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Spectral attenuation of sound in dilute suspensions with nonlinear particle relaxation

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Theoretical studies on the dissipation and dispersion of sound in two-phase suspensions have been briefly reviewed. Previous studies on the sound attenuation in particle-laden flows under Stokesian drag and conduction-controlled heat transfer have been extended to accommodate the nonlinear drag and heat transfer. It has been shown that for large particle-to-fluid density ratio, the particle Reynolds number bears a cubic relationship with $\omega\tau_d$ (where ω is the circular frequency and τ_d the Stokesian particle relaxation time). This dependence leads to the existence of a peak value in the linear absorption coefficient occurring at a finite value of $\omega\tau_d$. Comparison of the predictions with the test data for the spectral attenuation of sound with water injection in a perfectly expanded supersonic air jet shows a satisfactory trend of the theory accounting for nonlinear particle relaxation processes.

1 Introduction

Sound attenuation in fluids, representing the dissipation of acoustic energy from a sound wave, occurs through a number of physical processes involving molecular viscosity, thermal conductivity, and other dissipative or relaxation processes [1, 2]. In all these absorption mechanisms, acoustic energy is converted into thermal energy, and other mechanisms deflect or scatter acoustic energy [1]. When a fluid contains inhomogeneities such as suspended particles (solid particles, drops and bubbles) additional viscous and heat conduction losses occur in the immediate neighbourhood of the suspended particles [1-3].

Referring to Fig. 1, The acoustic intensity I of a plane wave propagating through an absorbing medium is expressed by

$$I = I_0 e^{-\alpha x} \tag{1a}$$

where x is the distance traversed, I_0 is the intensity at $x = 0$, and α the linear absorption coefficient for the medium. The quantity α depends on viscosity, thermal conductivity, and other factors such as molecular relaxation.

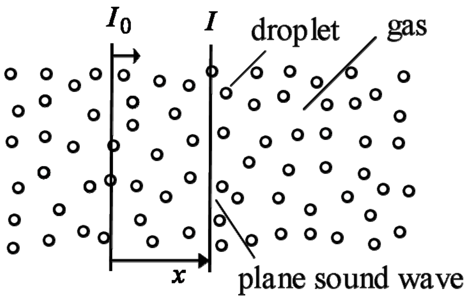

Fig. 1 Attenuation of a plane sound wave in a gas-droplet mixture.

Recently data on sound attenuation in supersonic air jets containing suspended water droplets reveal that the linear absorption coefficient displays a spectral peak. The particulate relaxation models for the sound attenuation are all based on Stokes drag (linear drag law) and pure conduction limit (linear heat transfer), and do not account or explain this attenuation behaviour. This article attempts to investigate this attenuation behaviour by considering nonlinear drag and heat transfer laws applicable to relatively large-sized droplets.

2 Review of Previous Work

Stokes [4] developed the first successful theory of sound absorption due to the effect of molecular viscosity of the fluid (internal friction). Helmholtz [5] theoretically investigated the effect of viscosity on the sound attenuation in a circular tube, considering the effect of friction near solid boundaries. Kirchhoff [6] theoretically treated sound attenuation due to the viscosity and heat conduction effects in an unbounded fluid medium and in narrow tubes. The so-called *classical sound absorption* in fluids includes Stokes' viscous contribution and Kirchhoff's thermal conduction contribution to attenuation in unbounded fluids.

Sound propagation in aerosols and fog has been studied experimentally and theoretically by several investigators. The first (early) observations by Tyndall [7] for sound propagation in a fog were rather inconclusive. Lord Rayleigh [8] estimated the scattering effect of small spherical obstacles in a non-viscous atmosphere, and showed that the effect depends on the number of scattering particles and the ratio of their diameter to the wavelength of the sound.

Sewell [9], in his pioneering work, theoretically considered the attenuation of sound in a viscous medium containing suspended cylindrical or spherical particles (obstacles) with perfectly rough surfaces. Sewell predicted the attenuation of sound by rigid particles suspended in a gas, assuming that the particles are immovable.

Epstein [10], in a theoretical treatment of the attenuation of sound by spherical particles suspended in liquids or gases, derived and extended Sewell's result by permitting the particles to move. The theory predicts attenuation of sound proportional to ω^2 at very low frequencies, and approaching the values given by Sewell's theory for very high frequencies. Epstein and Carhart [11] additionally considered heat conduction effects, and found that for fogs the effects of viscosity and heat conduction are both important, and approximately additive. The theory of Epstein and Carhart [11] consistently underpredicts the measurements of Knudsen et al. [12] for sound absorption by water droplets in air.

