Using wavelet transform to locate acoustic emission source in composite plate with one sensor

Amir Refahi Oskouei and Mehdi Ahmadi

Polytechnic of Tehran, AmirKabir University of Technology, Department of Mechanical Engineering, NDT Lab., 424 Hafez Ave, 15875-4413 Tehran, Iran
amir.refahi@cic.aut.ac.ir

In this paper a new method for locating the AE source with one sensor is presented. When acoustic emission signals propagate in wave-guided, they have multi-mode and dispersion characteristics. The separation of the modes at the sensors could make it possible to extract the exact information about the source that produced the waves. Based on the modal nature of AE, it can be understood that a good location would have two methods to determine the arrival times. One is determined on the same part of the waves (not only the same mode, but also the same frequency) at all sensors. The other is determined on the different modes at one sensor, which makes it possible to reduce the number of sensors needed. First, through modal analysis of the propagation of elastic waves in a thin plate, the dispersion characteristics of the modes are predicted. Second, the wavelet transform theory is briefly outlined and its application in elastic waves is explained. It is shown that by using the peak of the magnitude of the wavelet transform, the arrival times of the different modes can be determined. Additionally, experiments were undertaken using a lead break on the edge of the plate. These demonstrated the feasibility of the one sensor linear location scheme.

1 Introduction

The acoustic emission technique (AE) has for many years been considered as the prime candidate for structural health and damage monitoring in loaded structures, such as liquid petroleum gas tanks, fire extinguishers, oxygen gas tanks, etc. Source location has always been regarded as one of the main advantages of AE testing. Based on arrival time differences at a limited number of sensors and the velocity of the waves, the spatial location of the source event can be calculated. The key element in the location procedure is the determination of the arrival times of the AE waves at the different sensors [1-3]. Traditional AE systems determine arrival times by using a threshold: the arrival time of a wave at a sensor is the point where it first crosses the threshold. An attempt to provide a better theoretical background for AE testing is modal acoustic emission (MAE). MAE starts from the observation that AE waves are mechanical in nature and should therefore be treated as such. Following the general theory of wave propagation in solids, AE waves should propagate through a structure in a variety of modes, and also have the characteristics of dispersion and attenuation. The separation of these modes at the sensors could make it possible to extract the exact information about the source that produced the wave. The propagation characteristics of Lamb waves in composites, with emphasis on group velocity and characteristic wave curves, are investigated theoretically and experimentally by Wang et al that is related with MAE [4].

Based on the modal nature of AE, it can be understood that a good location would have two methods to determine the arrival times. One is determined on the same part of the wave (not only the same mode, but also the same frequency) at all sensors. The other is determined on the different modes at one sensor, and makes it possible to reduce the number of sensors needed.

Recently, the wavelet transform (WT) has been introduced to the time–frequency representation of transient waves propagating in a disperse medium for its good resolution both in the time and frequency domains, and it can be used to detect the arrival times of dispersive waves propagating in structures. Jiao et al [5] applied the WT using the Gabor wavelet to the time-frequency analysis of modal acoustic emission source location in thin plates. Sung et al [6] used WT to monitoring of impact damages in composite laminates. Ni et al [7] correlated the Wavelet transform of acoustic emission signals to fracture mechanisms of composites. Qi [8] developed wavelet-based acoustic emission (AE) analysis methods to characterization of composite material.

In this paper the MAE and WT are used to find the acoustic emission source location in thin plate composite materials.

2 Wavelet Transform

The continuous WT of f(t) is a function defined as [9,10]

$$WT_f(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t)\psi^*\left(\frac{t-b}{a}\right)dt \qquad (1)$$

Where a>0 and the superscript * denotes a complex conjugation. The analysis function for the WT can be defined as

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) \qquad (2)$$

Its elements are generated by shifting and scaling a basic wavelet $\psi(t)$. The parameters a and b stand for the scale and shift of the basic wavelet. In time, $\psi_{a,b}(t)$ is centred at b with a spread proportional to a. In this article, the relation between the scale variable a and the frequency ω is $\omega = \omega_0 / a$, where ω_0 is a positive constant. As such, the function $\psi_{a,b}(t)$ may be considered as a window function both in the time and frequency domains. The size of the time window is controlled by scale a: It is possible to change the window size either in the time or frequency domain. This multi resolution is a primary characteristic of the wavelet analysis. A basic wavelet satisfies the admissibility condition

$$\int_{-\infty}^{+\infty} \frac{\left|\hat{\psi}(\omega)\right|^2}{|\omega|} d\omega \prec \infty \qquad (3)$$

