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A mixin algorithm for geoacoustic inversion

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Shear wave velocities in the sediment can be estimated by using the dispersion curves of interface wave. The estimation method is based on matrix inversion: a regularized least-square algorithm solved by singular value decomposition (SVD). This method can only invert the shear wave velocities and requires knowledge of the thicknesses and densities of the sediment layers.
This paper presents a new algorithm for inversion of geoacoustic parameters based on dispersion curves that we have coined "mixin inversion". The mixin inversion combines the regularized least-square algorithm using SVD and global search using genetic algorithms (GA). In the mixin inversion the GA searches the depths and densities by doing a shear wave velocity inversion with SVD for each of its parameter values. This algorithm can be applied on cases where the depths, densities and shear wave velocities of the sediments are unknown. The advantage is that it is faster than using pure GA since the search space is much smaller and it can be applied on cases where pure SVD inversion fails because necessary information about depths and densities is not known.

1 Introduction

The parameters that enable us to quantitatively characterize the sea bottom in a geoacoustical way are the compressional-wave and shear-wave velocities, and the corresponding attenuation and density. They are usually expressed as functions of depth. Shear wave velocity profile in marine sediments is related to the shear strength of sediments, it can be used to evaluate how much load the sediments can stand. This closely associates with application to geotechnical site inspection in preparation for pipe laying operations and instillations of platforms and sub-sea production and processing modules. However, the shear wave velocity has been found a difficult property to estimate, and relatively few measurements in the field are available [1, 2].

Comparing to the *in-situ* measurement or by analyzing the samples of the bottom material in laboratories for obtaining the geoacoustic properties, remote measurement techniques have advantages for covering large area and improving the depth resolution. An estimate of the shear velocity as function of depth in the upper part of the sediment layers can be obtained from inversion of measured dispersion curves of the interface wave by using a regularized least-square algorithm solved by singular value decomposition (SVD) [3-6]. The convergence speed of SVD is very fast. However this method can only invert the shear wave velocities and requires knowledge of the thicknesses and densities of the sediment layers.

GA is based on simulating the evolution of a population of models through random processes that mimic genetic crossover (recombinations of existing models) and mutation (random variations) in a manner that favors models with a low mismatch (or conversely, a high match, referred to as fitness). GA has been applied to geoacoustic inversion by a number of authors [7, 8]. GA can inverse both layer thickness, density and shear wave velocity. However, GA uses a large search space and many evaluations of the forward model.

The objective of this paper is to present a new algorithm for inversion of geoacoustic parameters based on dispersion curves that we have coined "mixin inversion". The mixin inversion method combines the regularized least-square algorithm using SVD and global search using GA in an attempt to retain the advantages of each while overcoming their respective weaknesses. The GA searches the depths and densities by doing a shear wave velocity inversion with SVD for each of its parameter values. This algorithm can be applied on cases where the depths, densities and shear wave velocities of the sediments are unknown. The advantage is that it is faster than using pure GA since the search space is much smaller and it can be applied on cases where pure SVD inversion fails because necessary information about depths and densities is not known. In this paper the mixin technique is presented in Sec. 2. In Sec. 3 the mixin technique is applied to three testcases for estimating the shear wave velocity as function of depth in the upper part of the sediment and the layer thicknesses and densities of the sediment.

2 The mixin algorithm

Shear wave velocity can be estimated by using dispersion curves of interface wave along the water sediment boundary since the dispersion property of the interface wave is closely related to shear wave velocity variation as function of depth and the layer thickness of the sediment. The dispersion curves of interface waves can be obtained by time-frequency analysis. The principle component decomposition was used to multi-sensor data for obtaining the dispersion curves of phase velocity of the interface waves [9], while wavelet transform is a favourite method for single-sensor data for estimating the dispersion curves of the group velocity of the interface waves [6, 10].

