Condenser microphone as parametric electroacoustic system and its time-domain modelling via equivallent electrical circuit in SPICE software

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Condenser microphone usually works with constant polarization voltage. Where special attentions are prescribed for spectrum of self noise there may be useful to use high frequency voltage (pump) applied to transducer instead of DC charge. When frequency of pump is higher in orders than frequency of acoustic signals, large parametric gain occurs in the transducer. The method for describing such a system via equivalent electric network with approach of electronic filter design blocks was developed and special analogies were used for convergence improvement. The method was applied in time-domain simulation of pressure transducer with current-mode diode discriminator in SPICE computer analysis software. Simulations of transient behaviour after switching the system on were demonstrated.

1 Motivation

The pressure capacitor microphone is a widely used electroacoustic transducer. There are two distinct methods of its connection into electric circuit.

In the first one – constant charge operation – voltage on the transducer is stabilized to D.C. potential (polarisation voltage) in the orders from tenths to hunderths of volts [1]. Stabilization may be realised rather simply – by a resistor (with the value from fraction to units of gigaohms) or more ingeniously by using of nonlinear feedback network with semiconductor diodes [2] and finally in some cases transducers are prepolarised in the production – using electrets. In all those examples transducer was at the electrical branch rather high – impedance and wideband signal source. This may be limitation where transducer is used in the non-standard environment (high – humidity or temperature changes) or when special attention is paid to the noise performance of the microphone – preamplifier system.

The second method of connection – high frequency operation [3] – uses high frequency oscillator to pump high-frequency reactive power into transducer. Transducer then transpose input acoustic {or mechanic} signal into frequency near the pump oscillator. Signal output from the transducer is then high - frequency, with narrow relative bandwidth and reactive impedance of the capacitor transducer near pump oscillator is low. Due this low impedance the power gain occurs in transducer. We may note here, that capacitor microphone is (if properly designed) high quality air capacitor with quality factor of order of 1000 at the MHz range. Its quality supersedes the best semiconductor diodes - varactors ten times. In the paper we should introduce apparatus for simulation of this kind of transducer.

2 Dedicated electromechanical analogy

Let us have a new electromechanical analogy with these properties: Topological graph of electric circuit is equivalent to graph of mechanical circuit (rigid connection corresponds with the node). Distinct lumped elements of mechanical system, which are: Elastor, mechanical resistor and inertor we should represent by: resistor, capacitor, and double capacitor (or Frequency Dependent Negative Resistor)

This analogy will be topologically – equivalent with seldom used second electromechanical analogy [7]. Its advantage compared to classical analogies relies on temporary independence of stabilizing element. In capacitor microphone stabilizing element there is compliance and it corresponds with resistance in our equivalent circuit, which is time independent and thus analysis of transducer stability can be done via DC analysis. Analogous method was used for convergence improvement in simulation of strongly coupled MEMS via FEM analysis in ANSYS [6]. Let us introduce lumped elements of this analogy closer:

2.1 Elastor

Let the (mechanical) elastor be represented by (electrical) resistor. Equation U=R.I (Ohm's law - R stands for electric resistance) corresponds to equation: $x_M=C_M.f_M$ (Hook's law, where C_M stands for compliancy of elastor)

Fig.1: Elastor

2.2 Mechanical resistor

The mechanical resistor stands for the capacitor. Equation I=C.U` (C – electric capacity) stands for equation $f_M=r_M.x`_M$ (r_M-mechanical resistance) and apostroph ` stands for first derivative with respect to time.

Fig.2: Mechanical resistor

2.3 Inertor

Inertor stands for double capactior (or Frequency Dependent Negative Resistor). Equation I=D.U`` (D- Bruton's factor) stands for equation $f_M=M_M.x``_M$ (M_M – mechanical mass) and double apostroph `` stands for second derivative with respect to time.