Where $\hat{\psi}(\omega)$ denotes the Fourier transform of $\psi(t)$. Although one can choose any basic wavelets that satisfy the admissibility condition (3), the Gabor function was adopted in this study because it is known to provide a better resolution both in time and frequency domains than any other wavelets. The Gabor function is

$$\psi_g(t) = \frac{1}{\sqrt[4]{\pi}} \sqrt{\frac{\omega_0}{\gamma}} \exp\left[-\frac{(\omega_0/\gamma)^2}{2} t^2\right] \exp(i\omega_0 t) \qquad (4)$$

and its Fourier transform as

$$\hat{\psi}_g(\omega) = \frac{\sqrt{2\pi}}{\sqrt[4]{\pi}} \sqrt{\frac{\gamma}{\omega_0}} \exp\left[-\frac{(\gamma/\omega_0)^2}{2} (\omega - \omega_0)^2 \right] \quad (5)$$

Where γ and ω_0 are positive constants. Although the Gabor function does not satisfy condition (3), it does not present any problem in practice if we set $\gamma = \pi\sqrt{2/\ \ln 2} = 5.336$. The Gabor function (4) may be considered as a Gaussian window function centered at $t = 0$ and its Fourier transform (Eq. (5)) centered at frequency $\omega = \omega_0$. The function $\psi_g((t-b)/a)$ is then centered around $t = b$, and its Fourier transform $\left[a \exp^{(-ib\omega)} \hat{\psi}_g(a\omega) \right]$ is centered around $\omega = \omega_0/a$. The $WT_f(a,b)$ using the Gabor wavelet thus represents the time–frequency coefficients of $f(t)$ around $t = b$ and $\omega = \omega_0/a$. In this study, we set $\omega_0 = 2\pi$ such that $1/a$ is equal to the usual frequency $f = \omega/2\pi$.

3 Time–frequency analysis of dispersive waves

For the time–frequency analysis of dispersive waves, consider two harmonic waves of unit amplitude and of slightly different frequencies ω_1 and ω_2 propagating in the x-direction

$$u(x,t) = e^{-i(k_1 x - \omega_1 t)} + e^{-i(k_2 x - \omega_2 t)} \quad (6)$$

where k_i are the wave numbers corresponding to the frequency ω_i. Introducing

$$\begin{aligned} (k_1 + k_2)/2 &= k_c & (\omega_1 + \omega_2)/2 &= \omega_c \\ (k_1 - k_2)/2 &= \Delta k & (\omega_1 - \omega_2)/2 &= \Delta\omega \end{aligned} \quad (7)$$

Eq. (6) can be written as

$$u(x,t) = 2\cos(\Delta kx - \Delta\omega t) e^{-i(k_1 x - \omega_1 t)} \quad (8)$$

It can be seen that this resulting wave is comprised of two parts. The carrier wavelet represents, by the exponential term, propagation with the phase velocity $c_p = \omega_c/k_c$. On the other hand the modulated wave given by the cosine term travels with the group velocity $c_g = d\,\omega/dk$ in the limit $\Delta k \to 0$. When the Gabor wavelet is used as the basic wavelet, the WT of $u(x,t)$ is given by

$$\begin{aligned} WT_u(x,a,b) = \sqrt{a} \Big\{ & e^{-i(k_1 x - \omega_1 b)} \psi_g^*(a\omega_1) \\ & + e^{-i(k_2 x - \omega_2 b)} \psi_g^*(a\omega_2) \Big\} \end{aligned} \quad (9)$$

The magnitude of the WT obtained is

$$\begin{aligned} |WT_u(x,a,b)| = \sqrt{a} \Big\{ & \left[\hat{\psi}_g(a\omega_1) \right]^2 + \left[\hat{\psi}_g(a\omega_2) \right]^2 \\ & + 2\hat{\psi}_g(a\omega_1)\hat{\psi}_g(a\omega_2)\cos(2\Delta kx - 2\Delta\omega b) \Big\}^{1/2} \end{aligned} \quad (10)$$