The regularized least-square algorithm for estimates of shear wave velocity uses SVD to solve the propagator matrix for obtaining dispersion curves. The convergence speed of this algorithm is very fast. However this algorithm can only estimate shear velocity, not other geoacoustic parameters, such as layer thickness and density, which shear wave velocity is sensitive to. In order to obtain more accurate estimates of shear velocity as function of depth when using dispersion curves of interface wave, accurate information on layer thickness and density is needed. If there is no accurate information on these parameters they should be estimated when doing shear wave velocity inversion. Therefore other inversion schemes are needed.

GA is suitable for inversion of a large number of parameters, but many evaluations of the forward model are needed and therefore it is not optimal in this case.

The mixin inversion scheme combines SVD and GA, where the layer thicknesses and the densities are inverted by GA and the shear velocities are inverted by SVD. The mixin inversion works by replacing the objective function for the dispersion curves in the GA call by an objective function that also does the shear wave velocity inversion.

For each thickness and density the GA tries, the objective function does a shear velocity inversion. The advantage is that the GA does not have to try many improbable combinations of shear velocities and is therefore more efficient than a pure GA inversion.

3 Examples

In this section the mixin algorithm is applied for three cases. All of the three cases are synthetic and the geoacoustic parameters for the three cases are listed in Table 1, where *N* is the number of layers (including water layer and a half space basement), *d* is the layer thickness, ρ is layer density, v_p and v_s are the layer's *P*- and *S*-wave velocities, respectively. A multi-sensor method [9] is used for estimating dispersion curves of the of the interface waves. The synthetic dispersion data of phase velocity of the interface waves are generated by using a matrix method [4]. Two dispersion modes are selected as the observed data for the inversion. Since the dispersion curves are independent of the P-wave velocities, it is assumed that P-wave velocities are known parameters during the inversion for all of the there cases.

3.1 Testcase 1

The first testcase consists of three layers; the water layer, a sediment layer and the half space bottom. The geoacoustic parameters, listed in table 1, are the true values which are used for generating synthetic dispersion curves. Two dispersion modes are selected with the frequency from 0.1 to 6.25 Hz. In the mixin algorithm, GA inverses the thicknesses and the densities of the sediment layer, while the SVD inverses the shear wave velocity in the sediment layer and the bottom.

In the initial run for this case, the mixin algorithm inverted for sediment depths in the range 10 to 30 meters. The depth always converged to between 18 and 21 meters, but the sediment density was not successfully inverted. Since the depth was so well determined, a new run was done with the search range for the depth restricted to 15-25 meters. This time a better estimate for the density was obtained.

Fig. 1 shows the inversion results. The upper panels show the scatter plots for the thickness (left) and density (right) of the sediment layer, where the solid lines represent the true parameter values and the range of abscissa values indicates the parameter search interval. The lower left panel shows the measured dispersion curves (red dots) and the model fits (blue line) and the lower right plane shows the estimated shear wave velocities (blue) and the true values (red) of the sediment layer and the bottom.

The inversion results indicate that the layer thickness is very sensitive parameter and is very well determined as illustrated by the narrow "tornado-like" distribution in the upper-left panel, while the layer density is almost insensitive to the interface waves. Therefore it is difficult to determine the layer density by using the dispersion of interface waves.

