Forced vibrations of a cylindric panel with regular orthogonal system of stiffeners

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Forced vibrations of a cylindrical panel freely supported in contour, with regular orthogonal system of stiffeners (typical of aircraft fuselage structure) are investigated. The connection of all three components of displacement of the shell and of discrete stiffeners as well as of the respective elastic and inertial forces and moments are taken into account in the case of excitation by normal and tangential forces. This connection can be described correctly at application of the method of space-harmonics. The task of forced vibrations is solved directly except the procedure of solving the task in terms of eigen-values. Some illustrative examples of the aircraft fuselage panel vibration at different excitation fields are presented. They demonstrate a high efficiency of applying this method. Potential possibilities of using this method for solving the tasks related to vibrations of a framed shell modelling the aircraft fuselage section and to the sound field in the volume bounded by it are considered.

1 Introduction

A shell or a panel regularly strengthened by stiffeners in two orthogonal directions is the foundation of a wide range of engineer constructions, in particular of aircraft fuselage. A great number of works is known to exist which are devoted to vibrations of stiffened plates, shells and to excitation of periodical structure which are elucidated in reference [1-3] and in recent publications [4-8].

In the majority of work single-dimensional or quasi-single-dimensional systems stiffened in one direction are considered. Unfortunately, the number of works where stiffening in two directions is considered and which are of prime practical interest are rather limited. Here one can note works [5,6].

Work [5] considers the task related to eigen-frequency density of infinite plate with a regular orthogonal system of stiffeners. However, the relations obtained in the work cannot be directly used for determining the vibration velocities. The vibrations of cross-stiffened aircraft fuselage fragment are predicted in work [6] but without account for the connection between the tangential components of the shell and stiffener displacement. Moreover, the method proposed in [6] is not efficient for the regular systems and substantially limits the possibility of practical evaluations of vibrations in the high-frequency region.

The present work proposes the efficient analytical method based on space harmonic expansions [1] for predicting the forced vibrations of cylindrical panels regularly stiffened in two directions. The method is realized in the cases when the boundary conditions for a limited constructions permit considering it as a part of the infinite one. The solution obtained permits determining all the components of vibration velocities directly. In this case there is no necessity of preliminary solving the tasks on eigen-values.

This method takes into account an interconnection between three components of panel and stiffener displacement, stiffener responses to bend and torsion. Division of all the vibration forms accounted in the prediction into independent groups and reduction of the solution to the system of equations relative to stiffener responses permit predicting the vibrations for large aircraft fuselage fragments practically over the whole sound frequency range.

2 Prediction relations

Consider a limited thin cylindrical panel regularly stiffened by frames (or rings) in the circular direction and by stringers in the longitudinal one (Fig.1). The panel is freely supported on the edges. Let the panel of radius R consist of N^r spans of d^r length between the frames with N^s cells of d^s width between the stringers in each span.

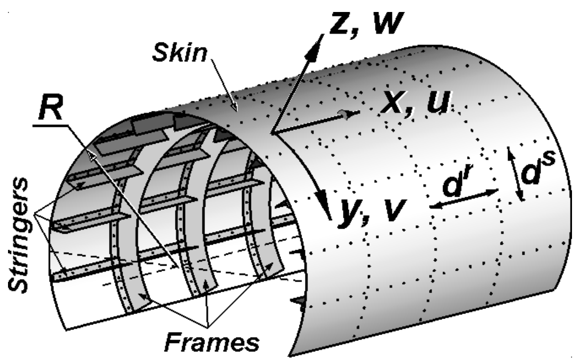


Fig. 1 Regularly stiffened cylindrical panel.

Three panel displacement components are connected with external distributed forces through vibration equations which can be written as follows:

$$\mathbf{Lw} = \mathbf{q}, \quad \mathbf{w}(x,y) = \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \ \mathbf{q}(x,y) = \begin{pmatrix} q_u \\ q_v \\ q_w \end{pmatrix}, \qquad (1)$$

where $\mathbf{L}$ is the elasto-inertial matrix shell operator with dimension 3×3. Here and below time multiplier $\exp(\mathrm{i}\omega t)$ is omitted.

