Boundary element model to study sound transmission provided by a single panel in the presence of an elastic interlayer

Andreia Pereira and António Tadeu

University of Coimbra, Department of Civil Engeneering - Pólo II da Universidade - Rua Luís Reis Santos, 3030-788 Coimbra, Portugal
apereira@dec.uc.pt

In this paper the Boundary Element Method is applied to develop a numerical model which may be used to study the airborne and impact sound insulation provided by a single panel of infinite extent bounded by two fluid media, with an elastic interlayer (joint) inserted into the partition. The elastic interlayer is placed in the panel, perpendicular to its surfaces and fully occupying the panel thickness. When the interlayer assumes the properties of a resilient material, it is able to reduce the wave propagation through the elastic medium of the partition. The model is excited by a harmonic line load that acts either in the fluid medium or in the elastic medium in the direction perpendicular to the panel's surface.
The model is developed following a direct frequency domain formulation which assumes full coupling between the fluid media and the elastic media. Analytical Green's functions for an elastic single layer bounded by fluid media are used to avoid having to discretize the horizontal surfaces of the partition.
The proposed model is verified by comparing the responses against those provided by a direct frequency domain Boundary Element algorithm, which uses Green's functions for full fluid and elastic media. This algorithm is only used for verification proposes, as it requires the discretization of all interfaces, which entails considerable computational effort.
Numerical simulations are then displayed in order to illustrate the applicability of the proposed model to the analysis of airborne and impact sound insulation when an elastic interlayer is inserted into a single partition.

1 Introduction

When waves propagating along building structures encounter changes of materials or configurations part of the energy is reflected and thus the energy propagating across these discontinuities may be attenuated.

The simulation of wave propagation through elastic connected plates was first studied by Cremer et al. [1] who used the wave approach to perform an analysis assuming normal incidence and the propagation of longitudinal and bending waves. Their research studied corner, T and cross-junction configurations. They concluded that changes in the direction of structural elements do not attenuate most structure borne sound. The general case of transmission relating to three wave types with random incidence for arbitrary L, T and cross junctions was obtained by Wöhle, Beckmann and Schreckenbach [2,3] and by Craven and Gibbs [4,5].

In order to obtain greater attenuation of wave propagation one method that has been used in building structures consists of inserting elastic interlayers made of acoustically soft materials. Cremer et al. [1] also addressed this problem for several configurations under the above assumptions. More recently other researchers such as Mees and Vermeir [6] have analysed sound transmission between plates connected by a hinge or an elastic interlayer.

In this paper a numerical model is developed to analyze the dynamic behavior provided by a single panel of infinite extent bounded by two fluid media, with an elastic interlayer inserted inside the partition. Note that this model assumes the propagation regarding all types of waves. The model makes use of the Boundary Element Method and assumes full coupling between the partition and the elastic interlayer. Airborne sound insulation and impact sound pressure level when a load acts along the y direction, assuming cylindrical line loads are calculated. Responses for different sets of receivers are obtained to assess sound transmission in the vicinity of the interlayer and further from the interlayer and the source. The analysis uses the responses provided by the panel without the interlayer as a reference. The next section of the paper describes the formulation of the model, followed by a description of the applications and the discussion of the responses.

2 Problem formulation

Fig. 1 shows a single layer of infinite extent, with thickness h, which separates an infinite homogeneous acoustic medium. In the acoustic medium with density ρ_f and Lamé constant λ_f propagate sound waves with a velocity α_f. In the elastic medium with density ρ_1, Poisson ratio ν_1 and shear modulus μ_1 propagate compression waves with velocity α_1 and shear waves with velocity β_1. An elastic interlayer with thickness e, and density ρ_2, is inserted inside the partition. In the elastic interlayer propagate compressional waves with velocity α_2 and shear waves with velocity β_2.

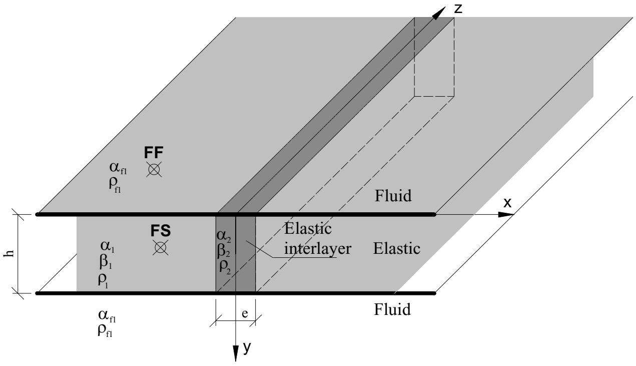


Fig 1 Geometry of the model which assumes an elastic interlayer inserted inside a single panel of infinite extent.