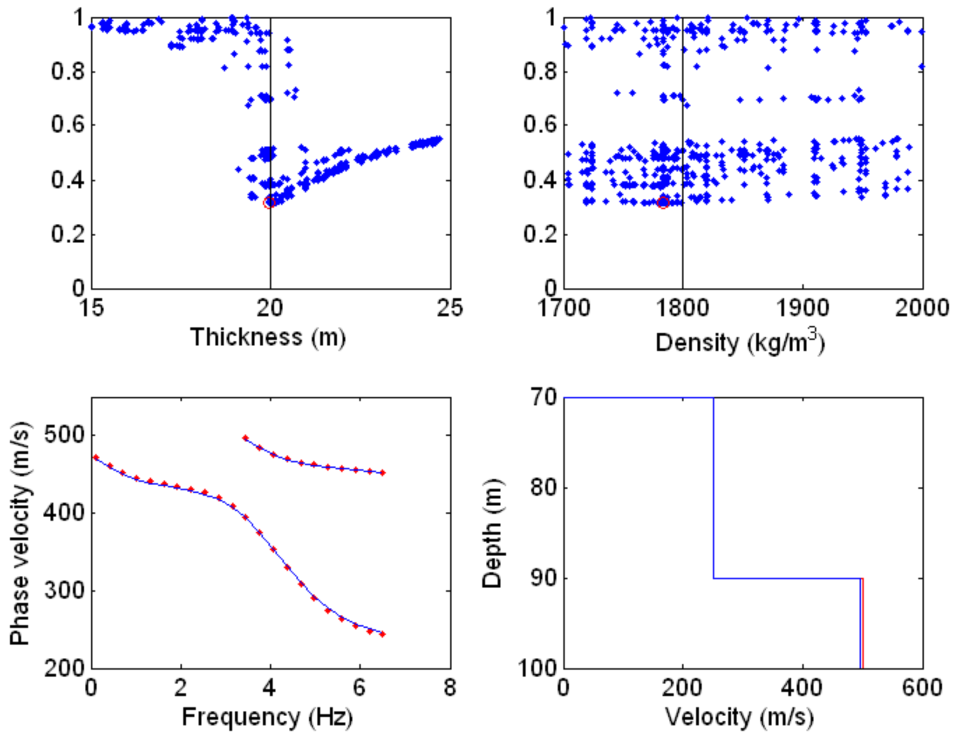

Fig.1 Inversion Results for the first test case. Upper: scatter plots for layered thickness (left) and density (right) where the solid lines represent the true parameter values and the range of abscissa values indicates the parameter search interval. Lower left: phase velocity data (red stars) sampled from the estimated dispersion data and the model fit (blue solid lines); Lower right: inversed shear wave velocities (blue) and the true values (red).

3.2 Test case 2

The second testcase consists of six layers including the water layer and the half space bottom. The geoacoustic parameters, listed in table 1, are the true values which are used for generating synthetic dispersion curves. Two dispersion modes are selected with the frequency from 0.5 to 15 Hz. The mixin algorithm inverts the layer thicknesses and densities for the four sediment layers and the shear velocities. The search space is set to be 7-30 meters for the layer thicknesses. The search spaces for the densities are between 1700-2100 kg/m^3 for the first sediment layer and 1800-2200 kg/m^3 for the second to fourth sediment layers. The inversion results are shown in Fig.2. The upper panels are the scatter plots for the layer thicknesses and the middle panels are the scatter plots for the layer densities. The lower left panel shows the measured dispersion curves (red dots) and the model fits (blue line) and the lower right plane shows the estimated shear wave velocities (blue) and the true values (red) of the sediment layer and the bottom. The solid lines in the upper and middle panels indicate the true values of the layer thickness and density. It can be seen that the thicknesses for the upper two layers are very well determined as indicated by the narrow "tornado-like" plots than that for the deeper layers given by the wider distribution. The wider scatter plots in the middle panels indicate that it is difficult to estimate the layer density by using interface waves. It can be seen in the bottom panels that the modelled data are well matched with the synthetic dispersion curves and the shear wave velocities as function of depth are very well estimated even though the layer densities are not well estimated.