If $\Delta\omega$ is sufficiently small such that $\hat{\psi}_g(a\omega_1) \approx \hat{\psi}_g(a\omega_2) \approx \hat{\psi}_g(a\omega_c)$, we obtain

$$|WT_u(x,a,b)| = \sqrt{2a} \left| \hat{\psi}_g(a\omega_c) \right| [1 + \cos(2\Delta kx - 2\Delta\omega b)]^{1/2} \quad (11)$$

This result indicates that the magnitude of the WT takes its maximum value at $a = \omega_0/\omega_c$ and $b = (\Delta k/\Delta\omega)x = x/c_g$ In other words, the location of the peak on the (a,b) plane indicates the arrival time of the group velocity c_g at frequency $\omega_c = \omega_0/a$ i.e. $f = 1/a$.

4 Dispersion relation of plate waves

In order to locate the AE source, the group velocity of the considered elastic wave modes at certain frequency must be known According to the theory of MAE, the AE signals are actually dispersive elastic waves. The propagation of the elastic waves in thin-walled structures exhibits multi-modes and dispersion characteristics, and have been studied by Achenbach, Graff and many others [11–13]. Fig. 1 is the disperse curve of theoretical prediction and experimental measurement of Lamb waves in the relatively thick laminate [+45°/-45°]s. Over the frequency range 0–250 kHz, there only exist two modes A0 and S0. Near 150 kHz, the group velocities of A0 and S0 are 1.87 and 5.8 m/ms, respectively. It is convenient from the disperse curves to get the velocity of any mode under the frequency range concerned.

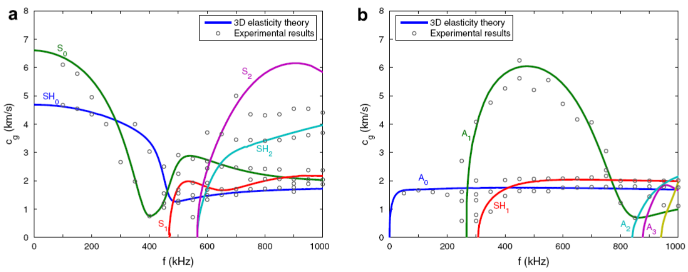


Fig.1 Theoretical and experimental results of group velocity dispersion curves traveling along $\theta = 30^\circ$ in the laminate $[+45^\circ/-45^\circ]_s$: (a) c_g of symmetric modes; (b) c_g of anti-symmetric modes.

5 One sensor AE source location test

A schematic representation of the experimental set-up is shown in Fig. 2. The source location was carried out on a composite plate, with dimensions 700 mm x 700. Lead breaks were used as the simulated AE source. For the amplitudes of the plate waves depended on the source orientation [14], the lead was broken on a certain angle to produce both the flexural modes and extensional modes. One broadband piezoelectric sensor was coupled to the surface of the plate with vacuum grease. A two-channel analyzer (PCI-2) was used for waveform acquisition. A PAC preamplifier, model 1220A, with a 40 dB gain and a 100–300 kHz bandwidth was used for signal conditioning. A wide band sensor, model PAC WD, with an operational range of frequencies between 100 and 1000 kHz and a resonant frequency at about 125 kHz was used. The surface of the sensor was covered with silicon grease in order to provide good acoustic coupling between the specimen and the sensor. The acoustic emission software used was the AEWin from Physical Acoustics Corporation 2006. The acquired waveforms were then stored on a computer for further analysis and display.

Fig. 2 also shows the sensor arrangement. For source location, the distance between the sensor and the lead break is unknown (Table 1). As mentioned previously, the location of the peak of the WT magnitude on the (a,b) plane corresponds to the arrival time $b = x/c_g$ of the wave on the frequency of $f = 1/a$. Therefore the distance between the sensor and lead break source can be calculated according to Eq. (12)

$$L = (b_{a0}(f) - b_{s0}(f))(c_{ga0} - c_{gs0}) \qquad (12)$$

The integral in Eq. (1) was evaluated numerically using the trapezoidal rule with a time step equal to the sampling rate Δt_s.

