Let us express D running in W_p as D^A and a wedge bounded by CE_p and D^A as W_p^D and call it as a partial wedge truncated by D^A. Similarly if O is rotated in the clockwise direction, it touches D in W_n where $n=d(\theta_O-\pi)$ is a nonpositive integer. Let us express D running in W_n as D^C and a wedge bounded by AE_n and D^C as W_n^D . Then the virtual space V can be formulated by

$$V = W_p^D \cup W_n^D \cup (\textstyle\sum_{i=n+1}^{p-1} \cup W_i). \qquad (14)$$

If either p=0 or n=0 holds, the third term of Eq.(14) becomes zero. The potentials on D^A and D^C are different for most cases and the potential in V is not continuous along D. Accordingly it has been called as a virtual discontinuity line.

3 Wave analysis in the virtual space

The Green's theorem is applied to V to derive a new expression for U in terms of U^E and the properties of diffracted waves are analyzed using the new expression.

3.1 Application of Green's theorem to V

Let us draw a closed curve C_i in W_i for $i=n,...,0,..,p$ as shown in Fig.2. Each C_i is composed by two circular arcs of radius ε (ε<<1) and γ (γ >>1) respectively and two segments connecting these arcs along the edges. The centers of curvature of arcs lie on Q and the radii ε and δ are common for all C_i . Then the following relation is obtained by applying the Green's theorem to C_i

$$\sum_{i=n}^{p} \int_{C_i} U(\boldsymbol{r}_\ell)\partial g(k\,|\,\boldsymbol{r}_\ell - \boldsymbol{r}_O\,|)/\partial \boldsymbol{n} - g(k\,|\,\boldsymbol{r}_\ell - \boldsymbol{r}_O\,|)\partial U(\boldsymbol{r}_\ell)/\partial \boldsymbol{n}\, d\ell = 0, \qquad (15)$$

where a circle of very small radius centered at O is included in C_0 and that centered at S_i in C_i . If S_n is not included inside C_n, no circle is added to C_n. The same story holds for S_p and C_p. As seen in Fig.2, two integral paths run parallel to the edge, that is, one in C_i and the other in C_{i+1} ($i=n,\ldots,p$-1). Since the potential is continuous at edges, these pairs of integrals are cancelled out each other. And there remain the integrals along D^A and D^C since D is the only edge in Fig.2 where the potential is discontinuous. In this case O lies on the extension of D and the relations

$$|\,\boldsymbol{r}_\ell - \boldsymbol{r}_O\,| = \ell + r_O\,,\ \partial g/\partial \boldsymbol{n} = 0\,. \qquad (16)$$

Hold. Thus at the limits of ε→0 and γ→∞, the integrals along D^A and D^C can be expressed as $-U^E(r_O,\theta_O+\pi)$ and $U^E(r_O,\theta_O-\pi)$ respectively and the integrals along the arcs become zeros. Consequently Eq.(15) can be rewritten as

$$U(\boldsymbol{r_O}) = F(S_p,W_p^D)U^F(\boldsymbol{r_O},\boldsymbol{r_{Sp}}) + F(S_n,W_n^D)U^F(\boldsymbol{r_O},\boldsymbol{r_{Sn}}) + \textstyle\sum_{i=n+1}^{p-1} U^F(\boldsymbol{r_O},\boldsymbol{r_{Si}}) - U^E(r_O,\theta_O+\pi) + U^E(r_O,\theta_O-\pi), \qquad (17)$$

where the direct waves are resulted from the integrals along the small circle centered at S_i and $F(S_p,W_p^D)$ takes 1 if $S_p \in W_p^D$ and 0 otherwise. Eq.(17) is the expression of the potential of waves in the wedge in terms of the elementally diffracted waves. The function F is included in Eq.(17) since the sources S_n and S_p may not be included in the truncated wedges. The sum of the first three terms in Eq.(17) comprises the geometrical optics waves. Then as seen from Eqs.(4) and (17) the diffracted waves is expressed by the sum of the fourth and fifth terms in Eq.(17), that is,

$$U^D(\theta_O) = -U^E(\theta_O+\pi) + U^E(\theta_O-\pi), \qquad (18)$$

where r_O is deleted in the expression for simplicity.

The physics of wedge diffraction is quite clear in Fig.(17). The first term in Eq.(17) changes discontinuously whenever S_p crosses D^A but this jump is compensated by the fourth term so that the potential changes continuously. The same story holds for the second term and it is compensated by the fifth term. The diffracted waves are expressed by the sum of the fourth and fifth terms as shown in Fig.(18). Accordingly the physics in wedge diffraction has been unclear in the convention theory of diffraction. Eq.(18) can be rewritten in terms of the original angles as

$$U^D(\theta) = -(-m)^{d(\theta+\pi)}U^E((\theta+\pi)^*) + (-m)^{d(\theta-\pi)}U^E((\theta-\pi)^*), \qquad (19)$$

where Eqs.(8),(9),(11) are used and in case of the soft wedge U^D depends only on the original angles.

