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A numerical study of the onset of granular avalanches

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In this contribution, we address the problem of interpreting the occurrence of precursors to avalanching in the case of tilted granular beds. Discrete numerical simulations are carried out in which a granular slope is slowly driven towards instability by a slow rotation in the gravity field. Focussing on the mobilization of friction forces, we show the existence of a stick-slip dynamics developing far from slope failure and generating precursors. These precursors become sparser close to the avalanche but of increasing amplitude. Evaluating the work dissipated by their occurrence, we observe that it grows exponentially with the slope. Although no time correlations could be evidenced between precursors occurrence, their amplitude is shown to obey a well defined distribution. Moreover, space correlations in the behavior of the grains reveal a slow structuring of the granular packing.

1 Introduction

Omnipresent in nature and in industrial processes, granular matter is of paramount interest in many fields, from civil engineering to physics, mechanics or geophysics. Yet, despite its central importance and the amount of research devoted to the subject, the behavior of granular matter still escapes modeling. This is for instance the case of the transition from static equilibrium to rapid flow, as the trigger of a surface avalanche. Theoretical models relying on analogy with phase transition proved to capture parts of the behavior [1]. Experimental works showed that the transition can be characterized by a meta-stable state and a sub-critical bifurcation [2]. Closer inspection of the grains behavior using numerical simulation allowed for the characterization of the meta-stability interval preceding the avalanche in terms of mobilization of friction at contacts and precursors occurrence [3]. Yet, although better understanding was achieved through these various works, predicting the destabilization of a granular slope remains as problematic as ever. An interesting perspective in this direction is the analysis of the acoustic signal emitted by a granular slope evolving towards avalanching. This would permit the identification and interpretation of the precursors, provided the latter have a clear acoustic signature. However, such experimental studies are still highly challenging.
In this contribution, we address the problem of interpreting the occurrence of precursors to avalanching in the case of tilted granular beds. Discrete numerical simulations are carried out in which a granular slope is slowly driven towards instability by a slow rotation in the gravity field. Focussing on the mobilization of friction forces, we show the existence of a stick-slip dynamics developing far from slope failure and generating precursors. These precursors become sparser close to the avalanche but of increasing amplitude. Evaluating the work dissipated by their occurrence, we observe that it grows exponentially while the slope approaches the limit angle of stability. Although no time correlations could be evidenced between precursors occurrence, their amplitude is shown to obey a well defined distribution. Moreover, space correlations in the behavior of the grains reveal a slow structuring of the granular packing.

2 Numerical simulations of the onset of a granular avalanche

2.1 The Contact Dynamics Method

The simulations were performed using the contact dynamics method (in 2D) which assumes infinitely rigid disks interacting through collisions and frictional contacts [4]. The equations of motion are solved for each grain taking into account each of his neighbors with which a contact exists. The contact laws are non-smooth and require an implicit solving of the contact forces acting between the grains. In addition, a coefficient of restitution e is introduced to control the amount of energy lost in the advent of a collision between grains.
The hard core assumption (ie the fact that the grains are infinitely rigid) implies that the granular system cannot accommodate external loading through the deformation of the disks, but only through local rearrangements of the packing geometry. In this process, the existence of friction forces at contact will be of major consequences: these forces control the possibility of the occurrence of slip motion between grains. The friction contact law is displayed in Figure 1: the Coulomb threshold is defined as the product of the normal force at the contact N and the coefficient of friction μ. When the tangential force at contact T is below this threshold, no slip motion is possible; on the contrary, when the threshold is reached, slip motion is incipient.

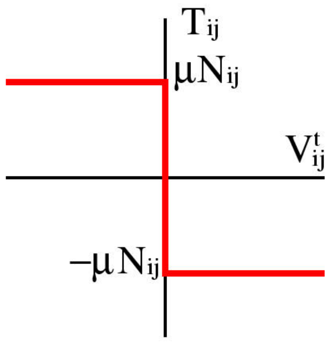

Figure 1: Coulomb friction law controlling slip motion at contacts between two grains i and j: N_{ij} and T_{ij} are the normal and tangential forces at contact respectively, μ is the coefficient of friction and V^t_{ij} is the tangential relative velocity.