For stringers (s) and frames (rings) (r) the vibration equations connect three displacement components and the angle of turning with the force vector and the momentum and this can be also presented in the matrix form:

$$\mathbf{L^s w^s} = \mathbf{q^s}, \quad \mathbf{w^s}(x) = \begin{pmatrix} u^s \\ v^s \\ w^s \\ \theta^s R \end{pmatrix}, \ \mathbf{q^s}(x) = \begin{pmatrix} q_u^s \\ q_v^s \\ q_w^s \\ m_y^s R^{-1} \end{pmatrix}, \qquad (2)$$

$$\mathbf{L^r w^r} = \mathbf{q^r}, \quad \mathbf{w^r}(y) = \begin{pmatrix} u^r \\ v^r \\ w^r \\ \theta^r R \end{pmatrix}, \ \mathbf{q^r}(y) = \begin{pmatrix} q_u^r \\ q_v^r \\ q_w^r \\ m_y^r R^{-1} \end{pmatrix}. \qquad (3)$$

Here $\mathbf{L}^s, \mathbf{L}^r$ are the elasto-inertial matrix operators of the stringer and of the frame, respectively, with dimension 4×4.

Now present the shell and stiffener vibrations and the forces exciting them in the form of an expansion in terms of harmonic vibration forms ($\boldsymbol{\varphi}(x), \boldsymbol{\psi}(y)$), satisfying the boundary conditions:

$$\mathbf{w}(x,y) = \sum_{\alpha,\beta} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \mathbf{W}_{mn}^{\alpha\beta} \boldsymbol{\varphi}_m^{\alpha}(x) \boldsymbol{\psi}_n^{\beta}(y),$$
$$\mathbf{q}(x,y) = \sum_{\alpha,\beta} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \mathbf{Q}_{mn}^{\alpha\beta} \boldsymbol{\varphi}_m^{\alpha}(x) \boldsymbol{\psi}_n^{\beta}(y), \quad (4)$$

$$\boldsymbol{\varphi}_m^{\alpha}(x) = \begin{pmatrix} \cos(k_m^{\alpha} x) \\ \sin(k_m^{\alpha} x) \\ \sin(k_m^{\alpha} x) \end{pmatrix}, \quad \boldsymbol{\psi}_n^{\beta}(y) = \begin{pmatrix} \sin(k_n^{\beta} y) \\ -\cos(k_n^{\beta} y) \\ \sin(k_n^{\beta} y) \end{pmatrix};$$

$$\mathbf{w}^{\mathbf{s}}(x,p^s) = \sum_{\alpha,\beta} \sum_{m=1}^{\infty} \mathbf{W}_m^{s\alpha\beta} \boldsymbol{\varphi}_m^{s\alpha}(x) \boldsymbol{\psi}^{s\beta}(p^s),$$
$$\mathbf{q}^{\mathbf{s}}(x,p^s) = d^s \sum_{\alpha,\beta} \sum_{m=1}^{\infty} \mathbf{Q}_m^{s\alpha\beta} \boldsymbol{\varphi}_m^{s\alpha}(x) \boldsymbol{\psi}^{s\beta}(p^s), \quad (5)$$

$$\boldsymbol{\varphi}_m^{s\alpha}(x) = \begin{pmatrix} \cos(k_m^{\alpha} x) \\ \sin(k_m^{\alpha} x) \\ \sin(k_m^{\alpha} x) \\ \sin(k_m^{\alpha} x) \end{pmatrix}, \quad \boldsymbol{\psi}^{s\beta}(p^s) = \begin{pmatrix} \sin(p^s \beta) \\ -\cos(p^s \beta) \\ \sin(p^s \beta) \\ \cos(p^s \beta) \end{pmatrix};$$

$$\mathbf{w}^{\mathbf{r}}(p^r,y) = \sum_{\alpha,\beta} \sum_{n=1}^{\infty} \mathbf{W}_n^{r\alpha\beta} \boldsymbol{\varphi}^{r\alpha}(p^r) \boldsymbol{\psi}_n^{r\beta}(y),$$
$$\mathbf{q}^{\mathbf{r}}(p^r,y) = d^r \sum_{\alpha,\beta} \sum_{n=1}^{\infty} \mathbf{Q}_n^{r\alpha\beta} \boldsymbol{\varphi}^{r\alpha}(p^r) \boldsymbol{\psi}_n^{r\beta}(y), \quad (6)$$

