Passive admittance synthesis for sound synthesis applications

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In physics-based sound synthesis, it is in general possible to incorporate a mechanical or acoustical immittance (admittance or impedance) in the form of a digital filter. Examples include modeling of the termination of a string or a tube. However, when digital filters are fitted to measured immittance data, care has to be taken that the resulting filter corresponds to a passive mechanical or acoustical system, otherwise the stability of the instrument model is at risk. This paper presents a simple method for designing and realizing inherently passive immittances. The immittance is composed as a linear combination of positive real (PR) functions, and the weights are determined by a constrained least squares optimization. The resulting filter is a parallel set of second-order sections. For wave-based modeling, such as digital waveguides (DWGs) or wave digital filters (WDFs), the immittance is converted to a reflectance filter. The parallel filter structure is retained during conversion, resulting in a numerically robust implementation. As an example, a guitar model based on the DWG approach is presented, using mechanical admittance measurements of a guitar bridge behavior. The model is implemented as an efficient real-time sound synthesis algorithm.

1 Introduction

In physics-based sound synthesis, the sound of an instrument is generated by modeling the instrument behavior rather than the sound itself, therefore the model blocks correspond to the main parts of the instrument (for an overview, see [1]). Depending on the modeling paradigm, these models can be parameterized in many ways. Typically, finite difference or finite element instrument models are parameterized by the geometry of the instrument (e.g., soundboard shape, thickness, etc.) and material properties (such as Young's modulus). On the contrary, the parameters of digital waveguide based models are usually determined from analyzing recorded sounds. Somewhere in-between, it is possible to parameterize parts of the instrument model by a measured mechanical or acoustical immittance. The effect of an immittance (e.g., the instrument bridge) connected to a string is that it changes the modal frequencies and decay times of the string compared to a rigid termination, and thus taking this into account results in more accurate modeling. Note that we will restrict ourselves to mechanical admittances, but the treatment is equally applicable to other passive (e.g., acoustical) systems and to impedances instead of admittances. Throughout the paper, only scalar admittance functions (point mobilities) will be considered.

The starting point of such a parameterization is a mechanical admittance measurement of the given part of the instrument (e.g., the bridge). Naturally, all parts of acoustical instruments are passive, that is, they can only dissipate energy that is introduced by the player. In theory, the measured admittance could be directly represented as an FIR or an IIR filter[1] fitted to the measured response. However, often the resulting digital filter is not corresponding to a passive termination, that is, at some frequencies it generates power instead of dissipating it. This can have two reasons: the measured impulse response itself may not be passive because of measurement errors, or due to the fact that the admittance is only approximated by the FIR or IIR filter fitted to the response.

Therefore, instead of straightforward filter design, such a design technique should be used that results in inherently passive admittance filter. In [2], passive admittance filters are constructed by manually tuning the modal frequencies and decay times of second-order resonators to produce a function similar to the guitar admittance. In [3], the mechanical admittance of a guitar up to 3 kHz is modeled by a set of mass-spring-damper elements (second-order resonators), and the matrix pencil method is used for parameter estimation. In the frequency-domain guitar model of [4], a standard modal analysis technique (circle fitting) is used up to 1.4 kHz, and above that a random number generator is applied to produce a statistically similar modal behavior as in the measured response. This was necessary because standard modal analysis techniques perform well only in the low frequency region where the modes are separated (up to 1–2 kHz in the case of the guitar), and they cannot easily capture the behavior at high frequencies, where the modal overlap is high.

This paper proposes an admittance filter design method that models the admittance accurately in the low frequency region (up to a few kHz), while at high frequencies, only the general trend of the admittance is modeled. This is motivated by the fact that in sound synthesis, low frequency admittance modeling should be more accurate, since this is the region that influences the decay times of the most important partials of the tone. The nonuniform resolution is achieved by determining the poles of the admittance filter by frequency warped filter design. Then, the transfer function is constructed as a weighted sum of passive (positive real) transfer functions with a relation to modal analysis, and the weights are found by nonnegative least squares optimization. For wave-based modeling, the paper presents a method for converting the admittance to a reflectance filter that retains the parallel structure of the admittance formulation.

2 Background

2.1 Passivity and positive realness

A system is passive if it cannot produce energy. For continuous-time systems, wide literature is available about the subject, as passivity is an important property in network analysis and synthesis as well as in nonlinear control. For passive systems, immittances are positive real (PR) [5].