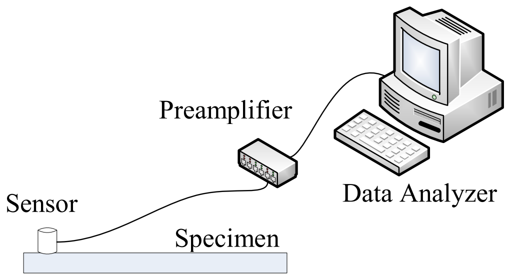


Fig.2 Setup configuration

Test number	The test distances between sensor and lead (mm)	The measured distances between sensor and lead (mm)	Distane error (mm)	Fractional error (%)
1	116.2	120	3.8	3.16
2	147.4	150	2.6	2.64
3	177.2	180	2.6	2.64
4	205.4	200	5.4	2.7
5	212.6	220	7.4	3.36
6	245.4	250	4.6	1.84

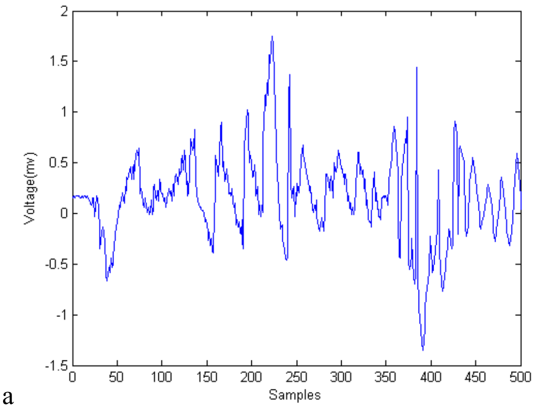
Table1 Result of one sensor leads breaks location experiments

The parameters a and b were discrete as follows

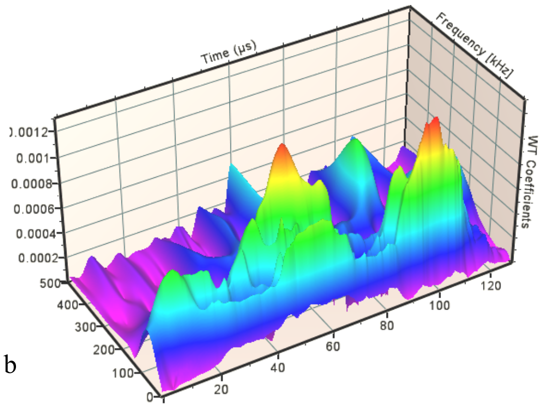
$$a = 1/(40000 + 5000n) \qquad\qquad b = n\Delta t_s$$

The lead was broken on the plate to excite acoustic waves where both the A0 and S0 modes are dominant. A typical waveform was shown in Fig. 3, which was collected by a sensor placed at some distance. The magnitude of the in-plane and out-of-plane components of these modes depends on the source orientation. As shown in Fig. 3(a), the two modes were separated in time for the different group velocity, and the separation time was in direct proportion to the distance between the sensor and the lead break source.

Fig. 3(b,c) showed the time-frequency distribution of the magnitude of the WT. The broken curve in the contour plot indicated the location of the peak of the magnitude for each value of frequency. As mentioned before, each peak represented the arrival time of a wave traveling with the group velocity. To determine the arrival time of simulated AE signals, the frequency point of 150 kHz was used to analyses. The frequency was chosen because the spectral analysis showed that it was a frequency that both S0 and A0 modes were typically present in the captured waveforms. The magnitudes of the WT at 150 kHz was shown in Fig. 3. This figure represented the variation of the spectral intensities of the 150 kHz components as a function of time. The peaks corresponded to the arrival time of the different modes at 150 kHz. Thus, the arrival time difference between S0 and A0 modes was easily determined.



a



b

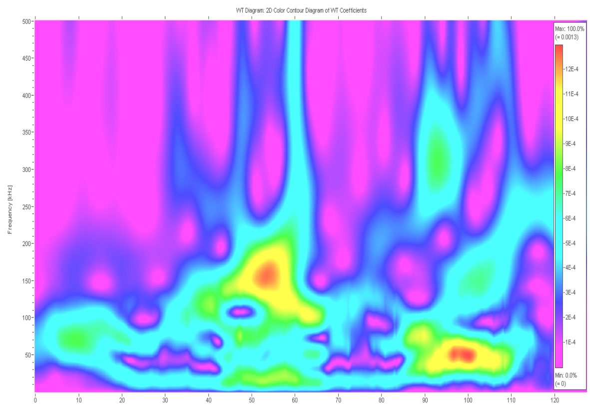

c

Fig.3 One sensor source location test waveform and its wavelet transform.