$$\boldsymbol{\varphi}^{r\alpha}(p^r) = \begin{pmatrix} \cos(p^r \alpha) \\ \sin(p^r \alpha) \\ \sin(p^r \alpha) \\ \cos(p^r \alpha) \end{pmatrix}, \quad \boldsymbol{\psi}_n^{r\beta}(y) = \begin{pmatrix} \sin(k_n^{\beta} y) \\ -\cos(k_n^{\beta} y) \\ \sin(k_n^{\beta} y) \\ \sin(k_n^{\beta} y) \end{pmatrix}.$$

Here $\mathbf{W}, \mathbf{Q}$ are the vector of generalized displacements and forces for panel or for stiffeners, p^s, p^r are the numbers of stringer and frame, respectively. The vibration forms are combined in $(N^s+1)(N^r+1)$ groups in which $\cos(k_m^{\alpha} d^r) = \cos(\alpha)$, $\cos(k_n^{\beta} d^s) = \cos(\beta)$. Wave numbers are determined as follows:

$$k_m^{\alpha} d^r = \begin{cases} 2\pi(m-1), \ \alpha = 0, \\ \pi \tilde{m} - (-1)^m \alpha, \ \ \alpha = \pi\{1,...,N^r-1\}/N^r, \\ 2\pi(m-1)+\pi, \alpha = \pi, \end{cases}$$
$$\tilde{m} = m + ((-1)^m - 1)/2;$$
$$k_n^{\beta} d^s = \begin{cases} 2\pi(n-1), \beta = 0, \\ \pi \tilde{n} - (-1)^n \beta, \ \ \beta = \pi\{1,...,N^s-1\}/N^s, \\ 2\pi(n-1)+\pi, \beta = \pi, \end{cases} \quad (7)$$
$$\tilde{n} = n + ((-1)^n - 1)/2.$$

One can show that at the regular arrangements of stiffeners for infinite systems or for systems with artificial “halves” of stiffeners on edges such groups of forms do not interact and are excited independently of each other.

For each set of indices for the panel and stiffeners we get our own frequency-dependent kind of matrix operator images $\mathbf{K}$ which connects the vectors of generalized displacements and forces. Later on the matrices of compliance $\mathbf{I}$ inverse to the matrices $\mathbf{K}$ are used:

$$\mathbf{K}_{mn}^{\alpha\beta} \mathbf{W}_{mn}^{\alpha\beta} = \mathbf{Q}_{mn}^{\alpha\beta}, \ \mathbf{W}_{mn}^{\alpha\beta} = \mathbf{I}_{mn}^{\alpha\beta} \mathbf{Q}_{mn}^{\alpha\beta}, \ \mathbf{I}_{mn}^{\alpha\beta} = \left(\mathbf{K}_{mn}^{\alpha\beta}\right)^{-1}; \quad (8)$$

$$\mathbf{K}_{m\alpha}^{s} \mathbf{W}_m^{s\alpha\beta} = \mathbf{Q}_m^{s\alpha\beta}, \ \mathbf{W}_m^{s\alpha\beta} = \mathbf{I}_{m\alpha}^{s} \mathbf{Q}_m^{s\alpha\beta}, \ \mathbf{I}_{m\alpha}^{s} = \left(\mathbf{K}_{m\alpha}^{s}\right)^{-1}; \quad (9)$$

$$\mathbf{K}_n^{r\beta} \mathbf{W}_n^{r\alpha\beta} = \mathbf{Q}_n^{r\alpha\beta}, \ \mathbf{W}_n^{r\alpha\beta} = \mathbf{I}_n^{r\beta} \mathbf{Q}_n^{r\alpha\beta}, \ \mathbf{I}_n^{r\beta} = \left(\mathbf{K}_n^{r\beta}\right)^{-1}. \quad (10)$$