When the above described system is excited by a spatially sinusoidal harmonic line load, acting in the fluid medium (FF) at (x_0, y_0), the incident pressure field at a point (x, y, z) is given in the frequency wave-number domain by:

$$\sigma^{full} = \frac{-\mathrm{i}A}{2}\mathrm{H}_0^{(2)}\left(k_{\alpha_{f1}}\sqrt{(x-x_0)^2+(y-y_0)^2}\right)\mathrm{e}^{-\mathrm{i}k_z z}, \qquad (1)$$

in which A is the wave amplitude; ω is the excitation frequency; $\mathrm{i}=\sqrt{-1}$; $k_{\alpha_{f1}}=\sqrt{\omega^2/(\alpha_{f1})^2-k_z^2}$ (with $\mathrm{Im}\,k_{\alpha_f}<0$); k_z is the spatial wavenumber along the z direction ($k_z=\frac{2\pi}{L}m$), and $H_n^{(2)}(...)$ are second Hankel functions of order n.

If a spatially sinusoidal harmonic line load acts at a point (x_s, y_s) of the elastic medium (FS) along the y direction, the resulting incident field can be expressed by the displacements $G_{y,k}^{full}$ (where the index, $k=x,y,z$, indicates the direction of the displacement) at a point (x,y) according to the following expressions [7]:

$$G_{yx}^{full}(\omega,x,y,k_z)=\gamma_x\gamma_y AB_2 \qquad (2)$$

$$G_{yy}^{full}(\omega,x,y,k_z)=A\left[\left(\frac{\omega}{\beta_1}\right)H_{0\beta_1}-\frac{1}{r}B_1+\gamma_y^2B_2\right]$$

$$G_{yz}^{full}(\omega,x,y,k_z)=\mathrm{i}k_z\gamma_y AB_1$$

where $A=\frac{1}{4\mathrm{i}\rho_1\omega^2}$; $\gamma_i=\frac{\partial r}{\partial x_i}=\frac{x_i}{r}$ and $i=1,2$ corresponds to the direction cosines; $B_n=(k_{\beta_1})^n H_{n\beta_1}-(k_{\alpha_1})^n H_{n\alpha_1}$; $H_{n\alpha_1}=\mathrm{H}_\mathrm{n}^{(2)}(k_\alpha r)$ and $H_{n\beta_1}=\mathrm{H}_\mathrm{n}^{(2)}(k_{\beta_1} r)$; $r=\sqrt{(x-x_s)^2+(y-y_s)^2}$; $k_{\alpha_1}=\sqrt{(\omega/\alpha_1)^2-k_z^2}$ with $\mathrm{Im}(k_{\alpha_1})\le 0$ and $k_{\beta_1}=\sqrt{(\omega/\beta_1)^2-k_z^2}$ with $\mathrm{Im}(k_{\beta_1})\le 0$.

Note that when k_z equals zero, Eqs. (1) and (2) allow the calculation of the incident field provided by cylindrical linear loads (corresponding to the pure two-dimensional case).

2.1 BEM formulation

The scattered field provided by the elastic interlayer is obtained by using a Boundary Element model. The numerical formulation of the BEM, applied to wave propagation, has been widely studied (Estorff [7], Banerjee [8]). The model used here has been developed using Green functions for single layered medium bounded by two fluid media derived by Tadeu et al. [9], therefore only the interlayer needs to be discretized. A similar formulation has been developed by the authors to study the wave propagation when a cylindrical circular heterogeneity is inserted inside a single partition [10]. In the present formulation the discretization has been performed using four boundaries S_1 to S_4, as illustrated in Fig. 2, in order to avoid numerical problems related to the integrations in the interfaces of the layer. The final system of equations is obtained after defining the continuity of stresses and displacements in the boundaries S_1 and S_3 (elastic/elastic interfaces). The continuity of pressure and normal pressure velocity is established in the boundaries S_2 and S_4 (acoustic/acoustic interfaces). These boundaries are discretized using constant boundary elements. To obtain the final system of equations, a set of integrals, listed in Table 1, must be evaluated at each element of the discretized boundary.