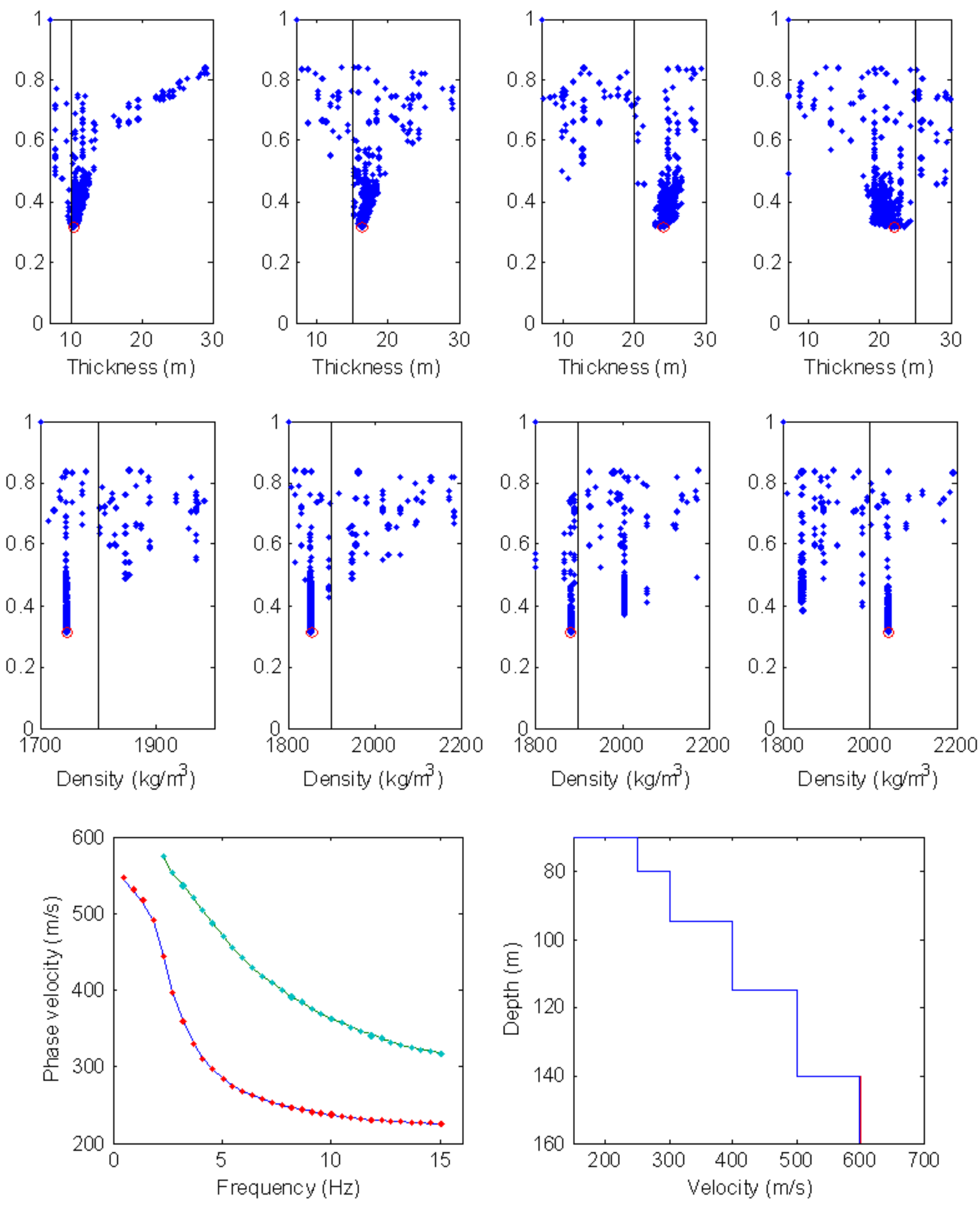

Fig.2 Inversion Results for the second test case. Upper: scatter plots for layered thickness where the solid lines represent the true parameter values and the range of abscissa values indicates the parameter search interval. Lower left: phase velocity data (red stars) sampled from the estimated dispersion data and the model fit (blue solid lines); Lower right: inversed shear wave velocities (blue) and the true values (red).