Now let us consider the connection of panel and stiffener vibrations. The generalized stiffener displacements depend only on the panel displacements of the same group and they can be presented in the following form:

$$\mathbf{W}_m^{s\alpha\beta} = \sum_n \mathbf{E}_n^{\beta} \mathbf{W}_{mn}^{\alpha\beta},$$
$$\mathbf{W}_n^{r\alpha\beta} = \sum_m \mathbf{E}_m^{\alpha} \mathbf{W}_{mn}^{\alpha\beta}. \quad (11)$$

Here $\mathbf{E}_n^{\beta}$, $\mathbf{E}_m^{\alpha}$ are the matrices with dimension 4×3 transforming the vectors of generalized panel vibration into the vectors for stiffeners. Note, that the first ones do not depend on longitudinal indices m, α and the second ones are independent of circular indices n, β. The generalized force exiting the panel form is made up of the external generalized force, one generalized response of frames with the same circular wave number k_n^{β} and one generalized response of stringers with the same wave number k_m^{α} and can be written as follows:

$$\mathbf{Q}_{mn}^{\alpha\beta} = \mathbf{Q}_{mn0}^{\alpha\beta} - \mathbf{F}_m^{\alpha} \mathbf{Q}_n^{r\alpha\beta} - \mathbf{F}_n^{\beta} \mathbf{Q}_m^{s\alpha\beta}. \quad (12)$$

Here $\mathbf{Q}_{mn0}^{\alpha\beta}$ is the generalized external force, $\mathbf{F}_m^{\alpha}$, $\mathbf{F}_n^{\beta}$ are the matrices with dimension 3×4 transforming the generalized vectors of support responses into the generalized vectors of forces affecting the panel.

One can exclude the panel displacements from equations (8-12) and obtain a system of equations related to unknown generalized forces affecting the stiffeners. The system can be written conveniently in the matrix form:

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} \begin{pmatrix} \mathbf{Q}^s \\ \mathbf{Q}^r \end{pmatrix} = \begin{pmatrix} \mathbf{W}_0^s \\ \mathbf{W}_0^r \end{pmatrix}, \quad (13)$$

$$\mathbf{A}_{mm} = \mathbf{I}_{m\alpha}^{s} + \sum_n \mathbf{E}_n^{\beta} \mathbf{I}_{mn}^{\alpha\beta} \mathbf{F}_n^{\beta}, \ \mathbf{B}_{mn} = \mathbf{E}_n^{\beta} \mathbf{I}_{mn}^{\alpha\beta} \mathbf{F}_m^{\alpha},$$
$$\mathbf{C}_{nm} = \mathbf{E}_m^{\alpha} \mathbf{I}_{mn}^{\alpha\beta} \mathbf{F}_n^{\beta}, \ \mathbf{D}_{nn} = \mathbf{I}_n^{r\beta} + \sum_m \mathbf{E}_m^{\alpha} \mathbf{I}_{mn}^{\alpha\beta} \mathbf{F}_m^{\alpha},$$
$$\mathbf{W}_{0m}^{s} = \sum_n \mathbf{E}_n^{\beta} \mathbf{I}_{mn}^{\alpha\beta} \bar{Q}_{mn0}^{\alpha\beta}, \ \mathbf{W}_{0n}^{r} = \sum_m \mathbf{E}_m^{\alpha} \mathbf{I}_{mn}^{\alpha\beta} \bar{Q}_{mn0}^{\alpha\beta}.$$

Here $\mathbf{A}$, $\mathbf{D}$ are the block-diagonal matrices. We restrict ourselves to a certain number of forms in group ($n < n_{\max}$, $m < m_{\max}$). Then the generalized forces acting on frames can be obtained as follows:

$$\mathbf{Q}^r = (\mathbf{D} - \mathbf{C}\mathbf{A}^{-1}\mathbf{B})^{-1}(\mathbf{W}_0^r - \mathbf{C}\mathbf{A}^{-1}\mathbf{W}_0^s). \quad (14)$$

Then the generalized forces acting on stringers are simply derived

$$\mathbf{Q}^s = \mathbf{A}^{-1}\left(\mathbf{W}_0^s - \mathbf{B}\mathbf{Q}^r\right). \quad (15)$$

Now, when the stiffener responses are known, we substitute them into Eqs. (8, 12) and will find the panel vibrations sought.