Fig.3: Inertor

3 Electromechanical conversion with electric field (electriostatic conversion)

We use energetic relations (sometimes called Lagrange-Hamilton formalism [8][9]) for simple and transparent deducton of fundamental equations of the electromechanical transduction. For potential energy of transducer holds [5][6]:

$$V = \frac{1}{2}\frac{q^2}{C} \tag{1}$$

Where q is electric charge and C is capacity. In the case of infinity plates with a constant distance (or microphone where square root of diaphragm surface is much higher than the distance between the diaphragm and the back plate) , for capacity holds:

$$C = \frac{\varepsilon A}{x} \tag{2}$$

Where A is effective electrical area of diaphragm, x is diaphragm displacement (assume piston – like movement) and ε is permitivity of vacuum. For potential energy of simple transducer thus holds:

$$V = \frac{xq^2}{2\varepsilon A} \tag{3}$$

Partial derivatives of (3) with a respect to its variables (which must be adequately chosen as *generalised coordinates*) gives us *generalised forces*. (We will use small letters for generalised coordinates and capital letters for generalised forces altough formulas holds generaly for DC and not only for small signal values and thus all letters should be capitals, the same holds for figs 1-3) We then get two equations correspondingly to two moments of freedom of transducer [5][6] .

$$F = \frac{\partial}{\partial x}V = \frac{q^2}{2\varepsilon A} \tag{4}$$

$$U = \frac{\partial}{\partial q}V = \frac{qx}{\varepsilon A} \tag{5}$$

Equations (4) and (5) contain variables which was used in our electromechanical analogy to be modelled by current (F) and voltage (x). Equation (4) thus may be simulated through controlled current source in the mechanical part of the transducer. It would be nice to make the electrical part similar to the mechanical part (5). It may be possible, but only if we introduce special electro – electrical analogy, where voltage will be used instead of charge and current will be used instead of voltage.

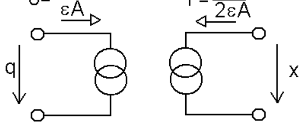

Fig.4: Admittance model of nonlinear elstat. transducer

Then we get twoport in the admittance equivallent circuit, which is especially benefitial for the simulations.

4 Dedicated electro- electrical analogy

Let us define supplementar analogy where voltage is used instead of charge and current is used instead of voltage. Resistor will be used instead of capacitor, because equation U=R.I is transformed into equation q=C.U. Transformation between these two analogies does twoport – mutator of the first kind [10].

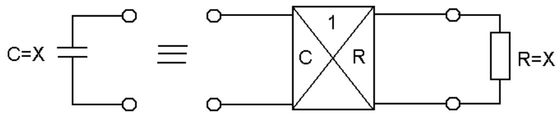

Fig.5: Property of mutator of the first kind and unity parameter

One property of mutator is change of operator impedance of circuit which is connected with one branch and seen from the other like as it was multiplied by derivative operator p and result inverted. If there will be a resistor connected with one branch of mutator, at the second one is seen the capacitor. If inductor is connected to one branch of the mutator, at the second one is seen FDNR. It can help us in the simulation of FDNRs in SPICE software without using of Laplace type controlled sources, which are not efficient in simmulations.

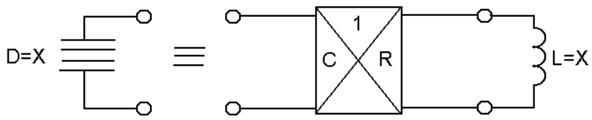

Fig.6: Realisation of FDNR via mutator

Mutator could be realised in SPICE via many ways. The simplest one is to use admittance model of gyrator with one Voltage Controlled Current Source of Laplace type. But this model is not suitable for time domain simulations. More sophisticated model uses three VCCSs and one inductance and this model will be used.

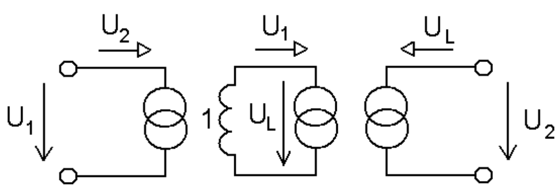

Fig.7: Realisation of the mutator

5 Pressure condenser microphone

Pressure capacitor transducer is realised in the form of two electrodes, where the first is light and compliant diaphragm and pressure wave incides at its outer side. Its iner side adjaces with back electrode, which is rigid and perforated. Volume between diaphragm and back plate is connected to outer space only via capillary which equalize static pressure changes. Volume between the back electrode and the diaphragm is extended by perforating of the back plate in a plurality of ways, because its maximisation maximise sensitivity of microphone. But its depth is limited by half of wavelength of maximal recorded frequency and motive (plane distribution of i.e. holes) of the back plate determines diaphragm damping. Back plate design is thus trade-off betwen the production cost and the microphone parameters

