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# The vibration sound absorption theory of flexible materials

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**Abstract**: During the past two years, a new theory has been established for the flexible materials that the vibration of materials brings the sound absorption, regardless they have pores in them or not. This theory is totally different from Classical theory, such as Rayleigh model and Zwikker and Kosten theory. Firstly, an empirical sound absorption coefficients formula of fibrous materials was found. Secondly, the theory sound absorption formula of thin fiber layers was obtained by the vibration sound absorption analysis and the applying of conservation law of energy. These two formulas well agrees with each other. Basing of these achievements and applying classical laws of conservation of momentum and conservation of energy, the sound absorption theory formula of membrane (diaphragm) was also obtained, which have been justified agree with the sound absorption spectra of one kind of plastic film. This paper will give the review and discuss of the main point about the vibration sound absorption theory and it's establishment.

**Keywords:** flexible materials, vibration sound absorption theory, review

# 1. Introduction

The conventional sound absorption theories for fibrous materials are unable to give satisfactory theoretic results in better agreement with practice. A new theory, which is totally different from Classical theory, such as Rayleigh model and Zwikker and Kosten theory, holding the sound absorption for fibrous materials originates from the vibration of materials, have been established. This viewpoint is able to interpret an acoustic phenomenon - that is, when a fabric is used as the wall facing or the face protective material of a sound absorber, the sound absorption coefficient is zero or very small, whereas when the fabric hangs independently or there is an air layer behind, there is a very high sound absorption coefficient. In addition, the sound absorption coefficient formula of the imperforated plate (membrane) has also been deduced.

# 2. An analysis on sound absorption of thin fibrous layer

## 2.1 Analysis on thin fiber layers using Rayleigh model

Classical Rayleigh sound absorption model for porous materials begin mainly with the air flow viscous resistance in a small pipe, thereby establishing the sound absorption theory. The motion equation of the sound wave in the pipe can be expressed as [1,2]

$$\rho_0 \frac{du}{dt} - \frac{\eta}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = \frac{\nabla p}{t} \qquad (2.1)$$

Solving this equation obtain the specific acoustic impedance. And then the sound absorption coefficient is obtained. It is find that the theoretical results are not agreement with practice especially for thin fiber layers which have also high sound absorption coefficients.

When looking back to the viscous theory, in Reference (2), suppose that the particle speed at $r = a$ ($a$ being the pipe radius), that is, on the pipe wall, is zero. This point is not applicable to the thin fibrous layer, because under the action of the sound wave, the thin fibrous layer will produce vibration inevitably and at this time, the particle speed at the position where $r = a$ is not zero. When using $u_a$ to stand for the speed on the pipe wall, according to the Newton law, $u_a = \frac{\nabla p}{j\omega M_s}$

where, we have $u = u_a$ when $r = a$. From Eq. (2.1), we may get the specific acoustic impedance

$$z = \frac{\nabla p}{\bar{u}} = r + jx \qquad (2.2)$$

in which

$$r = \frac{\{(\frac{7.8}{M_s} - \frac{6.4}{t})\times10^{-3} - \frac{0.016}{M_s}\}(1-2\times10^{-5}a^4\omega^2)a^2}{(\frac{7.8}{M_s} - \frac{6.4}{t})^2 a^4\times10^{-6} + (\frac{2\times10^{-5}a^4\omega^2-1}{\omega M_s})^2} \qquad (2.3)$$

$$x = \frac{(\frac{1.3}{M_s} - \frac{1.0}{t})\times10^{-4}a^2\omega + \frac{(2\times10^{-5}a^4\omega^2-1)^2}{\omega M_s}}{(\frac{7.8}{M_s} - \frac{6.5}{t})^2 a^4\times10^{-6} + (\frac{2\times10^{-5}a^4\omega^2-1}{\omega M_s})^2} \qquad (2.4)$$

Considering the calculation convenience, in the above two equations, *mm* is taken for the unit $a$.