3 Examples of calculations

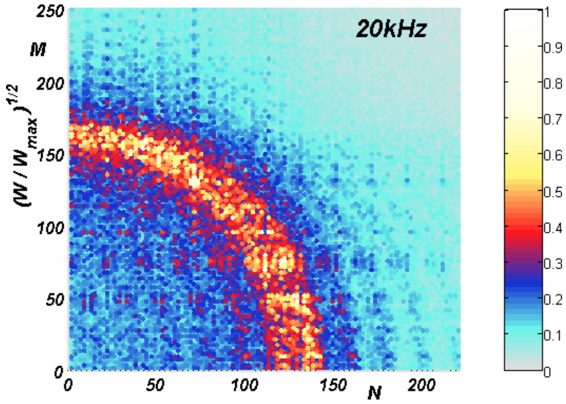
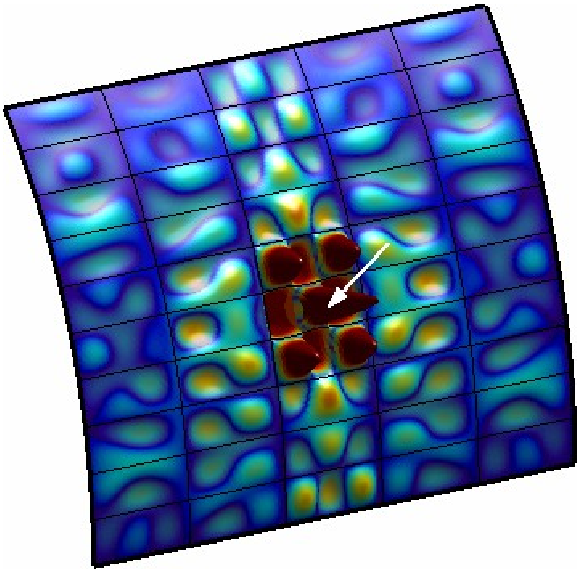


Fig. 2 An example of predicting the generalized displacements of panel n.1 under point excitation at the middle cell centre. f = 20 000 Hz.

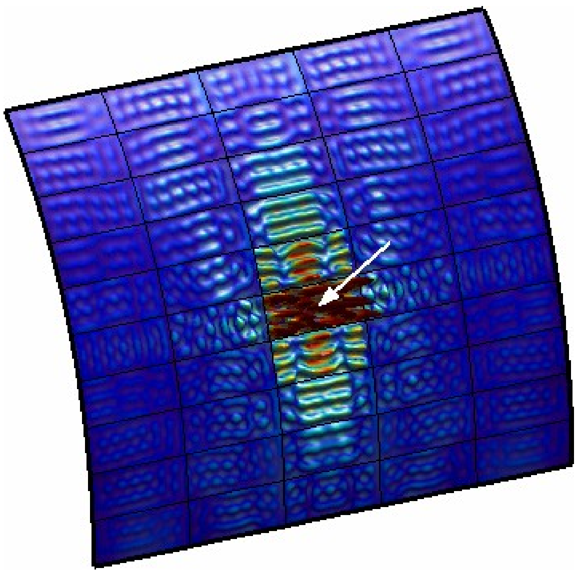
Now we demonstrate an application of the vibration calculation method to an example of the stiffened panel excitation. To make the calculations, the construction parameters were chosen which correspond to the fuselage fragments of large and small passenger aircrafts, some parameters of which are presented in table 1. The loss tangent took the value $\eta = 0.1$ for Figs. 3,4,6,7 and $\eta = 0.03$ for Figs. 2,5.