2.2 Setup

The numerical experiment consists of tilting a granular bed in the gravity field so as to slowly bring the slope of the free surface to the instability angle (denoted θ_{start} in the following) for which a surface avalanche is triggered. The tilting velocity is $\dot{\theta} = 0.2° s^{-1}$. The beds are prepared by random deposition of 4800 grains in the gravity field. Their diameter is uniformly distributed in a small interval ($d_{min}/d_{max} = 2/3$) to introduce grains size disparity and disorder; the mean grains diameter is denoted d, their mass is denoted m. The beds are periodic in the horizontal direction, of length $L = 100d$ and height $H = 48d$; an illustration is given if Figure 2.

In the following, we analyse the state of the granular packings while evolving towards avalanching, and more specifically the existence of precursors to the avalanche. Therefore, 20 independent runs were performed, with $e = 0.5$ and $\mu = 0.3$.

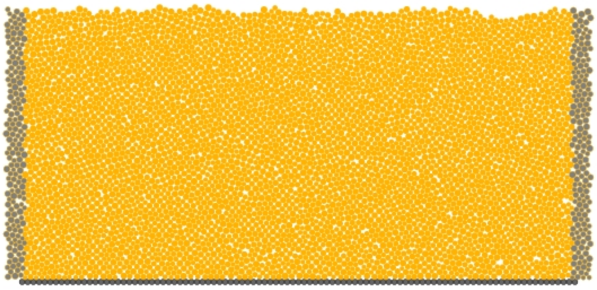

Figure 2: Example of a simulated granular bed of 4800 grains and periodic boundary conditions.

3 Stick-slip dynamics and precursors

While the granular beds are tilted, friction forces are gradually mobilized at contacts between grains. In order to be able to study this evolution, we focus on the contacts where the friction forces have reached the Coulomb threshold, and thus where slip motion is incipient; in the following, they will be referred to as critical. We define their density as the number of critical contacts to the total number of contacts in the packing: the latter is denoted ν.

Figure 3a shows a typical evolution of ν in the course of the tilting. The corresponding evolution of the mean translational velocity of the grains is shown in Figure 3b. The start of the avalanche can be located at $\theta_{start} \simeq 17.9°$. Before its onset, we observe a succession of peaks of ν showing periods of mobilization of friction forces at contacts followed by quick relaxation periods, characteristic of a stick-slip dynamics. Relaxation of the mobilization of friction forces coincides with peaks in the evolution of the mean grains velocity, giving evidence of the existence of dynamical rearrangements induced by slip motion at contacts. Similar observations were already reported in [3]. We thus observe here strong precursors to the avalanche event, comparable to the acoustic signal measured in [5] in experiments of granular shearing.

Considering now the set of 20 independent runs, we plot in Figure 4a each evolution of ν as a function of $\theta_{start} - \theta$, together with the corresponding average. We can see clearly that in the mean, the density of critical contacts ν increases at the approach of the avalanche; however, for each granular system, ν fluctuates far from this average (see Figure 4b). We identify the precursors to the avalanche as the fluctuations of ν regarding the mean: for each fluctuation, the size of the corresponding precursor is defined as the maximum deviation of ν from the average.

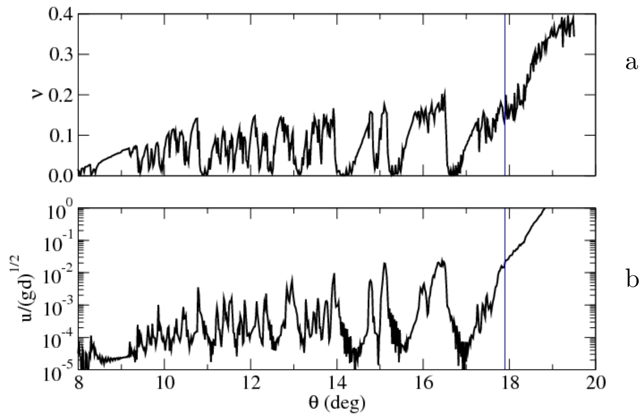

Figure 3: **a**: Example for one single run of the evolution of the density of critical contacts ν as a function of the slope angle θ and **b**: corresponding evolution of the normalized mean velocity of the grains.