For rational functions of s that do not have a pole on the closed right-half plane (that is, for asymptotically stable systems), $H(s)$ is PR if and only if

$$\mathrm{Re}\left\{H(j\omega)\right\} = \frac{1}{2}\left(H(j\omega) + H^{\star}(j\omega)\right) \geq 0 \qquad (1)$$

for all real ω [5]. Here * means complex conjugation. That is, the for rational transfer functions (in this paper, we only deal with immittances that can be written in rational form), passivity of the system can be checked by looking at the frequency response only. This is directly related to the physical interpretation of passivity, meaning that the real part of the power measured on the system should be positive if driven

[1] It is important to keep in mind that immittance itself is generally not a filter or transfer function but a constraint relation between quantities such as force and velocity, while wave-based reflectance is a filter in the sense of input-output relationship.

by a sinusoidal source at any frequency. It also follows from Eq. (1) that the phase of the immittance should be within $\pm\pi/2$ for all frequencies.

The PR condition for a digital transfer function $H(z) = H(e^{-j\vartheta})$ in a rational form with poles in the open unit disk (asymptotically stable systems) is similar to that for the continuous case [5]:

$$\mathrm{Re}\left\{H(e^{-j\vartheta})\right\} = \frac{1}{2}\left(H(e^{-j\vartheta}) + H^{\star}(e^{-j\vartheta})\right) \geq 0. \quad (2)$$

That is, it is enough to check positive realness on the unit circle, by looking at the frequency response. We only mention two additional theorems here that will be useful later. First, for all PR rational functions in z, the poles and zeros are in the unit disk, that is, all PR functions are minimum-phase (but not vice versa). Second, if a positive real function $H(s)$ is converted to a discrete time function $H(z)$ by the bilinear transform, it remains positive real [2, 6].

Fitting positive real functions to measurement data are frequently used in modeling and verification of integrated circuits, therefore, a wide range of continuous-time methods are available (see, e.g., [7, 8]). Most of these sophisticated algorithms could be modified for discrete-time systems. However, they did not find their way to the musical acoustics and sound synthesis community. This is probably due to their complexity, and also because the modal framework (outlined in Sec. 2.2) also provides passive models and it is better related to the physical structure of the instrument.

2.2 Modal framework

The quest for a PR transfer function can be simplified if some assumptions are made on the structure. However, we have to note that due to these underlying assumptions, the fit will be less accurate than the general estimation methods mentioned earlier.

In modal analysis, the general assumption is that the structure can be described as a set of masses which are connected by linear springs and linear dampers [9]. Then, the vibration of the structure can be decomposed to a sum of normal modes with different modal frequencies ω_r, decay rates σ_r and modal shapes $\mathbf{\Phi}_r$. If the damping is viscous and it is distributed proportionally to the mass and stiffness elements (referred as proportional damping in the literature), then the modal shapes $\mathbf{\Phi}_r$ are real, meaning that the different points of the structure reach their maxima and minima at the same time instant [9]. It has been shown that proportional damping is a reasonably good approximation for the violin [10] and for the guitar [4], and most probably for other similar instruments. In this case, the mechanical admittance (mobility) matrix of the system can be written as [10]

$$\frac{v_i(j\omega)}{F_k(j\omega)} = Y_{i,k}(j\omega) = j\omega \sum_{r=1}^{R} \frac{\Phi_i^r \Phi_k^r}{m_r(\omega_r^2 - \omega^2 + 2j\sigma_r\omega_r\omega)} \quad (3)$$

where F_k is a force acting on position k, v_i is the velocity of point i, m_r is the effective mass of mode r, and Φ_i^r is the ith element of the modal shape vector $\mathbf{\Phi}_r$ (thus, the superscript r does not refer to the rth power).

For our purposes, of particular interest is the case where $i = k$, which is called point mobility. This is the case when the velocity is measured at the same point and same direction as the applied force. In this case, the transfer function becomes

$$\frac{v_k(j\omega)}{F_k(j\omega)} = Y_{k,k}(j\omega) = j\omega \sum_{r=1}^{R} \frac{A_{k,k}^r}{m_r(\omega_r^2 - \omega^2 + 2j\sigma_r\omega_r\omega)} \quad (4)$$

where $A_{k,k}^r = \Phi_k^r \Phi_k^r \geq 0$ since Φ_k^r is real. This corresponds to an impulse response composed of exponentially damping cosine functions. It is easy to see that such a function is PR since it is composed of PR functions; the phase of the second-order terms span from 0 to $-\pi$, which is rotated back by $\pi/2$ by the leading term $j\omega$.