	Panel n.1	Panel n.2
Radius, R	3 m	1.3 m
Spans, $N^r \times d^r$	5×0.5 m	16×0.45 m
Cells, $N^s \times d^s$	10×0.2 m	40×0.15 m
Skin thickness, h	0.019 m	0.012 m

Table 1 Some panels parameters

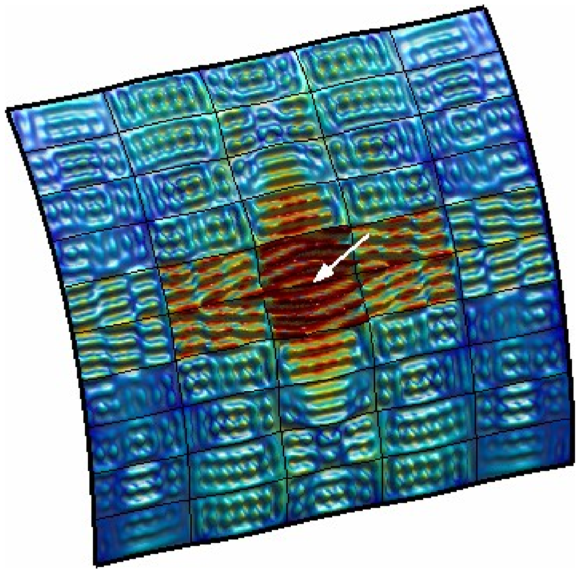


a) 400 Hz, cell centre

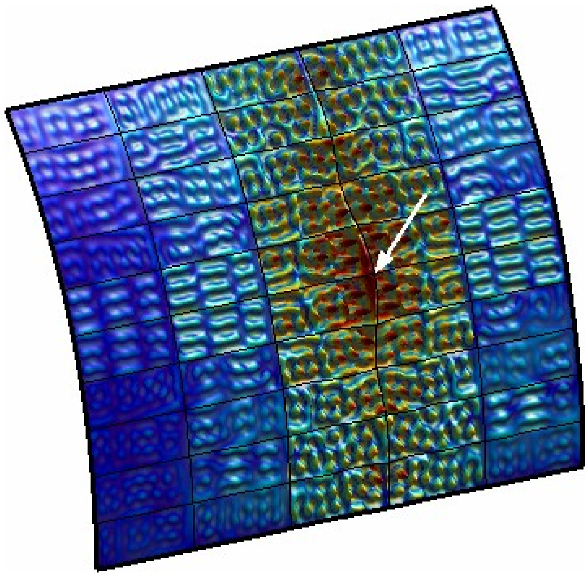


b) 4 000 Hz, cell centre

Fig. 3 Excitation of the stiffened panel n.1 at frequencies of 400 and 4 000 Hz by a harmonic point force applied to the cell centre.



a) 4 000 Hz, stringer middle



b) 4 000 Hz, stiffener intersection

Fig. 4 Excitation of the stiffened panel n.1 at a frequency of 4 000 Hz by a point force applied to the stringer middle and to the stiffener intersection.

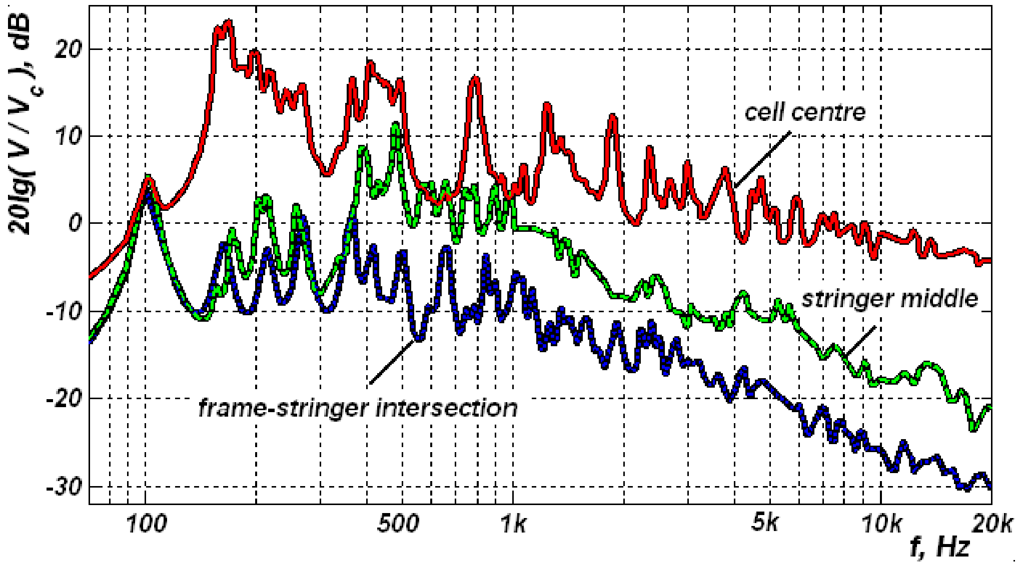


Fig. 5 The rms-velocities of the panel excited by a point force $qe^{i\omega t}$ applied to the cell centre, to the stringer middle and to the stiffener intersection. $V_c = q / m_c\omega_c$, m_c is cell mass, $\omega_c = 2\pi$ *136 Hz.