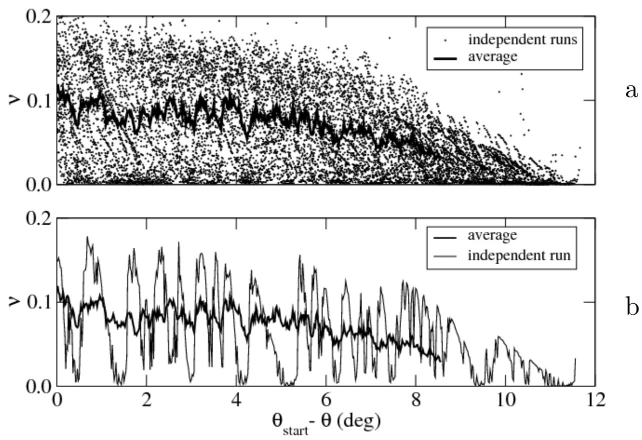

Figure 4: **a**: Evolution of the density of critical contacts ν for 20 independent runs (dots) and the corresponding average (full line) as a function of $\theta_{start} - \theta$; **b**: Example of the evolution of ν for a single run (normal line) and the average over 20 runs (bold line) as a function of $\theta_{start} - \theta$ (deg).

Analyzing each of the independent run, we can attempt a statistical picture of the occurrence of precursors. Essentially, the questions arising are: is the size of a given precursor dependent on the size of previous occurrences, or on their location in time? is the frequency of precursors, or their size, dependent on the proximity of the avalanche? The latter question is easily answered. Figure 5a shows the probability P of occurrence of a

precursor p as a function of $\theta_{start} - \theta$: the closer to the trigger of the avalanche, the sparser the occurrence of precursors. On the other hand, plotting the size A of a precursor p as a function of $\theta_{start} - \theta$, we observe that the precursors become of greater amplitude closer to the avalanche (Figure 5b). In other words, the evolution of a granular bed towards avalanching is characterized by frictional induced dynamical events less and less probable but of increasing size.
As for the time and size correlations between the precursors themselves, no clear tendency could be evidenced.

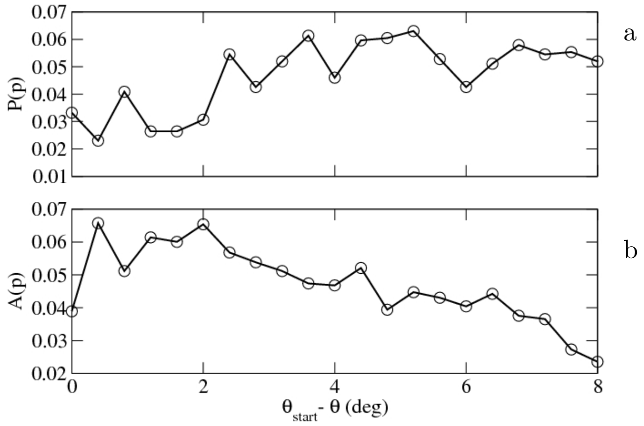

Figure 5: **a:** Probability P of occurrence of a precursor p and **b:** mean size A of precursors as a function of $\theta_{start} - \theta$ (deg).

4 Energy released by precursors

Focusing on the evolution and behavior of the density of critical contacts allows for the disclosure of the existence of friction induced precursors. However, a more accurate description of the evolution towards avalanching can be attempted when considering the amount of energy dissipated during the occurrence of these precursors. A possible approximation is achieved through the evaluation of the work of the friction forces at contacts in the advent of slip motion:

$$W = \sum_i T_i \times d\ell_i, \quad (1)$$

where the $T_i = \mu N_i$ and $d\ell_i$ are respectively the tangential force and tangential displacement at the contact i. Figure 6 shows the evolution of W normalized by mgd as a function of $\theta_{start} - \theta$ for a single run: we observe the occurrence of bursts of increasing size while the avalanche becomes incipient. Plotting on the same graph the behavior of W averaged over all the independent runs, we observe an exponential increase at the approach of the avalanche (with an exponent of $\simeq 0.5$).