3.2 Properties of diffracted waves

Let us introduce new variables x,y_A,y_C,b as follows

$$\begin{aligned} &x = \theta/\Phi, \\ &y_A = (\theta+\pi)^*/\Phi,\ y_C = (\theta+\pi)^*/\Phi, \\ &b = \pi/2\Phi, \end{aligned} \qquad (20)$$

where angles are normalized by Φ and the parameter b is used to describe the wedge shape from now on. Then the following relations

$$\begin{aligned} &y_A(x) = (-1)^{p(x)}(x+2b-2p(x)), \\ &y_C(x) = (-1)^{n(x)}(x-2b-2n(x)), \\ &p(x) = \mathrm{int}(x/2+b+\mathrm{sgn}(x+2b)/2), \\ &n(x) = \mathrm{int}(x/2-b+\mathrm{sgn}(x-2b)/2), \end{aligned} \qquad (21)$$

are derived from Eq.(8) and Eq.(9). And Eq.(19) can be rewritten as

$$U^D(x) = -(-m)^{p(x)}U^E(y_A(x)) + (-m)^{n(x)}U^E(y_C(x)). \qquad (22)$$

Let take x along the horizontal axis and y_A and y_C along the vertical axis as shown in Fig.3(a) and examine the relation between them graphically. As seen from Eq.(21), the following relations

$$p(0) = -n(0),$$
$$y_A(0) = -y_C(0) = y_0,\qquad (23)$$
$$y_0 = (-1)^{p(0)}(2b - 2p(0)),$$

hold at x=0. Then in the neighborhood of x=0 the following relations

$$y_A(x) = m_0 x + y_0,$$
$$y_C(x) = m_0 x - y_0,\qquad (24)$$

hold until either $|y_A|$ or$|$ $y_C|$ becomes greater than 1 where m_0=$(-1)^{p(0)}$ and m_0 and y_0 are shown in Fig.3(b) as a function of b. The two lines described in Eq.(24) are parallel to each other and the slope of the line is either 1 or -1 as seen in Fig.3(b). As a result of this inversion, the two lines cross at x=1 and x=-1 as shown in Fig.3(a). Thus y_A and y_C draw a rectangle that is inclined to either 45˚ or 45˚ and inscribed to a square of edge length 2 and centered at the origin.

3.3.1 Boundary condition

As mentioned in the previous section, the two original angles cross at θ=±Φ, that is, when O lies on the edges. Then as seen from Eq.(22), U^D=0 holds at the edges of soft wedge (m=-1), that is, the Neumann boundary condition is satisfied. In the case of the hard wedge (m=1), the two terms in the right side of Eq.(22) become the same at the edges since the reflection numbers p and $|n|$ are different by 1. Thus the Dirichet boundary condition $\partial U^D/\partial\theta$=0 is satisfied at the edges of hard wedge. Consequently it becomes clear that the boundary conditions are satisfied in Eq.(22). It is the necessary condition that the expression for diffracted waves should satisfy but no conventional expressions have satisfied it so far.

3.3.2 Nondiffractive wedge

Let q be a natural number (q=1,2,…). If b=q holds, y_0=0 is resulted from Eq.(23) and Eq.(24) becomes

$$y_A(x) = y_C(x) = m_0 x,\qquad (25)$$

and the reflection numbers do not change for $|x|$≤1. Then as seen from Eq.(22), U^D≡0 holds, that is, the diffracted waves are identically zero for the soft and hard wedges. Let us denote the wedges that satisfy b=q as the nondiffractive wedges and b=1 corresponds to a reflecting plane and b=2 to a concave wedge of Φ= π /4. This is the well-known result but in the conventional analysis of diffracted waves this fact is useless since there are no diffracted waves in these wedges [2]. In this analysis, however, they are very useful since U^E does exist in them and the potential in them can be expressed by the geometrical optics waves.