It is known, to enable the material to have a sound absorption action, the specific acoustic impedance $r$ must be greater than zero. The author has measured the fabric and nonwovens cloth with the micro pore diameter between 0.01*mm* and 0.25mm, with the first maximal sound absorption (the cavity depth 20cm) changing between 0.18 and 0.88. However, the specific acoustic impedance in the micro pore diameter scope worked out from Eq. (2.3) is negative-that is, the theoretical sound absorption coefficient is zero.

### 2.2 Application of the resonance theory

In this part, he resonance theory of the perforated plates applied to fabrics.

Let the area taken up by each resonator on the perforated plate be $S_0$, and the perforated area be *S*. The acoustic impedance of the resonator during the vertical incidence is the sum of the air layer acoustic impedance and that of the perforated pore, that is [3]

$$Z = \frac{1}{j\omega C} + j\omega m \qquad (2.5)$$

where $\omega = 2\pi f$ ; *m* – acoustic mass of the resonator, which is the sum of cavity acoustics mass $m_1$ and perforated pore acoustic mass $m_2$, namely [3]

$$m = m_1 + m_2 = \frac{\rho_o D}{3S_0} + \frac{\rho_o l_0}{S} \qquad (2.6)$$

C – cavity acoustic compliance $C = \dfrac{V}{\rho_0 c_0^2}$

in which *V* is the volume of the resonator cavity depth $V = S_0 D$ .From testing and calculating results of fabrics, the micro pore perforation sound mass $m_2$ of the micro pore of the fabric and nonwovens is basically not greater than 1/1000 of the cavity mass $m_1$.Therefore, the perforation sound mass $m_2$ can be assumed to be zero. The acoustic impedance during the resonance is zero. At this time, from the deducing of above equation, we get

$$f = \frac{\sqrt{3}}{2\pi}\frac{c_0}{D} = 0.27\frac{c_0}{D} \approx \frac{c_0}{4D} \qquad (2.7)$$

After conversion, we get

$$D \approx \lambda/4 \qquad (2.8)$$

### 2.3 Conclusion

From the conclusion that the specific acoustic impedance is negative, it is able to get an inference, namely, with respect to the thin fibrous layer, the speed of the fluctuation of the material itself is greater than the speed of the sound wave in the micro pore, and then the phenomenon that the acoustic impedance is negative occurs. According to the resonance sound absorption theory of the perforated plate and the existent acoustic impedance theoretic formula, it is able to get the first peak sound absorption frequency that approximates the measured results of the thin fibrous materials when not considering the sound absorption of the micro pores. Evidently, the sound absorption should originate from the sound energy loss by the fluctuation of materials instead of the viscous resistance of the micro pore to the sound wave. Hence, if the material is faced on the wall or other body, it's vibration will be limited. And the sound absorption coefficient will be zero or very small.

## 3 Empirical formula for sound absorption coefficient of fibrous materials

### 3.1 Sound absorption spectra of fibrous materials

The sound absorption coefficient was tested using the standing wave pipe test system (B&K2107) at distances of 5*cm*, 10*cm*, 20*cm*, 30*cm* and 40*cm* of the cavity depth behind the material. The thick layer materials were directly put in the standing wave tube and the thin layer soft material was placed in the standing wave pipe with the peripheral fixed with a ring support.

It can be seen that the maximal sound absorption coefficient appears when the cavity depth equal to an odd number multiplication of the 1/4 wavelength and the minimal sound absorption coefficient appears when that equal to an integer number multiplication of the 1/2 wavelength. This conclusions have also been obtained by H. Kuttruff(Germen), F.Ingerlev(Danmark) [4,5]

This fact is also the characteristic of the standing wave amplitude in the pipe. It is possible, hence, to assume that the sound absorption coefficient is in direct proportion to the standing wave amplitude. Considering the angular frequency $\omega = 2\pi f$ , and $f = c/\lambda$, ($f, c, \lambda$ representing frequency, sound speed and wavelength respectively),

we have

$$a = A\left|\sin \omega t\right| = A\left|\sin(2\pi ct/\lambda)\right|$$

In the standing wave pipe, if the rigid wall behind the material is taken as a zero point ,then at the position of the material, we get *D* = *ct* (*D* being the distance of the cavity depth). Thus

$$a = A\left|\sin(\frac{2\pi D}{\lambda})\right| = A\left|\sin(2\pi D\frac{f}{c})\right| \qquad (3.1)$$

### 3.2 The conclusions deduced from above formula

(1) An increase in material thickness is equal to an increase in the thickness of air layer behind.