The effect of transport processes on the attenuation and dispersion of sound in aerosols have also been reported by Soo [13], Chu and Chow [14] and Chow [15]. Soo [13] accounted for nonstationary effects including the Basset term and added mass terms. Chu and Chow [14] presented a theory for the dispersion of sound, which agrees with the data of Zink and Delgasso [16]. Chow [15] considered the attenuation of sound in dilute emulsions and suspensions with viscous dissipation and thermal conduction, and additionally included the effect of surface tensions. It was shown that the effects of surface tension are negligible for systems with solid particles or liquid droplets, but are important in the case of bubbly mixtures. Nonstationary effects, which become important at low particle Reynolds numbers, are further investigated by Gumerov [17].

The preceding theories are similar in the sense that the calculation of the acoustic field around the particles is determined by computing the potential of additional waves that appear when a plane wave falls upon a spherical obstacle (the so-called scattering approach). The attenuation of sound is then computed by considering the entropy increase owing to irreversible momentum and heat transfer between the droplet and the gas.

Temkin and Dobbins [18], in their classical work, theoretically considered particle attenuation and dispersion of sound in a manner which illustrates explicitly the relaxation character of the problem. The linear droplet drag and heat transfer are respectively obtained from

$$F_p = 6\pi\mu_g \left(u'_p - u'_g\right) \tag{2a}$$

$$Q_p = 2\pi d_p k_g \left(T'_p - T'_g\right) \tag{2b}$$

which correspond to the zero droplet Reynolds number limit ($\mathrm{Re}_p \to 0$), where

$$\mathrm{Re}_p = \rho_g \left|u_g - u_p\right| d_p / \mu_g \tag{2c}$$

In the above, u is the velocity, T the temperature, ρ the density, μ the dynamic viscosity, k the thermal conductivity, and d the diameter. The subscripts g and p stand respectively for the gas and the particle, and the primes denote fluctuations from the mean. The properties ρ_g, μ_g, k_g and ρ_p refer to the mean values.

At *low mass concentrations* $C_m = n_0 m_p / \rho_g << 1$, the theory of [18] yields

$$\overline{\alpha} = \left(\frac{c_0 \alpha}{C_m \omega}\right) = \frac{\omega\tau_d}{1+\omega^2\tau_d^2} + (\gamma-1)\left(\frac{c_{pp}}{c_{pg}}\right)\frac{\omega\tau_t}{1+\omega^2\tau_t^2} \tag{3a}$$

$$\overline{\beta} = \frac{\left[(c_0/c)^2 - 1\right]}{C_m} = \frac{1}{1+\omega^2\tau_d^2} + \frac{(\gamma-1)\left(c_{pp}/c_{pg}\right)}{1+\omega^2\tau_t^2} \tag{3b}$$

with
$$\tau_d = d_p^2 \rho_g / \left(18\mu_g\right) \tag{3c}$$

and
$$\tau_t = \frac{m_p c_{pp}}{2\pi d_p k_g} = \frac{\mathrm{Pr}\, c_{pp} d_p^2 \rho_p}{12\mu_g\, c_{pg}} = \left(\frac{3}{2}\right)\left(\frac{c_{pp}}{c_{pg}}\right)\mathrm{Pr}\,\tau_d \tag{3d}$$

In the above equations, c_p refers to the specific heat, $\mathrm{Pr} = c_{pg}\mu_g / k_g$ the Prandtl number of gas, c_0 the speed of sound in the gas phase, γ the isentropic exponent (specific-heat ratio), n_0 the mean number of particles per unit volume of mixture, and m_p the mass of one particle. The quantity $\overline{\alpha}$ refers to the attenuation per unit frequency per unit mass fraction, and $\overline{\beta}$ the dispersion coefficient.

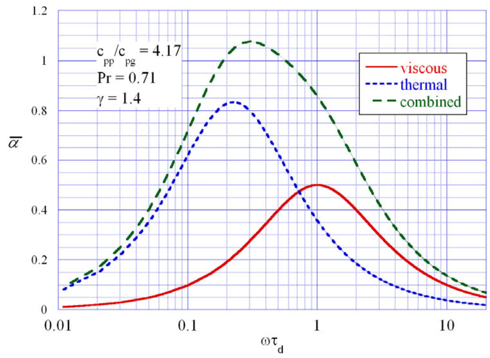

Fig. 2 Absorption coefficient per unit frequency for dilute suspensions according to Temkin and Dobbins [18].