Parameter	Testcase 1	Testcase 2	Testcase 3
N	3	6	Unknown between 3-6
d (m)	[70, 20, ∞]	[70, 10, 20, 15, 25, ∞]	[70, 18, 20, 15, 20, ∞]
ρ (kkg/m^3)	[1.0, 1.8, 2.0]	[1.0, 1.8, 1.9, 1.9, 2.0, 2.1]	[1.0, 1.8, 2.0, 2.1, 2.2, 2.3]
v_p (km/s)	[1.5, 1.9, 2.0]	[1.5, 1.8, 1.9, 1.9, 2.0, 2.1]	[1.5, 1.8, 2.0, 2.0, 2.1, 2.1]
v_s (km/s)	[0.0, 0.25, 0.5]	[0.0, 0.25, 0.3, 0.4, 0.6, 0.6]	[0.0, 0.275, 0.45, 0.55, 0.65, 0.75]

Table 1 The true parameter values and the parameter symbols are defined in the text.

3.3 Testcase 3

The third testcase is a case with unknown number of layers and the mixin algorithm is applied to determine the number of sediment layers. The true parameters are listed in Table 1. The program runs a loop over number of layers used in the inversion and does a mixin inversion. The overall best fitness is found for the case with the correct number of sediment layers. The range of number of layers is set to be from 3 to 6, that is the number of the sediment layers changes from 1 to 4. In this testcase the densities of the sediment layers are considered as known parameters since they are insensitive to the dispersion of the interface waves. Fig. 3 plots the final value of the fitness function as function of the number of sediment layers. The true value for the umber of the sediment layers is on the upper-right corner in each plot. It can be seen that the number of sediment layers corresponding to the minimum value of the fitness function is the same as the true value in each plot. Fig. 4 presents the probability distributions of layer thicknesses for the cases with 2, 3 and 4 sediment layers in the inversion plotted in (a), (b) and (c), respectively, while the true number of sediment layers is 3. The arrows/red lines indicate the true values of layer thickness.

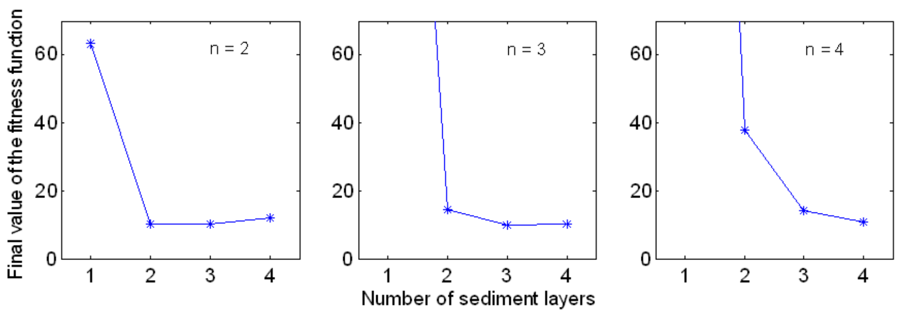

Fig.3 Final values of fitness function as function of number of sediment layers used in inversion. The value on the upper-right corner in each plot is the true value. The number of layers corresponding to the minimum value of the fitness function is the true value of number of sediment layers.