(2) Sound absorption spectra move in the direction of low frequency when the distance of the cavity depth increases.

(3) An increase in the distance of the cavity depth will cause the sound absorption coefficient at low frequency to increase.

(4) Sound absorption spectra move in the direction of low frequency during temperature drop.

The entire above conclusion can be justified by most of the sound absorption literature [1-6].

### 3.3 Relation between maximal sound absorption coefficient and air permeability

The air permeability of fabrics is a term in textile materials, whose definition is the air amount passing through a unit area of the material per second when the pressure difference on both sides is 100*Pa*, the unit being liter/second $\times m^2$ ( $L/(s\cdot m^2)$ ). Fig 3.1 gives the corresponding relationship between the maximal sound coefficient (the sound wave frequency being 400*Hz* and the cavity depth is 20*cm)*and air permeability of materials measured by the Y561 type fabric permeameter [9].

It can be seen from Fig.3.1 that the sound absorption coefficient and air permeability of fabrics and nonwovens become maximal when reaching a given value. As a result, we can work out a computational equation

$$A = 0.9 - \frac{|200 - Q|}{1000} \qquad (3.2)$$

Eq. (3.2) is actually an empirical formula of the maximal sound absorption coefficient $A$ in Eq.(3.1).Thus, a complete formula for calculating the sound absorption coefficient measured by the standing wave pipe has been obtained, that is

$$a = (0.9 - \frac{|200 - Q|}{1000})\left|\sin(\frac{2\pi D}{\lambda})\right| \qquad (3.3)$$

In which $a$ - sound absorption coefficient; $Q$ - air permeability $L/(s\cdot m^2)$; $f, c, \lambda$ express sound wave frequency, sound speed and wavelength respectively.

### 3.4 Conclusions

In the following, taking for example the sound absorption spectra of several materials at various distances of the cavity depth, let's verify the coincidence degree of the computational results from Eq.（3.3）with the actual case. It can be seen from Figs. 3.2 to 3.3that the results worked out in Eq. (3.3）tally well with measured results.

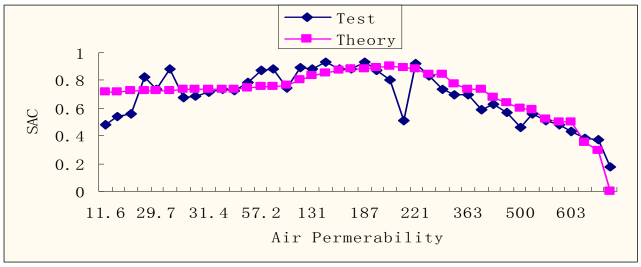



Fig. 3.1 Relation between sound absorption coefficient and air permeability of fabrics and nonwoven cloth with different textures

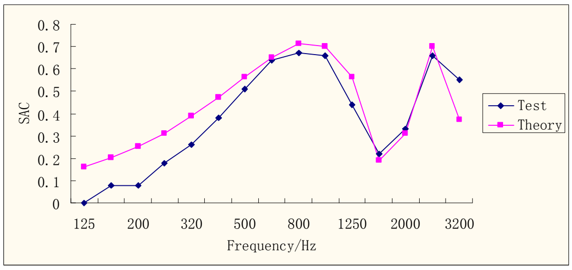



Fig. 3.2 Cotton plain fabric (thickness - 0.39*mm*; surface density - 121 $g/m^2$); cavity depth 10 cm