Displacements	Stresses
$\int_{C_l} G_{kl}^{surf,i}(\underline{x}_P,\underline{x}_Q,\omega)\,dC_l$	$\int_{C_l} H_{kl}^{surf,i}(\underline{x}_P,\underline{x}_Q,\nu,\omega)\,dC_l$
$\int_{C_l} G_{kf}^{surf,i}(\underline{x}_P,\underline{x}_Q,\nu,\omega)\,dC_l$	$\int_{C_l} H_{kf}^{surf,i}(\underline{x}_P,\underline{x}_Q,\omega)\,dC_l$
$\int_{C_l} G_{fl}^{surf,i}(\underline{x}_P,\underline{x}_Q,\omega)\,dC_l$	$\int_{C_l} H_{fl}^{surf,i}(\underline{x}_P,\underline{x}_Q,\nu,\omega)\,dC_l$
$\int_{C_l} G_{f}^{surf,i}(\underline{x}_\mathrm{P},\underline{x}_Q,\nu,\omega)\,dC_l$	$\int_{C_l} H_{f}^{surf,i}(\underline{x}_P,\underline{x}_Q,\omega)\,dC_l$

Table 1 Integrals of Green functions at the boundaries.

The parameters in Table 1 refer to: $G_{kl}^{surf,i}(\underline{x}_\mathrm{P},\underline{x}_Q,\omega)$ and $H_{kl}^{surf,i}(\underline{x}_\mathrm{P},\underline{x}_Q,\nu,\omega)$ are respectively, the Green's tensor for displacement and traction components in the elastic medium at point $\underline{x}_Q$ in direction $l=1,2,3$, when a virtual point source acts in the elastic medium at a source point source $\underline{x}_Q$, in direction $k=1,2,3$; $G_{kf}^{surf,i}(\underline{x}_P,\underline{x}_Q,\nu,\omega)$ and $H_{kf}^{surf,i}(\underline{x}_P,\underline{x}_Q,\omega)$ are the normal pressure velocity and pressure at point $\underline{x}_Q$ in the fluid medium, when a virtual load is applied at a point $\underline{x}_P$ of the elastic medium in direction $k=1,2,3$; $G_{fl}^{surf,i}(\underline{x}_P,\underline{x}_Q,\omega)$ and $H_{fl}^{surf,i}(\underline{x}_P,\underline{x}_Q,\nu,\omega)$ are the displacement and traction components in the elastic medium in direction $l=1,2,3$ at $\underline{x}_Q$ when a virtual load is applied in the acoustic medium at $\underline{x}_P$; $G_{f}^{surf,i}(\underline{x}_\mathrm{P},\underline{x}_Q,\nu,\omega)$ and $H_{f}^{surf,i}(\underline{x}_\mathrm{P},\underline{x}_Q,\omega)$ are normal pressure velocity and pressure in the acoustic medium at $\underline{x}_Q$ when the load is applied at a point $\underline{x}_P$ in the fluid medium; ν is the unit outward normal for the boundary segment C_l; the subscripts $k,l=1,2,3$ denote the normal, tangential and z directions, respectively; the subscript $i=1,2$ identifies the layered medium corresponding to the partition (layered medium 1) or the layered medium with the elastic interlayer (layered medium 2). Solving the resulting system makes it possible to obtain the nodal values.

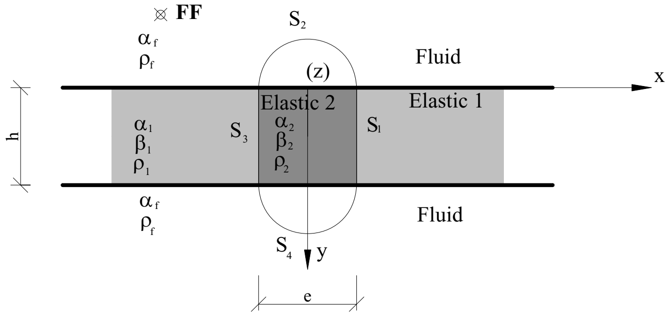


Fig. 2 Geometry of the BEM model where the discretized interfaces of the interlayer are defined.