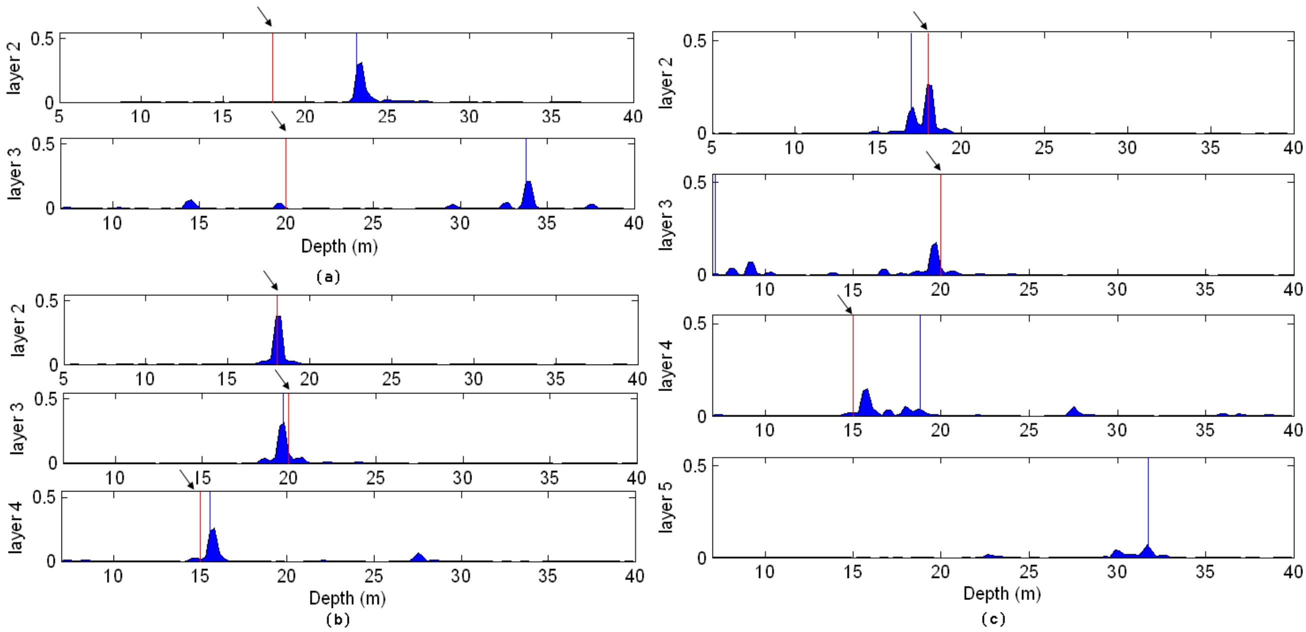

Fig. 4 Probability distributions of the thickness of sediment layers. The range of abscissa values indicates the parameter search interval. The arrows/red lines indicate the true values. Plots (a), (b) and (c) are corresponding to the cases with 2, 3 and 4 sediment layers in the inversion, while the true number of sediment layers is 3, which is corresponding to plots (b).

It can be seen that in the plots in Fig. 4 there are too few layers used in the inversion for the case shown in Fig. 4(a) and none of the layer thickness is good estimated for the layer thickness. And there are too many layers used in the inversion for the case shown in Fig. 4(c) and it seems that only the thickness for the first sediment layer can be estimated relatively weel, but not for other layers. While Fig. 4(b) shows clearly that this case has the best overall fitness and the number of the sediment layers corresponds 3 which is exactly the true number of the sediment layers.

4 Conclusions

The mixin algorithm is introduced in this paper. It is used in connection with the estimation of the shear wave velocity in situations when the information on layer thickness and density is unknown.

The mixin algorithm combines the SVD method for shear velocity inversion and GA for inversion of the layer thickness and density. Compared to GA, the search space for the mixin algorithm is smaller and therefore the computation time is reduced. Compared to SVD the mixin algorithm can also determine the shear velocity for cases when the number and thicknesses of the layers are unknown.

Three synthetic testcases are presented and it is shown that the mixin algorithm can be used to invert the shear velocities and sediment thicknesses effectively. The dispersion curves are dependent on the sediment densities, but it is difficult to invert the densities since the dependency is very weak.

Here only synthetic cases are presented and the applicability for inversion of real measurements is not known. However this method was inspired by an attempt to invert shear velocities using SVD on real measurements. During this attempt it was found that the result was very dependent on the guess of the layer thicknesses and number of layers. Therefore it was conceived that those unknown parameters could be determined by the mixin method.

The cases presented are for underwater, but this technique can just as well be applied for seismic.

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