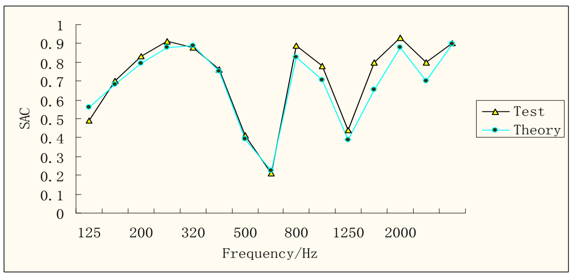



Fig. 3.3 Polyester fiber layer (thickness – 20mm; surface density – 1170 $g/m^2$); cavity depth 30*cm*

## 4 Vibration sound absorption theory

As shown in Fig. 4.1, suppose that the 1/4 wavelength sound wave air with mass $m$ acts on the material, the mass of which is $M$ . Again suppose that the material is a perforated material, its single pore area is $A$ . The material makes back and forth motion when pushed by the sound wave. The moving speed is $v_2$ . According to the kinetic energy conservation law, we have suppose that the sound absorption of the material is derived completely from the forced vibration of the material itself. Then, the sound absorption coefficient can be defined as

$$\alpha = \frac{\frac{1}{2}Mv_2^2}{\frac{1}{2}mv^2} = \frac{M}{m}\left(\frac{v_2}{v}\right)^2 \qquad (4.1)$$

In Fig. 4.1, the fluctuating speed of the material $v_2$ and the airflow speed $v_f$ are equal in magnitude but opposite in direction. In hydromechanics, the relation between the pressure difference on both sides of the pipe and the airflow speed is [4] $p = -\frac{8\eta L v_f}{R^2}$. According to Newton's Second law we have got

$$v_f = v_{af} e^{-\frac{B}{m_{pa}}t} \qquad (4.2)$$

where $v_{af}$ - flow speed maximal value. $B = \frac{8\eta L}{R^2}$

Considering the air mass acting on the material $m = (1-\sigma)m_a\left|\sin(\frac{2\pi D}{\lambda})\right|$ and the phase difference of the vibration speeds between the air particles and the material $\phi = 2\pi(D - \frac{\lambda}{4})/\lambda = \frac{2\pi D}{\lambda} - \frac{\pi}{2}$ ,we have got

$$\alpha = \frac{16 f \rho_0 c_0 M_s v_{af}^2}{(1-\sigma)p_a^2}\left|\sin(\frac{2\pi D}{\lambda})\right| \qquad (4.3)$$

where $M_s = \frac{M}{S}$ is the surface density of the material.

This is the theoretical formula getting from above analysis. If we let $A = \frac{16 f \rho_0 c_0 M_s v_{af}^2}{(1-\sigma)p_a^2}$ ,Eq.(4.3) will totally identical with our empirical formula of the sound absorption spectra formula(3.1).

From the definition of air permeability $Q$ we have $v_{af} = nQ$ .

$$\alpha \approx k\frac{M_s (nQ)^2 f}{(0.51 - 0.0012Q)}\left|\sin(\frac{2\pi d}{\lambda})\right| \qquad (4.4)$$

It can be seen from Eq.(4.4)that, for an certain wavelength $\lambda$, when the air permeability is increasing, in Eq. (4.4), the influence of the air permeability is far greater than the other factors. This conclusion is the same as that reached by the empirical formula of the sound absorption coefficient obtained in section 2.

As per Eq.(4.2), $B/m_{pa}$ can be termed speed attenuation coefficient. As the radius of the micro pores of most of the sound absorption fibers is below 0.1*mm*, the speed attenuation coefficient of most of the fibrous materials is above the magnitude order of $10^4$ - that is, the speed attenuation of the material is exceptionally rapid. This means that actually, the kinetic energy that the material acquires from the sound wave is eventually converted into the friction heat energy of the air in the micro pores of the material. In stand wave tube, if the too larger diameter or perforation of the micro pores of the material gives rise to too large air permeability, the freely vibrating material and micro pores would transfer and transmit the sound wave in large quantities and the sound wave energy might be reflected back by the rigid wall behind the material. In this way, the sound absorption will be lowered. This idea has been justified in Fig.3.1.