3.3.3 Symmetry of diffracted waves

If b=(2q-1)/2 , that is, b=1/2.3/2,… holds, y_0=±1 is resulted from Eq.(23) and the rectangle changes to a lozenge as seen in Fig.3(a). Then y_A and y_C become symmetric with respect to x=0 and U^D in the soft wedge

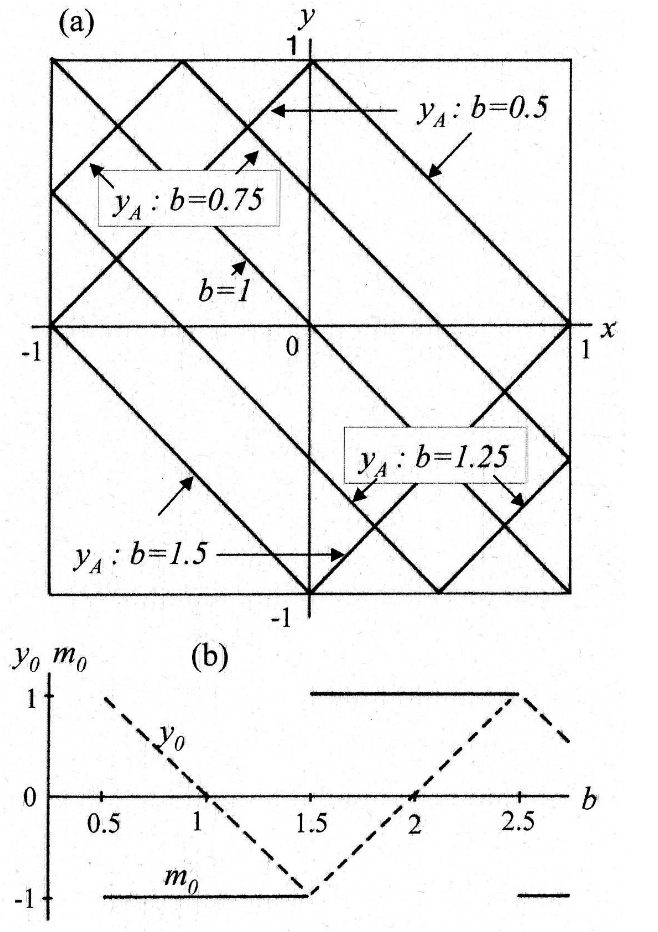

Fig.3 (a) two original angles for b=0.5,0.75,1.0,1.25,1.50. (b) y_0 and m_0 for y_A as a function of b.

becomes symmetric with respect to x=0 and that in the hard wedge antisymmetric. As seen from the diamond shape in Fig.3(a), U^D(0) is expressed by the sum of U^E(1) and U^E(-1) and they are the boundary values of U^E as discussed in the section 2.3. Thus U^D=0 and $\partial U^D/\partial\theta$=0 hold at x=0 for the hard and soft wedges respectively. These are the necessary conditions to maintain the continuity of the symmetric U^D .

4 U^E in the far field

In order to perform the integral given by Eq.(5) to derive the analytical expression of U^E, let us assume the nondiffractive wedge and express the potential as the sum of the direct waves from the source S_i, that is,

$$U(\boldsymbol{r}) = \sum_{i=0}^{2q-1} U^F(\boldsymbol{r}, \boldsymbol{r}_{Si}),\qquad (26)$$

where b=q and $\boldsymbol{r}_{Si}$ is given by Eq.(12). Let U^{EF} be the potential resulted by replacing $U(\boldsymbol{r}_\ell)$ in the integrand of Eq.(5) by $U^F(\boldsymbol{r}_\ell,\boldsymbol{r}_S)$, that is,

$$U^{EF}(\boldsymbol{r},\boldsymbol{r}_S) = \int_{L(\theta)} - g(k(r+\ell))\partial U^F(\boldsymbol{r}_\ell,\boldsymbol{r}_S)/\partial \boldsymbol{n}_E\, d\ell.\qquad (27)$$

Let us assume that O and S are placed in the far field of the wedge, that is, r, $r_S \rightarrow \infty$, then Eq.(27) becomes

$$U^{EF}(\boldsymbol{r},\boldsymbol{r}_S) = -(R/4)\cot((\theta_S - \theta)/2),$$
$$R = \exp(-jk(r+r_S)/(j2\pi k(rr_S)^{1/2}),\qquad (28)$$

where R is the normalization factor and the following approximations

$$H_0^{(2)}(x) \approx (2/\pi x)^{1/2} \exp(-jx + j\pi/4),$$
$$H_1^{(2)}(x) \approx (2/\pi x)^{1/2} \exp(-jx + j3\pi/4). \quad (29)$$

are used. Then U^E in the nondiffractive wedge of b=q can be calculated as

$$U^E(\theta) = -(R/4)\sum_{i=0}^{2q-1} m^i \cot((\theta_{Si} - \theta)/2). \quad (30)$$

If Eq.(12) and the following relation [3]

$$\sum_{i=0}^{q-1} \cot(i\pi/q + x) = q\cot(qx), \quad (31)$$

are used, then Eq.(30) can be rewritten as

$$U^E(\theta) = -(bR/4)\{\cot(\pi(\theta_S - \theta)/4\Phi)$$
$$+ m\tan(\pi(\theta_S + \theta)/4\Phi)\}. \quad (32)$$