Therefore, if the continuous-time modal data is available, it can be converted to a passive discrete-time filter by the bilinear transform or other similar methods that preserve the PR property. However, since the measured data is available in discrete time, it seems more reasonable to design a discrete-time admittance filter directly, and this also avoids the errors introduced by the continuous-to-discrete-time filter transformation.

3 The passive parallel filter

The basic idea of the admittance formulation proposed here is the decomposition of the transfer function to a set of simple PR functions (damped cosines), which correspond to the discrete-time versions of Eq. (4). After bilinear transform the general form becomes

$$\frac{v(z)}{F(z)} = Y(z) = b_0 + \sum_{r=1}^{R} \frac{b_r(1 - z^{-2})}{(1 - p_r z^{-1})(1 - p_r^{\star} z^{-1})} \quad (5)$$

which is PR if $|p_r| \leq 1$ (system is asymptotically stable) and $b_r \geq 0$. The second-order functions have been extended by a real constant term $b_0 \geq 0$, which is useful for modeling the constant envelope of the modal response at high frequencies (see Fig. 1 in the range of 10 kHz). Note that the bilinear transform is used here only to construct the general form of the second-order prototype. Therefore, correcting the frequency warping inherent in the bilinear transform is not necessary, contrary to transforming an analog filter to a digital one.

3.1 Parameter estimation

The parameter estimation proposed here is similar to the fixed-pole design of the parallel second-order filter [11], with an additional constraint on the filter weights b_r. The steps are the following:

1. **Preprocessing**: The measured admittance response is made minumum-phase and this will be the target $y_{\mathrm{t}}(n)$ for the design. This is motivated by the fact that all PR functions are minimum-phase (i.e., it is a necessary condition).
2. **Pole positioning**: The goal is to model the admittance more precisely at low frequencies compared to high frequencies. This has to be reflected by the pole positioning, since the poles determine the frequency resolution of the design, similarly to Kautz [12] and parallel filters [11]. Here we propose to fit a warped IIR (WIIR) filter [13] to

the measured admittance, then the poles $\tilde{p}_k$ of this WIIR filter are "dewarped" by the expression

$$p_r = \frac{\tilde{p}_r + \lambda}{1 + \tilde{p}_r} \tag{6}$$

where λ is the warping parameter, with which the WIIR filter was designed. If unstable poles p_r are found, they can be replaced by $1/p_r$.

Note that the nonuniform frequency resolution can also be achieved in other ways, such as estimating the poles in subbands, but frequency warping is probably the simplest method for this purpose.

3. **Weight estimation**: The final step is to estimate a model of Eq. (5) with $b_r \geq 0$, which is a linear-in-parameter problem with nonnegativity constraints. Here we estimate the parameters in the time domain, but it could also be done in the frequency domain in a similar way. The impulse response $y(n)$ of the admittance is

$$y(n) = \sum_{r=0}^{R} b_r u_r(n) \tag{7}$$

where $u_0(n) = \delta(n)$ is the unit impulse, and $u_r(n)$ for $r = 1, \ldots, R$ is the impulse response of the function $(1 - z^{-2})/[(1 - p_r z^{-1})(1 - p_r^\star z^{-1})]$. Writing this in a matrix form yields

$$\mathbf{y} = \mathbf{M}\mathbf{b}. \tag{8}$$

where the rows of the modeling matrix $\mathbf{M}$ contain the modeling signals $u_r(n)$, the parameter vector $\mathbf{b}$ is composed of the b_r parameters, and $\mathbf{y}$ is the vector of the resulting impulse response $y(n)$. If the error between the filter and target is minimized in the mean squares sense, the formulation becomes

$$\begin{gathered} ||\mathbf{M}\mathbf{b} - \mathbf{y}_\mathrm{t}||_2 = (\mathbf{M}\mathbf{b} - \mathbf{y}_\mathrm{t})^T(\mathbf{M}\mathbf{b} - \mathbf{y}_\mathrm{t}) \rightarrow \min \\ \mathbf{b} \geq 0 \end{gathered} \tag{9}$$

where $\mathbf{y}_\mathrm{t}$ is a vector constructed from the target admittance response $y_\mathrm{t}(n)$. Equation pair (9) is a standard nonnegative least-squares optimization problem.