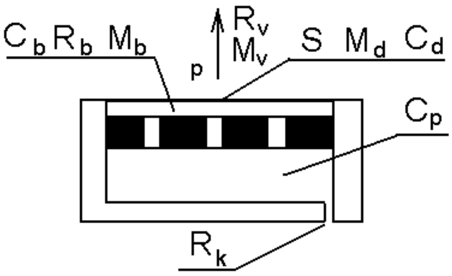

Fig.8: Pressure condenser microphone

Elements in the fig. 8 are: S – diaphragm area, M_d – diaphragm mass, C_d – diaphragm compliancy. R_v and M_v – radiation impedance, C_b, R_b and M_b - lumped parameters of the air between back plate and diaphragm, C_p is compliance of air in the cavity in the back plate and R_k is acoustical resistance of the capillary.
Fig. 8 could be redrawn into equivallent scheme in general or more simplified form.

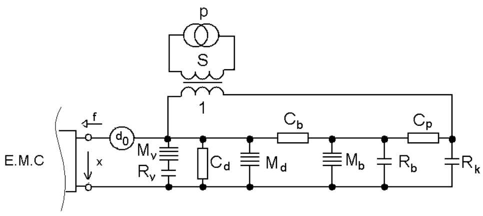

Fig.9: Equivallent model of pressure condenser microphone

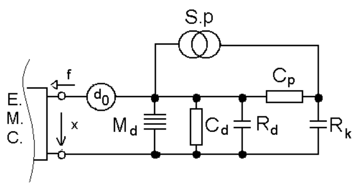

Fig.10: Simplification of model in fig. 9

We must not forget, that this transduction model is nonlinear and membrane displacement at zero voltage (assembly distance) is one of its main parameters. Displacement is modelled through voltage source in figs. 9 and 10.

6 Electronics

As an example we choose simple ring diode discriminator in the current mode. It is the simplest variant of electronic circuit which uses capacitor microphone in the parametric – amplifier mode. We use both sidebands and decoupling of pump and output is done via circuit symmetry. Let us note, that this circuit is not optimal from the self-noise view, (circuit from lit. [11] would be better) but it is simple and all the interesting effects can be seen from it.

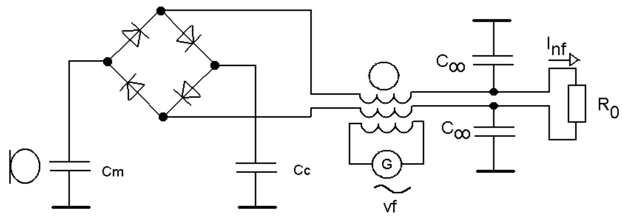

Fig.11: Simple discriminator

In the fig. 11 pump voltage from high frequency generator is fed into trifilar transformer. Transformer is used only for decoupling of D.C. signal current with A.C. pump voltage. In the case when the capacity of microphone Cm is equal to capacity of reference capacitor Cc, and all diodes are identical, then no D.C. current flows between windings of the transformer. If the symetry of circuit is disturbed, D.C. current starts to flow between transformer winding, which is proportional to the capacitor difference, pump voltage and pump frequency.
Interesting phenomenon is the growth of sensitivity (output current / input pressure ratio) linearly with pump frequency. It is quasistatic case of so called „Manley-Rove“ equations, [12] which are rudiments for analytical solving problems with a gain obtained by nonlinear capacitance twopoles. This holds for parametric amplifiers with varactor.
The natural condition for optimal circuit work is to have A.C. blocking capacitors short for pump frequency and open for audio signal. Next condition is to have load resistor small enough so that modulator diodes cannot open due to voltage drop.

7 Example of simulation

For simulation of specific circuit we choose freeware SPICE simulator with graphical environment „SwitcherCAD“, a product the of company „Linear Technology“, version 2.21 [13]. As example impulse response of the diaphragm displacement onto switching pump signal on was calculated. Parameter of nonlinear transducer 1/(εA) was set to 5.10^{14} (diaphragm area $2{,}26\text{cm}^2$). Equivalent compliance of the diaphragm was set to 50 µm/N, compliance of the air volume under diaphragm was 10 µm/N. Diaphragm is loaded by signal frequency of 1kHz with force 20mN pp, which corresponds with effective acoustic pressure of 31 Pa. (Rather high pressure - 124 dB) At the electric side was applied pump voltage 127V effective with frequency 1 MHz. Overall scheme of simulated circuit is in Fig. 12. In the Fig.13 (result) we can see, how diaphragm with assembly distance 20 µm moves closely to the back plate and is stabilized on the displacement determined by nonlinear transducer properties (quasistatic stability [1]). The frequency characteristic of overall system sensitivity is.presented in the Fig. 14.