We can thus characterize the evolution of the granular bed through the distribution of values taken by W for each independent run. Since W varies over several orders of magnitude, we rather consider the logarithm of the normalized work $w = \log_{10}(W/mgd)$. The distribution $P(w)$ is plotted in Figure 7 for all the runs and over the slope interval $[\theta_{start} - 8°, \theta_{start}]$. We observe a peaked distribution, but with a wide interval of

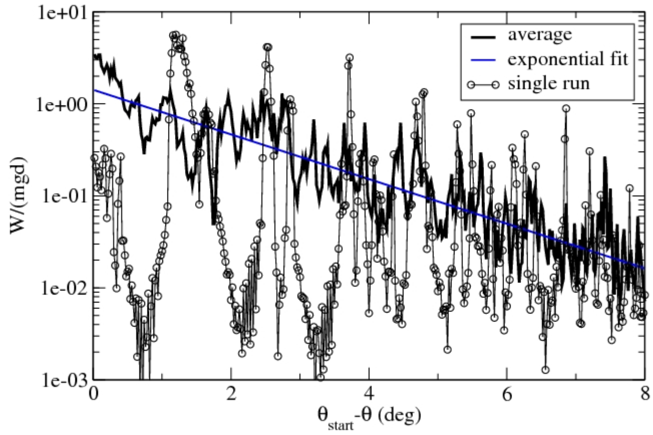

Figure 6: Evolution of the normalized work performed by friction forces as a function of $\theta_{start} - \theta$ (deg), for a single run, and averaged over 20 independent runs.

w obeying an exponential decrease: for $w \in [-2, 1]$, the probability P of having an event of size w obeys the following law:

$$\ln(P(w)) = a - bw,$$

where $b = 1.04$, expressing the fact that the probability of an event is exponentially decreasing with its size.

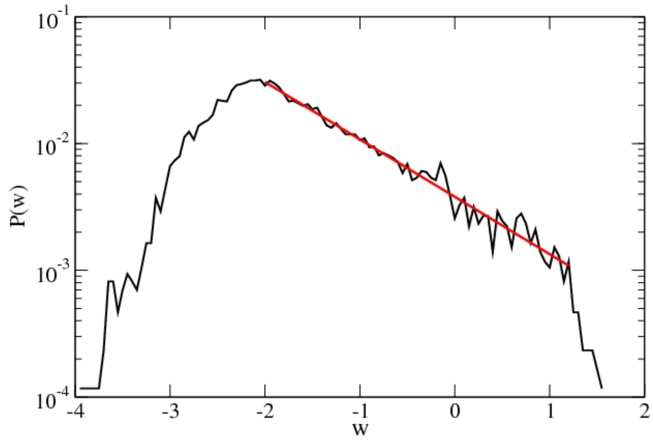

Figure 7: Distribution P of the possible values of the logarithm of the normalized work $w = \log_{10}(W/mgd)$ over the slope interval $[\theta_{start} - 8°, \theta_{start}]$.

5 Space structuring

As previously stated, the existence of time correlations in the occurrence of precursors to the avalanche could not be evidenced; however, we can show the existence of space correlations. Since precursors are induced by the mobilization of friction forces at contacts, and hence dissipate energy essentially through the subsequent slip motions, we will focus on the cumulative rotation experienced by each grain in the course of the tilting. Lets denote ω this quantity. Figure 8 shows snapshots of several granular beds close to avalanching, with the value of ω for each grain encoded in gray. In spite of the irregularity of the pattern, we can see the existence of clusters where the rotation of the grains is maximum, pointing out areas where precursors are expected to be the most probable. We define a pair correlation function for the value of ω for the grains just before the avalanche starts:

$$F(r) = \frac{\sum_{i,j} \omega_i \omega_j \Pi(x_{ij} - r)}{\sum_{i,j} \Pi(x_{ij} - r)}, \qquad (2)$$

where $\Pi(x)$ is a step function taking the value 1 where $|x| < d/2$, and 0 otherwise, and x_{ij} is the distance between the two grains i and j. Averaging over all the runs, we obtain for F the behavior shown in Figure 9. Approximating it by an exponential decrease (which can be achieved with a good correlation coefficient as long as $r < 10d$) gives a typical length scale $\lambda = 3.7d$ characterizing the spatial distribution of ω. The existence of precursors to the avalanche can thus be related to a slow structuring of the granular beds, in which zones where equilibrium is precarious slowly build up.