Then U^E for the hard wedge (m=1) is expressed as

$$U^E(\theta) = (bR/2)\cos(\pi x/2)/\{\sin(\pi x/2)$$
$$-\sin(\pi x_S/2)\}, \quad (33)$$

and for the soft wedge (m=-1)

$$U^E(\theta) = (bR/2)\cos(\pi x_S/2)/\{\sin(\pi x/2)$$
$$-\sin(\pi x_S/2)\}, \quad (34)$$

where the normalized angles $x=\theta/\Phi$, $x_S=\theta_S/\Phi$ and the following relation

$$\cot A \pm \tan B = 2\cos(A \mp B)/\{\sin(A+B) + \sin(A-B)\} \quad (35)$$

are used. As seen from Eqs.(33) and (34), U^E normalized by $bR/2$ is expressed as a function of x and x_S, that is, it is independent on b. And it is shown in Fig.4(a) for the hard wedge and Fig.4(b) for the soft wedge. The normalized U^E shows the discontinuity at $x=x_S$ and the curves in Fig.4(a) and (b) show the almost same behavior near $x=x_S$ but different at $x=\pm1$ as discussed in the section 2.3.. Since the amplitude of the source is constant and U^E is spread out over W_0, it would be reasonable to assume that the amplitude of U^E is proportional to the area of W_0 , that is, to b as seen in Eqs.(33) and (34).

The relations given by Eqs.(33) and (34) are proved to hold for $b=q$, that is, b=1,2,… . It would be natural, however, to assume that they also hold for any $b\geq1/2$. Then U^D for any b can be calculated by inserting U^E into Eq.(18) and Eq.(18) can be rewritten in terms of the normalized angle and b as

$$U^D(x) = -U^E(x+2b) + U^E(x-2b). \quad (36)$$

In the case of the hard wedge the following relation

$$U^D(x) = \frac{bR}{2}\{-\frac{\cos(\pi x/2+\pi b)}{\sin(\pi x/2+\pi b)-\sin(\pi x_S/2)}$$
$$+\frac{\cos(\pi x/2-\pi b)}{\sin(\pi x/2-\pi b)-\sin(\pi x_S/2)}\}, \quad (37)$$

is derived by inserting Eq.(33) into Eq.(36). In the case of

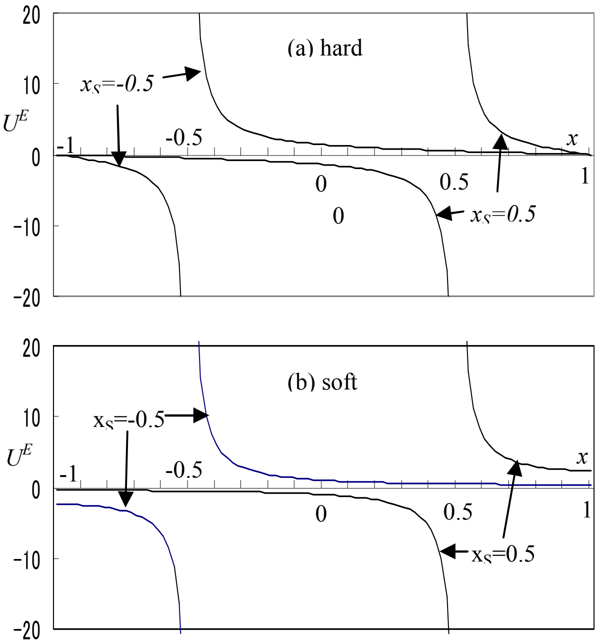

Fig.4 Graphs of normalized U^E for x_S=±0.5.

the soft wedge the following relation

$$U^D(x) = \frac{bR}{2}\{-\frac{\cos(\pi x_S/2)}{\sin(\pi x/2+\pi b)-\sin(\pi x_S/2)}$$
$$+\frac{\cos(\pi x_S/2)}{\sin(\pi x/2-\pi b)-\sin(\pi x_S/2)}\}, \quad (38)$$

is obtained and these relations agree with the rigorous ones literally [4]. The above statement about Eqs.(33) and (34) is justified by this agreement. In addition the top-down physical principle of VDPD is justified by this calculation since complex relations for diffracted waves are reproduced by the simple analytical calculation based on the principle.

5 Conclusion

The physics of wedge diffraction is made clear by expressing the wave field in terms of elementally diffracted waves. The elementally diffracted waves have simple structure and the far field rigorous solutions of waves diffracted by the wedge are reproduced with the help of this simple structure. By this result VDPD is justified as the top-down physical principle for analyzing waves diffracted by a wedge and probably by a polygon since it is composed by wedges.

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