The principal complexity of calculations is associated with matrix $(\mathbf{D}-\mathbf{CA}^{-1}\mathbf{B})$ inversion in expression (14). Dimensions of this matrix are $(4n_{\max} / N^s)\times(4n_{\max} / N^s)$. The number of forms sufficient for a correct prediction is determined according to test results of generalized displacements prediction.

Fig. 2 presents an example of such a prediction of generalized displacements, or more precisely, their radial components for point harmonic force applied to the centre of the middle cell of panel n.1 at frequency of 20 kHz. The generalized forces of point excitation are regularly distributed over the whole range of wave numbers or form indices, therefore it is convenient to use it as a test field of forces. One quarter of ellipse formed by the dominating forms in Fig. 2 corresponds to wave numbers determined from the relation for running waves in infinite plate $(k_x^2 + k_y^2)^2 = m\omega^2 / D$. It is seen in figure that for panel n.1 at this frequency and all the lower frequencies it is sufficient to account for $\mathrm{n}\times\mathrm{m} = 220\times 250 = 55\,000$ forms. Each group of forms in this case includes $22\times 50 = 1100$ forms. The matrices requiring the inversion $(\mathbf{D}-\mathbf{CA}^{-1}\mathbf{B})$ have dimensions 88×88.

Figure 3 shows the panel displacements at some moment of time. The exciting point force is applied to the cell center indicated by the arrow. At frequency of 400 Hz (Fig. 3a) both the neighboring and the distant cells are excited. With a frequency increase (Fig. 3b) the vibrations become more localized within the cell and the vibrations laterally to frames are transmitted in a weaker degree.

Figure 4a shows the force applied to a stringer in the span middle. At a frequency of 4000 Hz an intensive vibration propagation along the stringer is observed, though the vibrations themselves are significantly weaker, than at excitation at the cell centre. The minimum vibrations are observed when the point force is applied to the stringer and the frame intersection (Fig. 4b), despite the fact that they well propagate along the frame.

Figure 5 presents panel rms-velocities predicted for three cases of excitation shown in the previous figures for the frequency range of 70 Hz – 20 kHz. In the vicinity of 100 Hz the lowest eigen-form of the construction is excited. The point of force application in this case plays no significant role. At the frequencies higher than the eigen-frequency of the individual freely supported cell (136 Hz) the construction is more excited by the force applied to the cell centre. Beginning with the frequency a little less than the stringer span eigen-frequency (440Hz), there is a substantional difference between the excitation by the force applied to the stringer middle and that applied to the stiffener intersection.

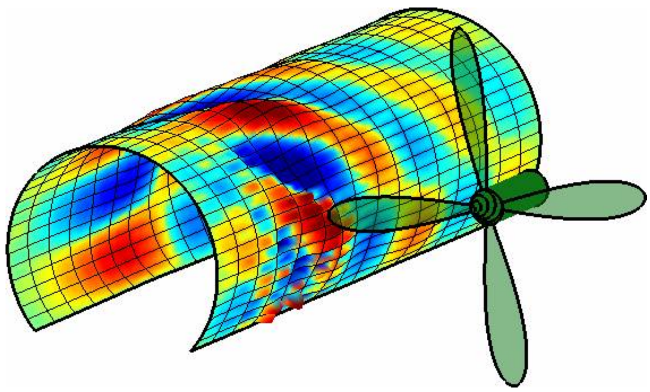
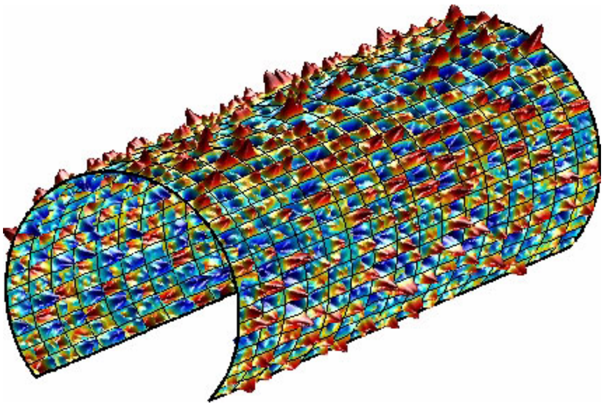