3.2 Example: guitar admittance modeling

Figure 1(a) shows the guitar bridge admittance function measured on an acoustic guitar (Gibson, from 1960's) by exciting the bridge with wire breaking technique [4] and measuring the movement by a miniature accelerometer. Figure 1(b) is a 40th order warped IIR filter fitted to measured response with warping parameter $\lambda = 0.85$, which follows the target response quite well but is not guaranteed to be passive.

On the other hand, the passive parallel filter estimated from the same poles (Fig. 1(b)) is inherently passive. The optimization of the b_r parameters was done by the `lsqnonneg` function in MATLAB. This resulted in 14 nonzero b_r parameters, corresponding to a constant term b_0 and 13 second-order filters in parallel (a filter order of 26). The passive filter is slightly inferior compared to the unconstrained filter design, but this is mostly due to the lower filter order. This is illustrated by Fig. 1(d), which is a 40th order passive parallel filter estimated from the poles of a 58th order warped IIR filter (not shown). This filter has the same computational complexity as a "dewarped" version of the warped filter of

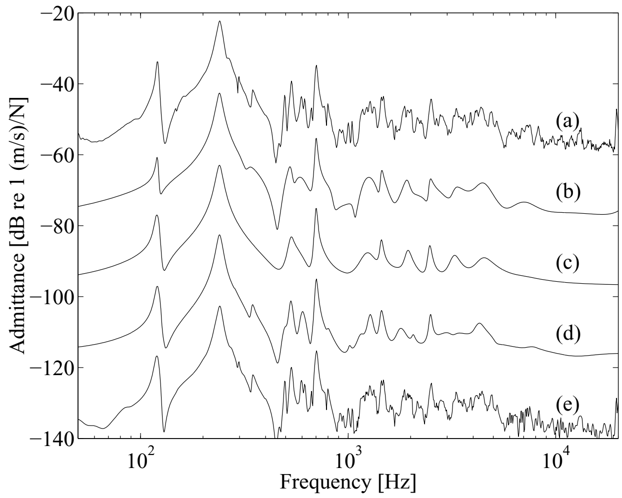

Figure 1: Modeling of guitar bridge admittance: (a) measured admittance response, (b) 40th-order warped IIR filter, (c) 26th-order passive parallel filter estimated by using the poles of (b). A 40th-order passive parallel filter (d) is presented for performance comparison with the 40th-order WIIR filter (b), while (e) shows a 360th-order design. For clarity, the curves (b)–(e) are offset by multiples of -20 dB.

Fig. 1(b). Figure 1(e) displays the response of a 360th-order passive parallel filter to show the robustness of the design. Note that this accuracy is usually not required for sound synthesis applications.

4 Wave-based modeling

A passive admittance function gives the relation between force and velocity for a mechanical system. It seems that it could therefore be directly applied as a termination of a finite difference string model, where the force acting on the termination is computed by the string, then this force is filtered by the admittance form as a filter giving the velocity of the termination, which is used in string model for the next iteration. However, interconnecting passive elements in such a way often results in unstable systems, unless special measures are taken to ensure numerical energy conservation [14].

This problem is automatically avoided in wave-based modeling [14], when the admittance is formulated as a function of wave variables instead of the Kirchhoff variables. In this case, it will be a reflectance filter producing a reflected wave to an incident wave (see the footnote in Introduction).

4.1 Digital waveguide termination

Digital waveguide modeling is the most efficient paradigm for modeling the 1-D wave equation. It is based on spatial and temporal discretization of the travelling wave solution for the wave equation [2].

Here we derive a reflectance filter for the case where a single string with a characteristic admittance Y_0 is connected to a termination having an admittance $Y(z)$. The reflectance transfer function for velocity waves can be written as [2]

$$H_\mathrm{v}(z) = \frac{v^-}{v^+} = \frac{Z_0 - Z(z)}{Z_0 + Z(z)} = \frac{Y(z) - Y_0}{Y(z) + Y_0} \tag{10}$$

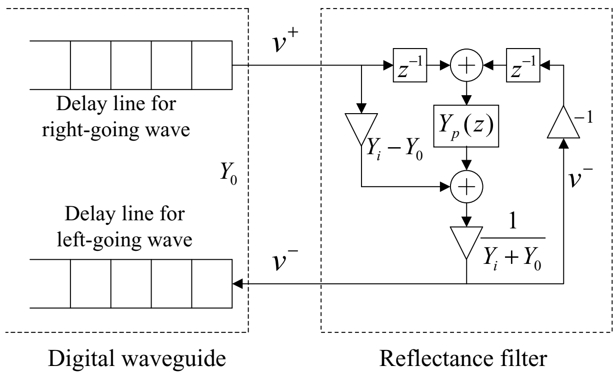

Figure 2: Reflectance filter connected to a digital waveguide string model.