With the thin fibrous layer, it produces vibration under the action of the sound wave and the air layer behind is equal to an elastic body. As for the thick fibrous layer, its surface layer material produces vibration under the action of the sound wave and the material behind is also equal to an elastic body. The magnitude of the air permeability dominates the magnitude of the compression module of the elastic body. The greater is the air permeability, the greater the is porosity, the easier is for the material to be compressed and the less is the compression module; the less is the porosity, the more difficult is for the material to be compressed and the greater is the compression module. In this way, the sound absorption coefficient of both of the thin and thick fibrous materials can be determined by the air permeability. This suggests that the two kinds of materials are able to apply the same sound absorption theory.

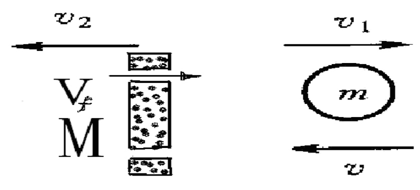



Fig. 4.1 Diagram of action process

# 5 Sound absorption formula of imperforated plate (membrane)

The theory of interaction between the sound wave and material is as shown in Figs 5.1.Under the condition that there is a rigid wall behind, the air layer between the rigid wall and material is equal to a spring when the material is vibrating under the action of sound pressure. The 1/4 wavelength wave air is regarded as a rigid solid, in which we have it's mass $m$ of, speed $v$; material mass $M$. After air collides with the material, we get air speed $v_1$ and material speed $v_2$ .The material sound absorption coefficient can be defined as follows:

$$\alpha = \frac{\frac{1}{2}Mv_2^2}{\frac{1}{2}mv^2} = \frac{M}{m}(\frac{v_2}{v})^2 \qquad (5.1)$$

At the moment air collides with the material, the spring does not work. At this time, the laws of conservation of momentum and conservation of energy are satisfied. We have got

$$\alpha = \frac{4mM}{(m+M)^2} \qquad (5.2)$$

Due to the spring action of the air layer behind the material, the moving speed of the material will be $v_2 = v_{2a}\cos(\omega t)$ .

Considering the air mass acting the materials as well as the resonance frequency, then,

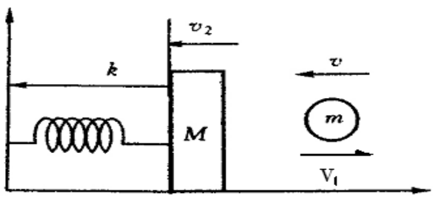
$$\alpha = \frac{4m_s M_s}{(m_s+M_s)^2}\cos^2(2\pi f_0 / f) \qquad (5.3)$$

Where,

$$m_s = \frac{m}{S} = m_{sa}\left|\sin(\frac{2\pi D}{\lambda})\right| = \frac{\rho_0 c_0}{4f}\left|\sin(\frac{2\pi D}{\lambda})\right|$$

$M_s = \frac{M}{S}$ ; $f_0 = \frac{1}{2\pi}\sqrt{\frac{\rho c^2}{MD}} = \frac{60}{\sqrt{MD}}$ in which, $S$ is the acted material area.

Comparing the test results, it is concluded that the tested spectrum of one kind plastic film is agreement with the results calculating from Eq.(5.3)




5.1 Diagram of interaction between the sound wave and material

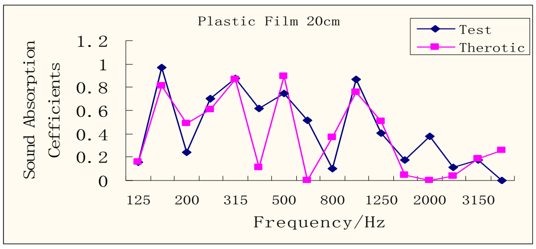



Fig.5.2 Comparison between measured spectrum and theoretical spectrum (20cm cavity depth) of the plastic film (thickness 0.12mm and surface density 0.08 $kg/m^2$ )

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