Fig. 6 Excitement of panel n.2 at the third harmonic of propeller excitation field, f = 264 Hz.

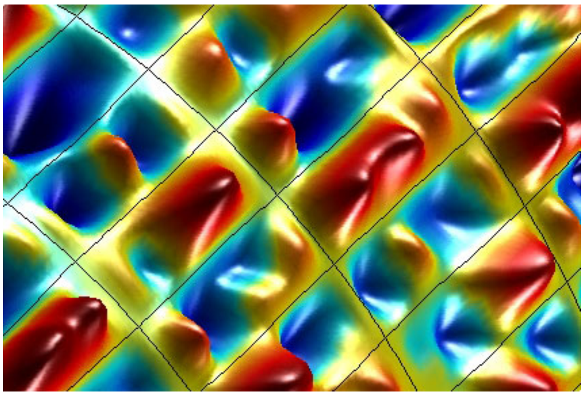
In considering models of aircraft fuselage, the prediction of its vibrations under real excitation fields such as the field from a propeller and field of pressure fluctuations of the turbulent boundary layer is of practical interest.

At lower harmonics from the rotating propeller one can use a simple orthotropic shell model which doesn't account for stiffener discreteness. However, starting with the frequency equal to the first eigen-frequency of the separated cell (here 145 Hz) the discrete properties of stiffeners begin to manifest themselves.

Fig. 6. shows an example of predicting the vibrations of panel n.2 under excitation by the third harmonic of the acoustic field of the propeller at the frequency of 264 Hz. The shell behavior is practically orthotropic, but in the region of the maximum excitation the independent vibrations of the cells are already reveled.



a) full shell excitement



b) enlarged fragment

Fig. 7 Momentary picture of fuselage surface vibrations exited by pressure fluctuations of the turbulent boundary layer.

Fig. 7a gives a snap of fuselage surface vibrations exited by pressure fluctuations of the turbulent boundary layer under flight condition. The pressure field parameters correspond to flight condition of M = 0.7, height 5 km and 20 m from the aircraft nose and they are identical for the whole shell surface. The second figure (7b) presents the vibrations on an enlarged scale. According to this figure, the forms with half-wave lengths of the cell dimension order dominate in the shell vibrations at the excitation by the boundary layer. These figures are obtained by summing the vibrations at the frequencies from 100 to 1100 Hz with a step of 25 Hz.

Solution of the task related to fuselage vibrations excited by the fields of external forces is of importance by itself and as a part of solution of the tasks of internal acoustic. The benefits of the proposed method based on special harmonic expansions consists also in the fact that the result of vibration predictions is presented in the form of amplitudes of sinusoidal forms. This permits passing directly to solving the tasks related to sound wave radiation or to internal acoustic mode excitation, using the methods of predicting the internal noise, worked out earlier and with account for the layers of sound-insulating material and radiation and absorption losses [7, 8].

5 Conclusion

The task related to forced vibrations of the cylindrical panel with an orthogonal system of stiffeners is solved with a correct account for their discreteness and elasto-inertial properties. The solution which is compact and permits determining all the components of construction vibration velocities directly under excitation by normal and tangential fields of forces practically over the whole sound frequency range is obtained. These components are presented in the form of special double trigonometric series. Such a presentation of panel velocities substantially simplifies the solution of subsequent tasks related to acoustic radiation of panels and to forming the acoustic field inside a closed volume.

Acknowledgments

The work is realized with a support of Russian Foundation for Basic Research (grant 06-02-16243)

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