8 Conclusions

In the paper was described elements needed for simulation of nonlinear electrostatic tranasducer in the time-domain. An example of application of the method onto type of microphone was introduced. This analogy was also applied by the author for analysis of nonlinear behaviour of normal DC polarised condenser microphone [15] (Simillar problem was solved analytically in [14]).

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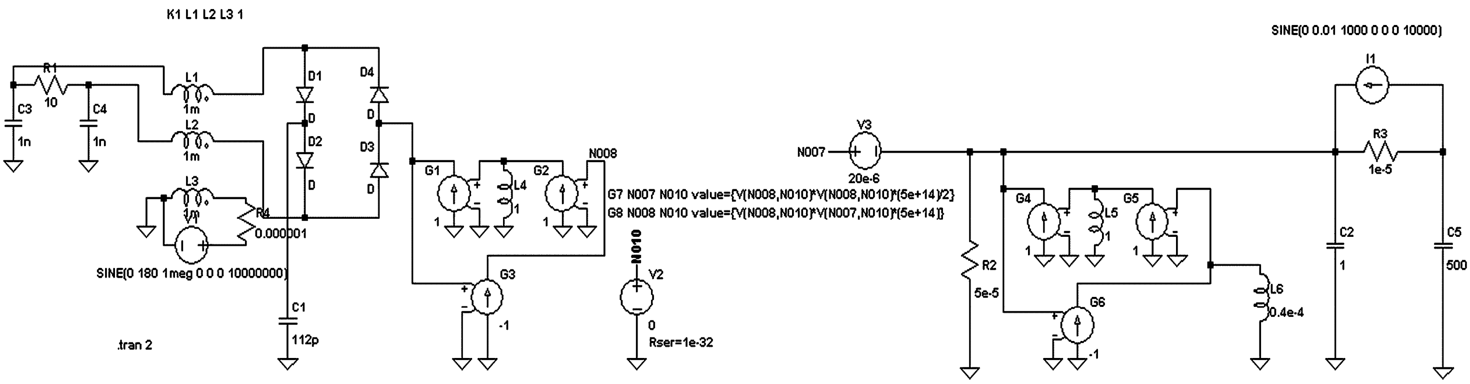

Fig.12: Schematic diagram of the overall system. Note, that nonlinear controlled sources are added as SPICE directive for clarity and lack of schematic element.