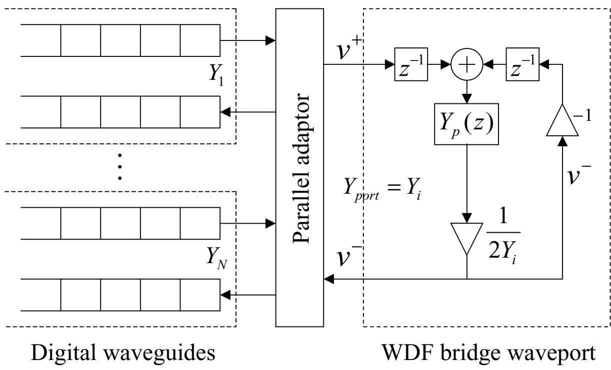

Figure 3: WDF reflectance filter connected through an adaptor to several DWG string waveguides.

where v^+ is the incident wave and v^- is the wave reflected from the termination. The string is characterized by the characteristic impedance Z_0 or admittance $Y_0 = 1/Z_0$. Similarly, the termination is described by the frequency dependent impedance $Z(z)$ or admittance $Y(z) = 1/Z(z)$. In theory, the parameters of $H_\mathrm{v}(z)$ could be computed by inserting Eq. (5) into Eq. (10) and rearranging it to a rational form, but in that case the numerically robust parallel structure is lost. Moreover, the conversion cannot be performed in practice for filter orders above 10–20, because the resulting filter becomes unstable due to the numerical inaccuracies during conversion. Indeed, the 26th order admittance formulation of Fig. 1(b) results in an unstable reflectance filter after conversion in MATLAB. This is because the pole density at low frequencies is high due to the warped pole positioning (the problem is similar to "dewarping" the warped FIR or IIR filters to direct form filters).

Therefore, we suggest constructing the reflectance filter in such a way that preserves the parallel structure of the admittance formulation. First, the admittance form is decomposed to the immediate response Y_i (which equals to $y(0)$, the first sample of the admittance impulse response) and to the response which depends only on past inputs $z^{-1}Y_\mathrm{p}(z)$ (where $Y_\mathrm{p}(z)$ is the z transform of $y(n-1)$ with $n \geq 1$), giving

$$Y(z) = Y_\mathrm{i} + z^{-1}Y_\mathrm{p}(z). \tag{11}$$

The decomposition can be done for the second-order terms in Eq. (5) separately [15]:

$$\frac{b_r(1-z^{-2})}{1+a_{r,1}z^{-1}+a_{r,2}z^{-2}} = b_r + z^{-1}\frac{b_{r,1}+b_{r,2}z^{-1}}{1+a_{r,1}z^{-1}+a_{r,2}z^{-2}} \tag{12}$$

with $b_{r,1} = -b_r a_{r,1}$ and $b_{r,2} = -b_r - b_r a_{r,2}$ for $r = 1, \ldots, R$. Thus, the two parts of the admittance filter become

$$Y_\mathrm{p}(z) = \sum_{r=1}^{R} \frac{b_{r,1}+b_{r,2}z^{-1}}{1+a_{r,1}z^{-1}+a_{r,2}z^{-2}} \tag{13}$$

$$Y_\mathrm{i} = b_0 + \sum_{r=1}^{R} b_r. \tag{14}$$

In Eq. (14), b_0 is written separately from the sum only to emphasize that it equals to the constant part of Eq. (5), while the other terms appear due to the decomposition from $Y(z)$ to Y_i and $Y_\mathrm{p}(z)$. Then, substituting Eq. (11) into Eq. (10) yields

$$v^- = \frac{1}{Y_\mathrm{i}+Y_0}\left[z^{-1}(v^+ - v^-)Y_\mathrm{p}(z) + v^+(Y_\mathrm{i} - Y_0)\right]. \tag{15}$$

This is illustrated in Fig. 2. The non-computable delay-free loop is avoided because of the decomposition to Y_i and $Y_\mathrm{p}(z)$, leading to the z^{-1} terms in Fig. 2. The digital waveguide string model should also incorporate a loop filter that models string losses and dispersion, and an allpass filter for tuning [2]. These filters are not depicted in Fig. 2.