In Fig. 3 the arrival times of A0 and S0 modes were 52.6 and 110.2 μs, respectively. As mentioned above the group velocity of A0 and S0 modes were 1.87 and 5.8 m/ms, respectively. Therefore, the distance between the sensor and lead can be calculated according to Eq. (12). The result is 226.4 mm, which is close to the measured distance of 250 mm. Repeated experiments also demonstrated good agreement between the calculation and measurement, as shown inTable1 and the location errors is usually less than 5%.

6 Conclusions

The results of a lead break location in a thin plate have demonstrated the feasibility of the one sensor source location concept, based on the wavelet transform and modal analysis of mechanical waves. The advantages of this type of source location concept in practical applications are twofold. First, it decreases the numbers of sensors needed, thus contributing to a reduction in the cost of AE testing.

Second, it can offer a solution in cases where only limited access is possible to a structure and traditional location techniques are not feasible.

In this article, the method of the wavelet transform and modal analysis is firstly used for linear location, and a broader application is in prospect. It is believed that further investigation into the possibilities of this type of analysis will lead to more intelligent and quantitative AE results.

Acknowledgements

The author wishes to thank the Department of Mechanical Engineering at Amirkabir University of Technology for providing facilities to carrying out the tests.

References

[1] C. Jomdecha, A. Prateepasena, and P. Kaewtrakulpong, V. Krum, "Study on source location using an acoustic emission system for various corrosion types*", NDT & E International.* 40, 584-593 (2007)

[2] Efstratios M. Lympertos and Evangelos S. Dermatas, "Acoustic emission source location in dispersive media*", Signal Processing.* 87, 3218-3225 (2007)

[3] Matthew Geoffrey Baxter, Rhys Pullin, Karen M. Holford, and Sam L. Evans**,** "Delta *T* source location for acoustic emission*", Mechanical Systems and Signal Processing.,* 21, 1512-1520, (2007)

[4] Lei Wang, F.G. Yuan, 'Group velocity and characteristic wave curves of Lamb waves in composites: Modeling and experiments", *Composites Science and Technology*, 67, 1370–1384(2007)

[5] Jingpin Jiao, Cunfu He, Bin Wu, Renyuan Fei, Xiuyan Wang, "Application of wavelet transform on modal acoustic emission source location in thin plates with one sensor", *International Journal of Pressure Vessels and Piping,* 81, 427–431 (2004)

[6] Dae-Un Sung, Chun-Gon Kim, Chang-Sun Hong, "Monitoring of impact damages in composite laminates using wavelet transform", *Composites Part B: Engineering,* 33, 35-43, (2002)

[7] Qing-Qing Ni, Masaharu Iwamoto, "Wavelet transform of acoustic emission signals in failure of model composites*", Engineering Fracture Mechanics.* 69 717–728 (2002)

[8] G. Qi, "Wavelet-based AE characterization of composite materials", *NDT&E International.* 33, 133–144 (2000)

[9] C. Sidney Burrus, "Introduction to wavelets and wavelet transforms: a primer", *Prentice Hall*, (1998)

[10] Y. Nievergelt, "Wavelets made easy", *Birkhauser*, (1999)

[11] J. D. Achenbach, "Wave Propagation in Elastic Solids", Elsevier, (1975)

[12] K. F. Graff, "Wave motion in elastic solids", (1975)

[13] A. H. Harker, "Elastic waves in solids with application to nondestructive testing of pipelines", (1998)

[14] M. Ohtsu and K. Ono, **"**AE source location and orientation determination of tensile cracks from surface observation*", NDT International*, 21, 143-150, (1988)