2.2 Verification of the model

The BEM algorithm used in this work was implemented and verified by comparing the results with a BEM model where 2.5D Green functions for a full space are used. This model requires the discretization of the interfaces of both the layer and the interlayer. In order to limit the number of boundary elements used to discretize the interfaces of the layer, complex frequencies with an imaginary part are used ($\zeta = 0.7\,\dfrac{2\pi}{T}$). This considerably attenuates the contribution of the responses from the boundary elements placed at $L = 2\alpha T$, reducing the length of the interface to be discretized [11].

Several verifications were performed, for interlayers of different materials and thicknesses. The responses provided by an elastic interlayer ($\alpha_2 = 431.3$ m/s; $\beta_2 = 282.9$ m/s; $\rho_2 = 140.0$ kg/m^3) with thickness $e = 0.50$ m, inside an elastic layer were chosen to illustrate the accuracy of the model (see Fig. 3).

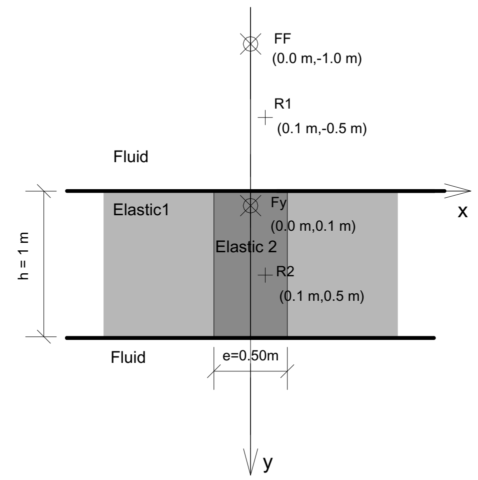


Fig. 3 Geometry of the verification.

The elastic layer ($\alpha_1 = 2182.2$ m/s; $\beta_1 = 1336.3$ m/s; $\rho_1 = 1400.0$ kg/m^3) of thickness $h = 1.0$ m divides an infinite acoustic medium with a density $\rho_f = 1000.00$ kg/m^3, allowing a dilatational wave velocity of $\alpha_f = 1500.0$ m/s. The geometry was subjected to a dilatational harmonic line load applied at point $(0.0\text{ m}; -1.0\text{ m})$ of the fluid medium with $k_z = 0.5$ rad/m. The responses were calculated at a receiver R1 placed in the fluid medium ($(0.1\text{ m}; -0.5\text{ m})$) and at a receiver R2 placed in the elastic medium ($(0.1\text{ m}; 0.5\text{ m})$). Computations are performed in the frequency range 2.0 Hz to 256.0 Hz, with a frequency step of 2.0 Hz. The BEM model using Green's functions for a full space assumes each surface of the single layer discretized with 600 boundary elements and an interlayer modelled with 180 boundary elements. Using the BEM model described in this paper, only the interlayer is discretized, using 320 boundary elements (200 boundary elements for the elastic/elastic interfaces and 160 boundary elements for the acoustic/acoustic interfaces). In order to illustrate the responses obtained in the verification, Figs. 4a and 4b display the scattered pressure recorded at receiver R1 and scattered displacements in the y direction at receiver R2 obtained with the two models. In this figure, the solid line represents the solution obtained by the BEM model where all the interfaces are discretized (labelled in the plots as Model 1), and the marks illustrate the solution provided by the BEM model used in this work (labelled in the plots as Model 2). Analysis of the results confirms a good agreement between the two solutions. Equally good results were achieved for loads placed in the elastic medium.