Fig. 2 displays the absorption coefficient $\overline{\alpha}$ (scaled by wavelength) as a function of $\omega\tau_d$, as given by Eq. (3a) due to Temkin and Dobbins [18] for water droplets in air ($\rho_p / \rho_g \approx 840$). The individual contributions due to particle drag and heat transfer on sound absorption are of the same order of magnitude, with the heat transfer contribution somewhat greater than the drag contribution. The theory agrees well with the data of [16, 19]. The quantity $\overline{\alpha}$ is very small for $\omega\tau_d << 1$ and $\omega\tau_d >> 1$, and becomes important at intermediate frequencies with a spectral peak at $\omega\tau_d \approx 1$. This trend is generally characteristic of sound attenuation in fogs, clouds, and artificially produced smokes.

For of $\omega\tau_d << 1$, the particles very rapidly adapt to changes in the fluctuations of the carrier fluid (flowfield) and move almost in equilibrium with the gas (particles follow the fluid motion perfectly). In the other extreme case of $\omega\tau_d >> 1$, Stokes linear drag law is not strictly applicable, as alluded to in Temkin and Dobbins [18], and particles are scarcely disturbed by the gas fluctuations. Under such circumstances, the particles remain almost fixed in space, and the gas executes oscillations around the particles. According to Temkin and Dobbins [18], Stokes linear drag law can be justified for $0 \le \omega\tau_d \approx 1$, provided that $\rho_g / \rho_p << 1$ and $\left(\omega d_p^2 / 8\nu_g\right)^{1/2} << 1$.

Following Temkin and Dobbins [18], limited to the case of an inert dispersed phase, Marble and Wooten [20] studied sound attenuation in a condensing vapor. It is shown that under some important circumstances, the effects of condensation and vaporization dominate the damping mechanism. Marble and Candler [21] studied sound absorption due to liquid droplet vaporization. Propagation of acoustic waves in a two-phase vaporizing and reacting media were treated by Dupays and Vuillot [22].

3 Present analysis

The present analysis extends the work of Temkin and Dobbins [18] for dilute suspensions to accommodate the nonlinear drag and heat transfer laws, which become important at high particle Reynolds numbers Re_p and at high frequencies. Only sound attenuation is considered here, with sound dispersion excluded from consideration.

3.1 Nonlinear particle drag and heat transfer

Without any loss of generality the attenuation of sound for large particle Reynolds numbers with nonlinear particle relaxation may be expressed by relations similar to Eq. (3a) as follows:

$$\overline{\alpha} = \left(\frac{c_0 \alpha}{C_m \omega}\right) = \frac{\omega \tau_{d1}}{1+\omega^2 \tau_{d1}^2} + (\gamma - 1)\left(\frac{c_{pp}}{c_{pg}}\right)\frac{\omega \tau_{t1}}{1+\omega^2 \tau_{t1}^2} \quad (4)$$

Here the relaxation times τ_{d1} and τ_{t1} correspond to the relaxation times under nonlinear drag conditions (generally representative of large-sized particles). They are related to the Stokesian relaxation times by the relations

$$\tau_{d1} = \tau_d \psi_1(\mathrm{Re}_p), \quad \tau_{t1} = \tau_t \psi_2(\mathrm{Re}_p) \quad (5a)$$

where $\psi_1(\mathrm{Re}_p) = C_{D1}/C_D$, $\psi_2(\mathrm{Re}_p, \mathrm{Pr}) = Nu_1/Nu$ (5b)

with C_{D1} standing for the nonlinear drag coefficient, and Nu_1 for the nonlinear heat transfer. The drag coefficient and the Nusselt number in Eq. (5b) are defined by

$$C_D = 2F_D \Big/ \left(\rho_g \frac{\pi}{4} d_p^2 u_g^2\right), \quad Nu = h_g d_p / k_g \quad (5c)$$

where h_g refers to the gas-droplet convective heat transfer coefficient.