Besides numerical robustness, a further advantage of constructing the reflectance filter in the proposed way is that if the modal parameters of the admittance are varied in real-time, Y_i and $Y_\mathrm{p}(z)$ can be constructed from $Y(z)$ by very few operations, as opposed to computing a reflectance filter $H_\mathrm{v}(z)$ by Eq. (10). For the conversion, two multiplications and two additions are needed per second-order section.

Similar derivations can be performed for the case when more strings are connected to the same termination. However, that case is handled in a more flexible way by constructing a WDF admittance element and connecting it to the waveguide string models by a parallel adaptor as shown below.

4.2 Waveport in WDF modeling

While DWGs focus on wave propagation in digital waveguides and scattering (reflection in our guitar bridge case) at waveguide junctions, wave digital filters are another paradigm where modeling is based on waveport elements and adaptors to connect them in parallel or series [1, 2]. WDF elements have a properly specified *port resistance* that make their wave ports reflection-free, i.e., their output values are available before the input values are required. Without this specific choice of port resistances, the WDF model would contain non-computable delay-free loops.

In addition to basic WDF elements, such as resistances, capacitances and inductances, or their mechanical/acustical equivalents, so-called consolidated elements for arbitrary immittances have been introduced in [15]. The principle is very similar to the DWG reflectance realization above, except that the WDF waveport is made free of immediate reflection and its formulation is independent of (waveguide) impedance(s) connected to it through adaptors. This is achieved by selecting port admittance (reciprocal of port impedance) $Y_\mathrm{port} = Y_\mathrm{i} = b_0 + \sum_{r=1}^{R} b_r$ so that the immediate path in Fig. 2 can be eliminated and the WDF bridge waveport becomes as shown on the right-hand side of Fig. 3. Now a parallel adaptor can connect this to any number of other waveports (digital waveguide string models in Fig. 3).

4.3 Realtime implementation

The DWG and WDF realizations of the parallel filter bridge admittance and a guitar string waveguide with first-order loss filter were tested in BlockCompiler software[2], which is a real-time modeling tool for physics-based modeling. The DWG model of Fig. 2 for the 26th-order bridge admittance filter [corresponding to Fig. 1 (c)] takes 5.1% of CPU time of a 1.67 GHz PowerPC processor (Macintosh G4) at sample rate of 44100 kHz, while the extreme case of the 360th-order bridge admittance filter [Fig. 1 (e)] takes 82% of CPU time. The computational load is proportional to the order of the bridge filter, which is common to all strings in the case of Fig. 3, so that a 6-string guitar model takes only little more CPU time than the single-string model.

5 Conclusion

This paper has presented a simple and robust method for constructing passive admittances that can be used in real-time sound synthesis. The model follows the target (measured) admittance curve more precisely where it is needed (i.e., at low frequencies), while in the high frequency range only the spectral envelope of the modes is followed. This is accomplished by estimating the poles of the admittance filter through a warped IIR filter design. Then, the admittance transfer function is composed as a weighted sum of positive real transfer functions. The weights are found by nonnegative least squares optimization. The admittance formulation is converted to a reflectance filter in such a way that retains the parallel filter structure. The parameters of the reflectance filter are computed from the admittance form parameters by only a few operations, which can be utilized if the admittance parameters are required to vary in real-time. The reflectance filter can be used both in digital waveguide and wave digital filter modeling paradigms. Due to its robustness, the proposed method can be used for the design and implementation of high-order admittance models (filter orders up to 360 were tested).

This work has considered scalar admittances only. A natural extension of the research is the modeling of passive admittance matrices, such as the bridge of the guitar with six strings and two transversal polarizations, resulting in a 12 by 12 admittance matrix. The parameter estimation algorithm could be improved by optimizing the poles in parallel with estimating the zeros, at the expense of added complexity.

Acknowledgments

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[2]see http://www.acoustics.hut.fi/software/ BlockCompiler/

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