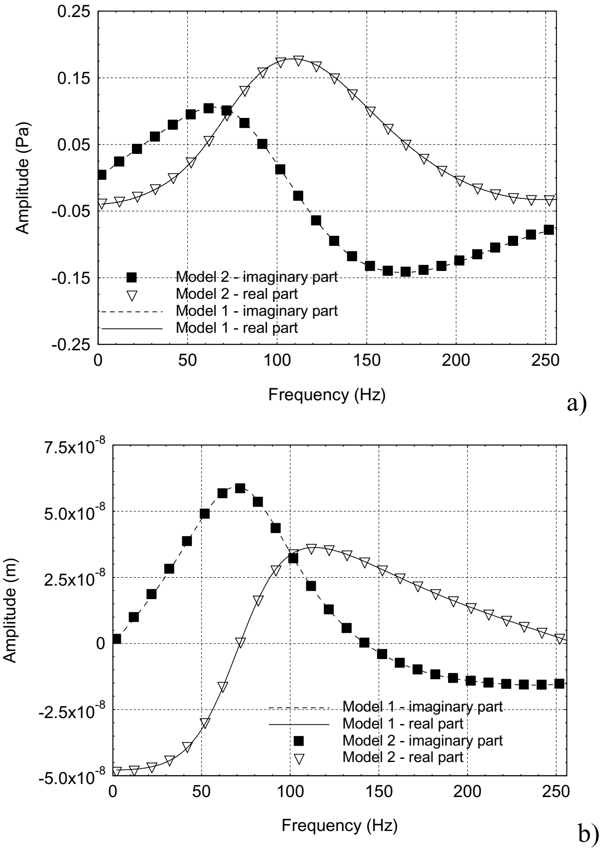
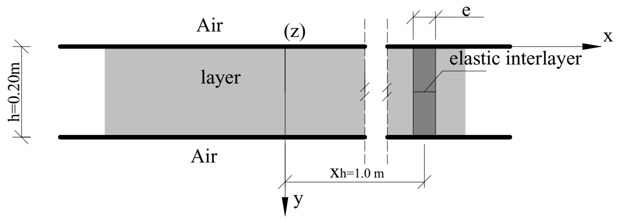


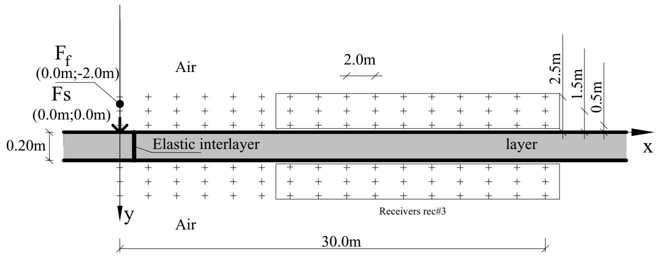
Fig. 4 Verification of the solution: a) Pressure response at receiver R1; b) Pressure response at receiver R2.

3 Applications

In the simulations, a single concrete partition ($\alpha_1 = 3498.60$ m/s; $\beta_1 = 2245.00$ m/s; $\rho_1 = 2500.00$ kg/m^3; $\eta_1 = 6.00 \times 10^{-3}$), 0.20 m thick, divides an acoustic medium with the air properties ($\rho_f = 1.22$ kg/m^3; $\alpha_f = 340.00$ m/s), as shown in Fig. 5. An elastic interlayer made from cork ($\alpha_2 = 431.3$ m/s; $\beta_2 = 282.9$ m/s; $\rho_2 = 140.0$ kg/m^3; $\eta_2 = 0.15$), with thickness $e = 0.10$ m, is inserted inside the panel at position $x_h = 1.0$ m.



a)



b)

Fig. 5 Geometry of the simulations: a) Position of the elastic interlayer inside the partition; b) Position of source and receivers.

The model is excited by a cylindrical line load that acts in the acoustic medium (F_f) at $(0.0\ \text{m}; -2.0\ \text{m})$ or in the elastic medium along the y direction (F_s) at $(0.0\ \text{m}; 0.0\ \text{m})$.

The calculations were performed in the frequency range $[2.0;\ 8192.0\,\text{Hz}]$ using a frequency step of $2.0\,\text{Hz}$. The responses were calculated at the grid of receivers shown in Fig. 5b.

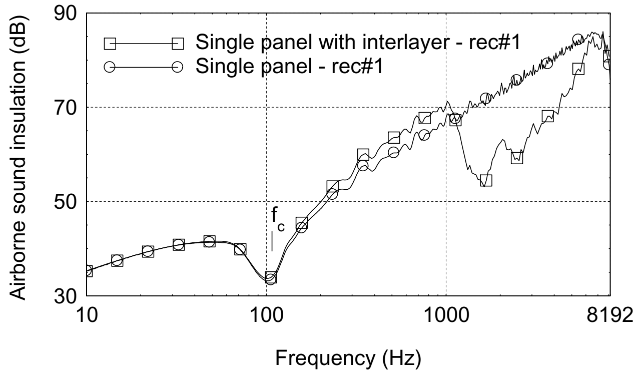
The elastic interlayer is discretized using constant boundary elements whose number varies with the excitation frequency. The boundary element is at least seven times shorter than the wavelength of the incident wave. In order to avoid numerical errors the size of each boundary element is also four times smaller than the thickness of the interlayer.