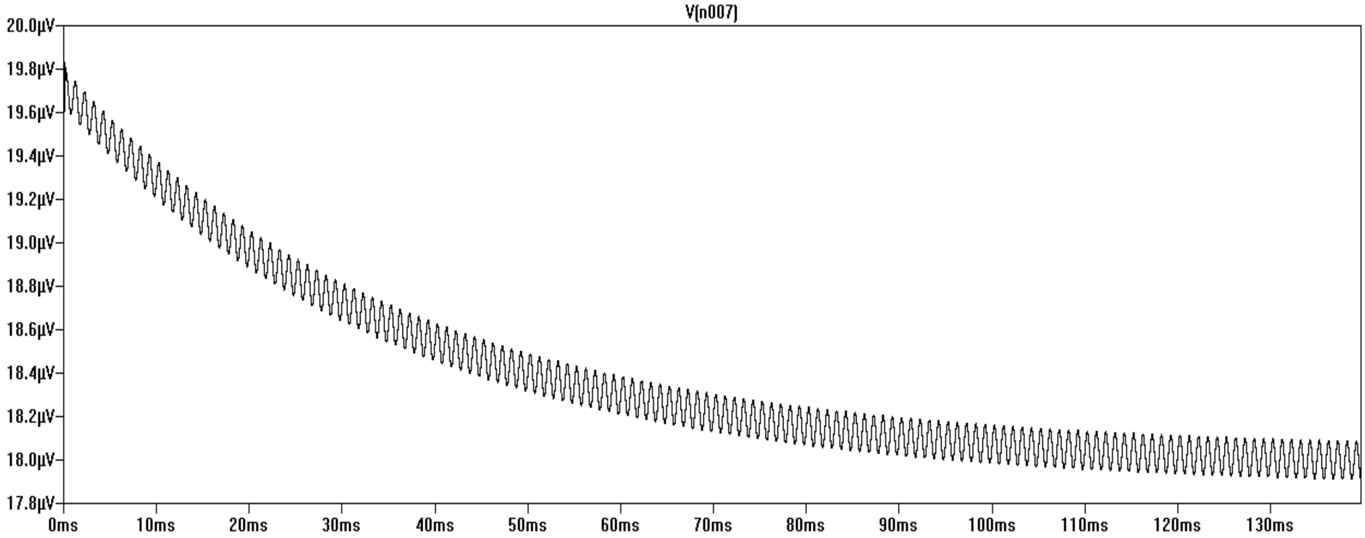

Fig.13: Displacement of diaphragm after switching system on (microphone is also excited with harmonic acoustic wave with frequency of 1 kHz)

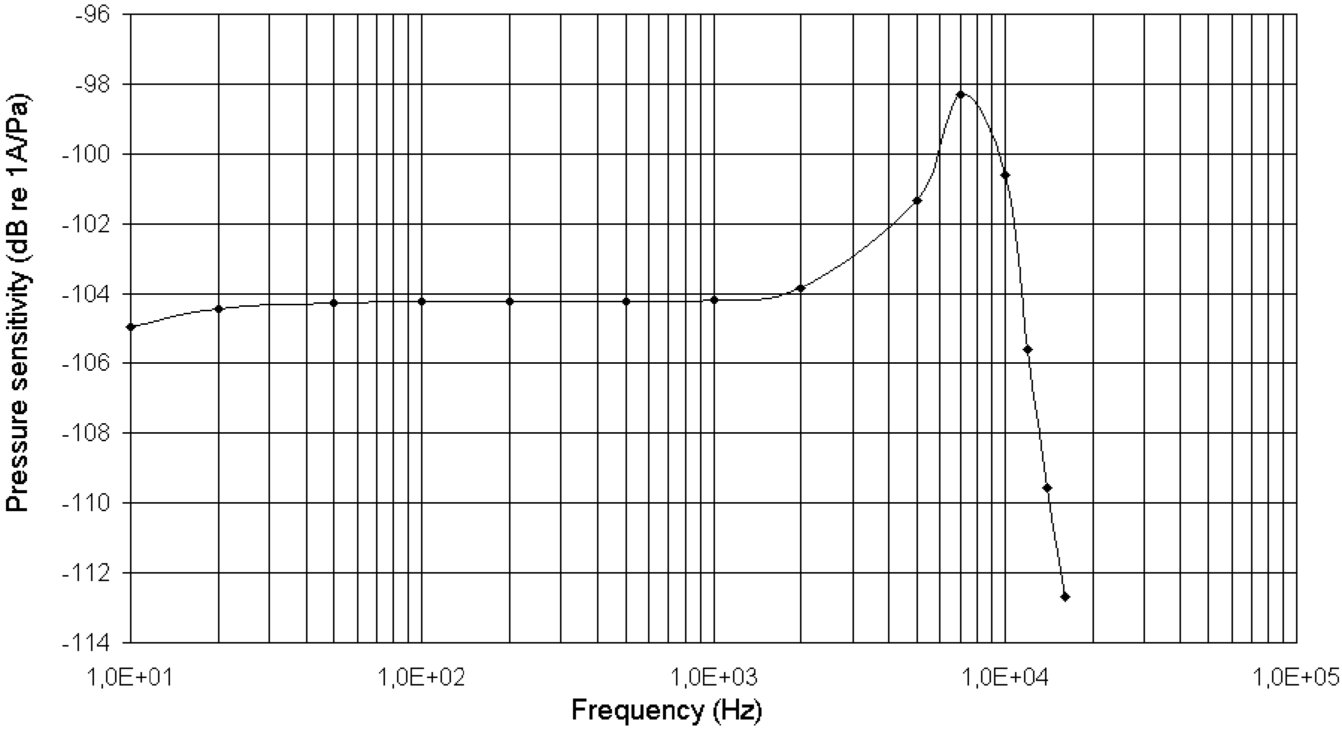

Fig.14: Frequency characteristics of pressure sensitivity of the microphone model on Fig.12 (Values are taken from steady-state of time domain simulation waveforms (like that on fig.13)