A good approximation (curve fit) for the drag coefficient C_{D1} is recommended as [23]

$$C_{D1} = \frac{24}{\mathrm{Re}_p} + \frac{6}{1+\sqrt{\mathrm{Re}_p}} + 0.4 \quad (6)$$

The first term on the RHS of Eq. (6) is the Stokesian drag coefficient defined by

$$C_D = 24/\mathrm{Re}_p \quad (7)$$

which is valid for particle Reynolds numbers less than about one. Eqs. (5a) and (5b) yield an expression for ψ_1 as

$$\psi_1(\mathrm{Re}_p) = 1 + \frac{\mathrm{Re}_p}{24}\left(\frac{6}{1+\sqrt{\mathrm{Re}_p}} + 0.4\right) \quad (8)$$

With regard to the particle heat transfer, a good correlation for heat transfer (by conduction and convection) is expressed by the well-known Ranz-Marshall correlation [24]

$$Nu_1 = 2 + 0.6\,\mathrm{Re}_p^{0.5}\,\mathrm{Pr}^{0.33} \quad (9)$$

In the pure conduction limit, we have $Nu = 2$, so that the function ψ_2 in Eq. (5b) becomes

$$\psi_2 = Nu_1/Nu = 1 + 0.3\,\mathrm{Re}_p^{0.5}\,\mathrm{Pr}^{0.33} \quad (10)$$

It now remains to find a relation for the droplet Reynolds number Re_p in terms of the density ratio and the particle relaxation time.

3.2 Droplet Reynolds number

The determination of particle Reynolds number required in the evaluation of the functions ψ_1 and ψ_2 in Eqs. (8) and (10) respectively is exceedingly complex. There exists relatively little information on the dependence of particle Reynolds number on the particle characteristics in two-phase flows. A study of turbulent diffusion of droplets in a gaseous medium [25, 26] based on Tchen's theory [27], indicates a *plausible* relationship of the form

$$\mathrm{Re}_p = f(\rho_g/\rho_p, \omega\tau_d) \quad (11)$$

Since Eq. (4) is applicable to large particle to fluid density ratios, we postulated here that the particle Reynolds number depends only on the particle relaxation time, and is independent of the particle to fluid density ratio:

$$\mathrm{Re}_p = f(\omega\tau_d) \quad (12)$$

In the present investigation, the following power law relation is proposed such that a peak in the linear absorption coefficient (as indicated by the measurements) is realized (an exponent of 2.5 or greater for the relaxation time ensures this peak):

$$\mathrm{Re}_p = c(\omega\tau_d)^3 \quad (13)$$

The adjustable constant c is determined from a correlation of the theory with the test data. A value of $c = 10$ is found to be satisfactory based on the data of Norum [28] for water droplets in a supersonic air jet.

4 Results and comparison

4.1 Effect of Nonlinear Particle Relaxation

The effect of nonlinear particle relaxation on the absorption coefficient per unit frequency is demonstrated in Fig. 3a for comparison with the theory of Temkin and Dobbins [18] for dilute concentrations. The results suggest that below $\omega\tau_d \approx 0.7$ the nonlinear particle relaxation effects are seen to be unimportant. Beyond $\omega\tau_d \approx 0.7$, the nonlinear theory departs from the linear theory.

Fig. 3b shows a comparison of the predicted linear spectral attenuation of sound under nonlinear particle relaxation. The theoretical result by Temkin and Dobbins [18] for dilute concentrations is also presented for a comparison. The theory suggests that at high frequencies the linear absorption coefficient predicted by the linear theory decreases with increasing frequency in accordance with experimental trend for large particle sizes.

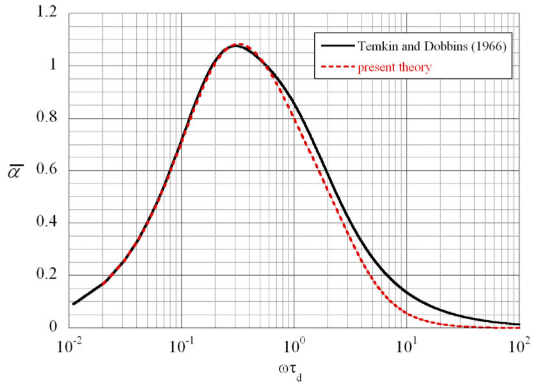

Fig. 3a Predicted absorption coefficient per unit frequency with nonlinear particle relaxation processes.