3.1 Airborne sound insulation

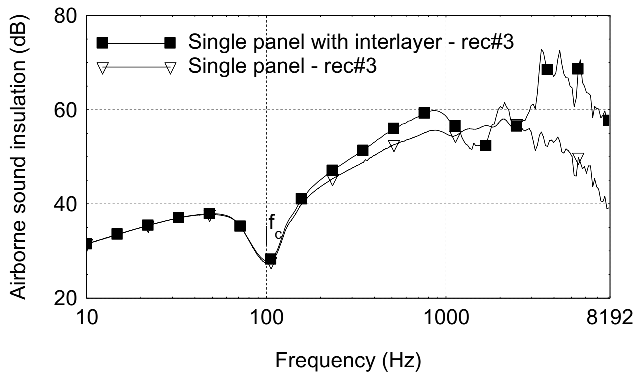
Fig. 6 shows the average airborne sound insulation provided by a single concrete layer, with and without the cork interlayer $e = 0.10$ m thick, placed at $x_h = 1.0$ m. The responses are obtained by calculating the logarithmic average of the sound pressure level in the emitting and receiving medium. The airborne sound insulation is then determined by calculating the difference between the average sound pressure level in the source and the receiving room.

In this figure the average airborne sound insulation responses were obtained using all receivers (referenced rec#1) and the receivers located on the right of the interlayer (referenced rec#3). The responses provided by the single panel without the interlayer are used as reference to draw conclusions.

The responses displayed in Fig. 6a refer to the average sound insulation obtained at the set of receivers rec#1. Analysis of this figure shows that in the low and medium frequency range the airborne sound insulation curves provided by the single panel with and without the interlayer are similar. At higher frequencies, however, the airborne sound insulation decreases when the interlayer is inserted inside the partition, since the sound is transmitted to the receiving room through this heterogeneity.



a)



b)

Fig. 6 Airborne sound insulation responses provided by a single panel with and without the cork interlayer with $e = 0.10$ m placed at $x_h = 1.0$ m : a) rec#1; b) rec#3.

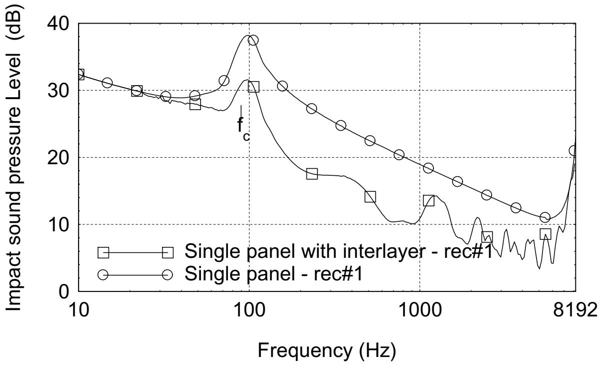
Fig. 6b displays the responses provided by receivers rec#3. At the higher frequencies, again, the airborne sound insulation provided by the single layer decreases. But at the end of the calculated range of frequencies the curve provided by the layer when the interlayer is present increases due to the energy that propagates inside the partition, which is attenuated by the presence of the interlayer.

3.2 Impact sound pressure level

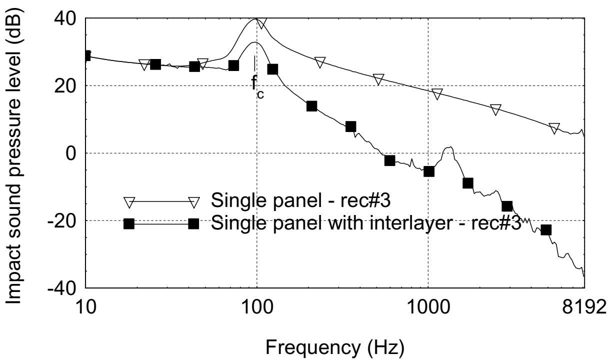
Fig. 7 displays the logarithmic average of the sound pressure level recorded in the receiving medium when a unit load acts within the elastic medium in the y direction using set of receivers (rec#1 e rec#3).