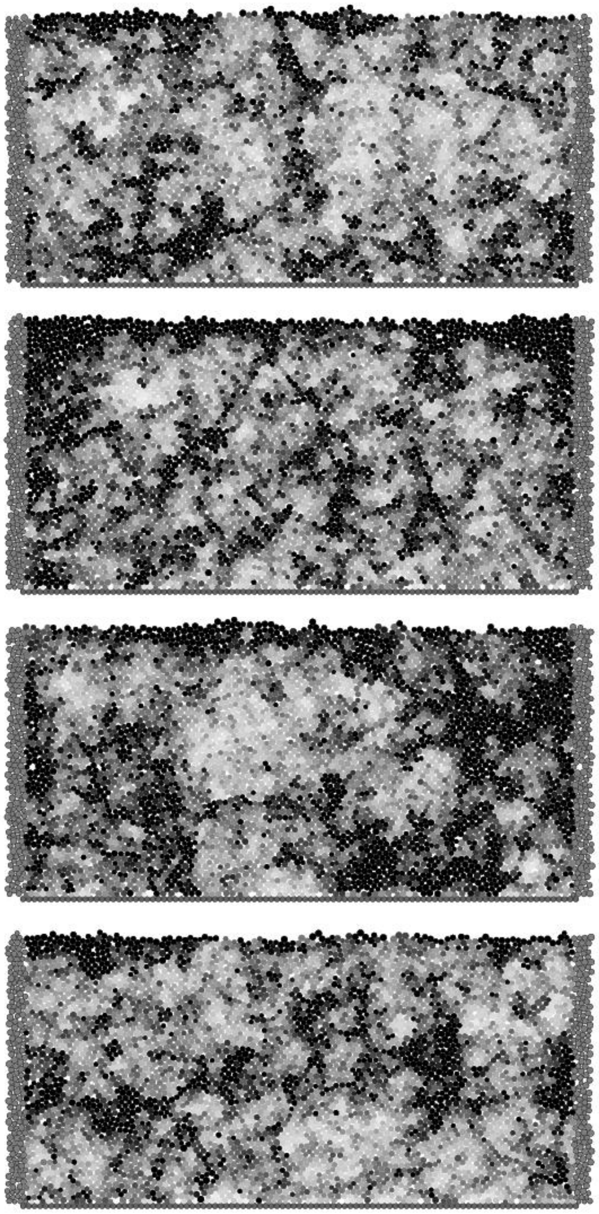

Figure 8: Snapshots of four granular beds close to avalanching; the black color indicated large cumulated rotation ω of the grains. The gray scale is linear, with an upper cut-off.

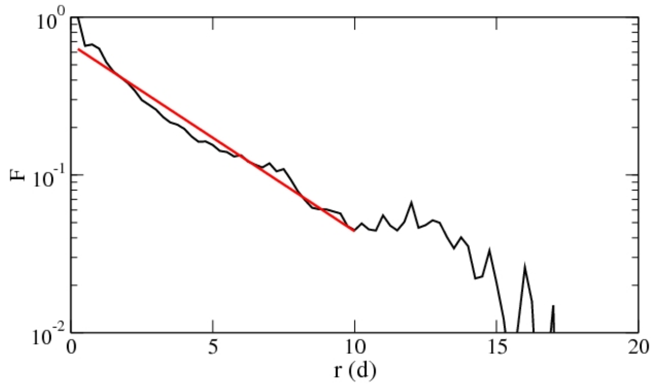

Figure 9: Pair correlation function $F(r)$ of the grains rotation ω averaged over all independent runs and exponential fit (slope 0.27).

6 Conclusion

The simulations show that the behavior of the precursors is affected by the proximity of the avalanche both in size and frequency. Moreover, the energy released by the occurrence of precursors obeys a well defined distribution, the shape of which should be more closely studied in relation to the proximity of the avalanche. Therefore, better statistic sampling would be needed, pointing out the necessity of performing a greater number of simulations. This would as well allow for a better characterization of the structuring of the granular packing, in which areas where local instabilities are more probable gradually build up. Finally, an essential question in the perspective of predicting avalanches and more generally failures, is the influence of the loading mode on the shape of the precursors. The identification of the relevant features describing the proximity of the destabilization irrespective of the type of loading would bring new prospects in the possibility of predicting failure.

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