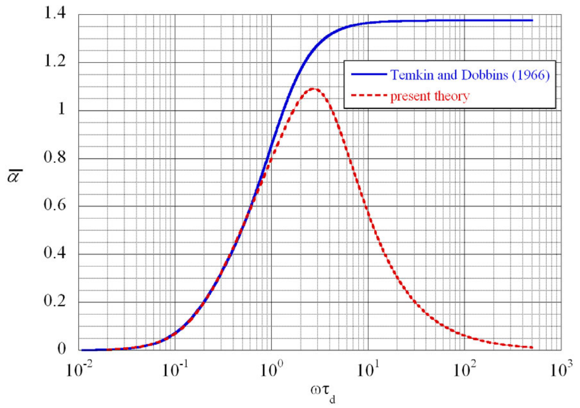

Fig. 3b Predicted linear absorption coefficient with nonlinear particle relaxation processes.

4.2 Comparison with experimental data

A direct comparison of the present theory with the measured spectral attenuation by Norum [28] for water droplets in a perfectly expanded (jet exit pressure equals the ambient pressure; thus shock-free) supersonic jet is displayed in Fig.4. The data correspond to hot supersonic jet of air from a convergent-divergent (CD) nozzle operation at a jet total temperature $T_t = 867$ K, and a jet exit Mach number M_j=1.45. The jet Mach number is defined as $M_j = u_j / c_j$, where the subscript j refers to the nozzle exit conditions. The mass flow rate (maximum considered) of water to that of the jet is about 0.85. The angle θ is measured from the jet inlet axis. The jet exit Reynolds number $\mathrm{Re}_{j1} = u_j d_j / \mu_{gj}$ is about 1.3x10^6, where d_j is the jet exit diameter. At this condition, supersonic turbulent jet mixing noise [29, 30] dominates upstream noise radiation, and Mach wave radiation [31] dominates the downstream noise radiation. In the data, water is injected at 45 deg. The data include spectra measured at angles of 45 deg, 90 deg and 135 deg, thus highlighting directivity effects.

From the measured ΔSPL (reduction in Sound Pressure Level) at a given frequency due to water injection, the linear absorption coefficient is deduced as

$$\alpha / \alpha_{\max} = \Delta\mathrm{SPL} / \Delta\mathrm{SPL}_{\max} \qquad (14)$$

where the subscript refers to the peak spectral reduction.

The comparisons suggest that the proposed theory based on the nonlinear particle relaxation processes satisfactorily describes the measurements for the spectral attenuation of sound, indicating a spectral peak. The inclusion of directionality effect (dependence on the angle of observation) on the spectral absorption is beyond the scope of the present work.

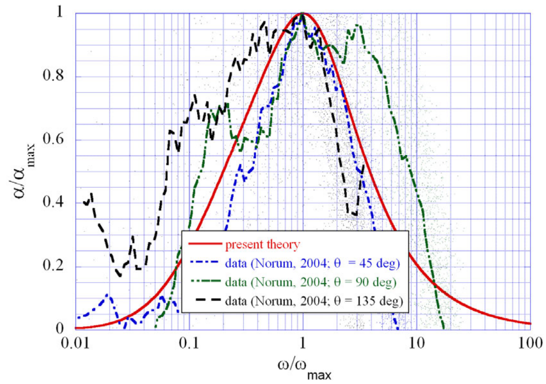

Fig. 4 Comparison of the predictions for the linear absorption coefficient with test data of Norum [28].

The spectral behavior shown in Fig. 4 is also noted in the experiments of Krothapplai et al [32] and of Castelain et al. [33] on sound attenuation with microjet injection (with low mass flow rates of water of the order of 10 percent of the jet mass flow rate). Some related experimental data on sound suppression with water injection include those of Zoppellari and Juve [34].

It should be remarked, however, that the data on supersonic jet noise considered here for comparison are not directly pertinent to plane waves of sound for which the theory has been developed. Since noise from turbulent jets may be regarded as a superposition of plane waves of differing frequency, the absolute values of the absorption coefficient in the test data could be different from that expressed by the theory. Finally, the effect of droplet evaporation on the spectral attenuation of sound, as indicated by the theory of [22], is left out of account in the present analysis.

5 Conclusion

The theory proposed here for sound attenuation in dilute suspensions with nonlinear particle drag and heat transfer is shown to satisfactorily represent the test data for noise reduction with water droplets suspended in a supersonic jet. It is found that the nonlinear particle relaxation processes are primarily responsible for reduction in the linear absorption coefficient at high frequencies. Extended comparisons are needed to further validate this finding.

Acknowledgments

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