The results provided by set of receivers rec#1 show that in the low frequencies the impact sound pressure level without and with the presence of the interlayer are similar. However in the vicinity of the critical frequency (referenced in the plot as fc) the sound pressure level amplitudes provided by the layer in the presence of the cork interlayer decrease and this behavior is seen also in the higher frequencies. The sound pressure level amplitudes provided by set of receivers rec#3 when the cork interlayer is inserted inside the layer decrease, allowing the conclusion that the presence of the interlayer is able to attenuate the wave propagation which reaches the receivers placed further away from the source.

In fact the presence of the interlayer allows a distinct vibration of the panel on the left and on the right of the interlayer. As the source is placed on the left of the interlayer this side of the panel radiates more energy in the receiving medium. On the other hand the energy is transmitted by the interlayer to the panel on its right defines a lower vibration level.



a)



b)

Fig. 7 Impact sound pressure level in the receiving medium provided by a single panel with and without a cork interlayer with $e = 0.10$ m placed at $x_h = 1.0$ m : a) rec#1; b) rec#3.

5 Conclusion

In this paper a BEM model was developed to compute airborne sound insulation and impact sound pressure level when an elastic interlayer is inserted into a single partition of infinite extent. The model was used to predict the acoustic behavior of a concrete partition in the presence of a cork interlayer. From the analysis of the responses it was concluded that at low and medium frequencies, the airborne sound insulation is not changed by the presence of the interlayer. At the higher frequencies a fall in insulation occurred due to the propagation of sound through the interlayer. It was also found that at the higher frequencies, for receivers placed further from the source and the heterogeneity, the resulting sound pressure level in the emitting and receiving spaces was reduced. When the load acts in the elastic medium along the vertical direction, the responses we found to be influenced by the presence of the cork interlayer in the medium and high frequencies. As a result, at this range of frequencies, sound pressure level in the receiving medium decreased in relation to the sound level obtained for the single layer.

The analysis allows the conclusion that the present model may be used to predict wave propagation in the presence of a single layer with a interlayer.

References

[1] L. Cremer, M. Heckl, E. Ungar, “Structure-Borne Sound “, Berlin, Springer-Verlag (1988).

[2] W. Wöhle, T. Beckmann, H. Schreckenbach, “Coupling loss factors for statistical energy analysis of sound transmission at rectangular structural slab joints, part I”, *Journal of Sound and Vibration*, 77 No. 3 323-334, (1981).

[3] W. Wöhle, T. Beckmann, H. Schreckenbach, “Coupling loss factors for statistical energy analysis of sound transmission at rectangular structural slab joints, part II”, *Journal of Sound and Vibration*, 77 No. 3, 335-344 (1981).

[4] P. G. Craven, B. M. Gibbs, “Sound transmission and modal coupling at junctions of thin plates, part I: representation of the problem”. *Journal of Sound and Vibration*, 77(3), 417-427 (1981).

[5] B. M. Gibbs, P. G. Craven, “Sound transmission and modal coupling at junctions of thin plates, part II: parametric survey”. *Journal of Sound and Vibration*, 77 No. 3, 429-435 (1981).

[6] P. Mees, G. Vermeir, “Structure-borne sound transmission at elastically connected plates”, *Journal of Sound and Vibration*, 166 No. 1, 55-76 (1993).

[7] A. Tadeu, E. Kausel,” Green’s functions for two-and-a half dimensional elastodynamic problems”, *Journal of Engineering Mechanics ASCE*, 126 (10), 1093-1097 (2000)

[8] O. Estorff, “Boundary elements in acoustics: advances and applications”, Witpress, Southampton (2000).

[9] P. K. Banerjee, “The Boundary Element Methods in Engineering”, McGraw-Hill Book Company, New York, (1994).

[10] A. Tadeu, J. António, “2.5D Green functions for elastodynamic problems in layered acoustic and elastic formations”, *Journal of Computer Modelling in Engineering and Sciences (CMES)*, 477-495, (2002).

[11] A. Tadeu, A. Pereira, “Analysis of Airborne Sound Insulation and Impact Sound Pressure Level Provided by a Single Partition Containing a Heterogeneity”, *Journal of Sound and Vibration*, 300, 800-816 (2007).

[12] E. Kausel, J. M. Roesset, “Frequency domain analysis of undamped systems” *Journal of Engineering Mechanics* ASCE 118, 721-734 (1992).