Analysis of wavefronts for the piston source acoustic fields

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The development of modern acoustic methods is based to a great extent on a thorough investigation of the structure of acoustic fields and on the refinement of methods of their analysis and calculation. The study of the local features of acoustic field and its space-time structure is an urgent problem of physical acoustics. This work is dedicated to analytical investigation of space-time structure of wavefronts corresponding to the acoustic fields which are radiated by the piston source into elastic media. Two various spatial distributions of acoustic field over the surface of the transducer are taken into consideration, namely, uniform and Gaussian.

1 Introduction

The development of modern acoustic methods is based to a great extent on a thorough investigation of the structure of acoustic fields and on the refinement of methods of their analysis and calculation. The study of the local features of acoustic field and its spatio-temporal structure is an urgent problem of physical acoustics. The widely used practical methods of analyzing the acoustic fields imply the application of radiators and receivers of acoustic waves with their sensitivity and resolution being often limited by the wavelength of elastic waves. At the same time, for obtaining objective information on the fine structure of the field, it is necessary to use instruments capable of operating in a wide frequency band and of measuring the field parameters on a small spatial scale.

Laser ultrasonic interferometry fully meets these requirements. It is one of the most advanced techniques for studying and visualizing the distributions of acoustic fields in elastic media [1]. The contactless data acquisition combined with a high resolution determined by the diameter of the laser beam, which, as a rule, is much smaller than the wavelength of acoustic waves, allows one to obtain the necessary information on the spatio-temporal structure of acoustic field on the scales much smaller than the acoustic wavelength and in presence of internal and surface discontinuities of the material.

For adequate interpretation of results obtained with the use of the laser interferometer, it is necessary to use the mathematical models allowing to calculate the geometrical parameters of the wavefronts of acoustic waves. Because they determine, in particular, the spatio-temporal dynamics of acoustic fields on the surface of a specimen where the optical detection of acoustic oscillation is fulfilled.

2 Theoretical model

Now, we consider the model of the wave process description by taking into account the finite size of the radiator of acoustic waves and the spatial distribution of the field over its aperture [2]. For this purpose, we will use the method of expansion in plane waves. Let us consider a radiator with the circular aperture. We introduce a cylindrical coordinate system (r, θ, z), where the z axis is directed along the normal to the plane sample surface, on which the radiator is mounted; r and θ are the radial and axial coordinates, respectively, in the plane perpendicular to the z axis. Let us assume that the field of the transducer is axially symmetric, i.e., is independent of the θ coordinate. Then, the solution to the wave equation for the scalar potential $\varphi(r,z)$ can be expressed using the inverse Fourier–Bessel transform in the form [3]

$$\varphi(r,z) = \exp(-i\omega t)\times$$

$$\times\int_0^\infty \Phi(k_r)\exp\left(iz\sqrt{k^2-k_r^2}\right)J_0(k_r r)k_r dk_r\,, \qquad (1)$$

where $\omega = 2\pi f$ is the angular frequency corresponding to the frequency f, $k = \omega/c_l$ is the wave number of the longitudinal acoustic wave, and k_r is the radial wave number. The function $\Phi(k_r)$ is the result of application of the direct Fourier–Bessel transform to the scalar potential $\varphi(r,0)$ defined in the plane of the radiator position (z = 0):

$$\Phi(k_r) = \int_0^\infty \varphi(r,0)\; J_0(k_r r)\; r\; dr\,. \qquad (2)$$

Let the transducer plate vibrate in the direction of the z axis. Assuming that the radius of the transducer aperture r_a considerably exceeds the wavelength of the longitudinal acoustic wave, we will neglect the effects of transformation of longitudinal waves into transverse ones at the edges of the transducer plate. Thus, we will assume that the radiator excites longitudinal waves in the elastic medium with nonzero components u_z of the displacement vector and τ_{zz} of the elastic stress tensor. Then, the equation of motion [3] takes the form

$$\frac{\partial \tau_{zz}}{\partial z} = \rho\frac{\partial^2 u_z}{\partial t^2}\quad, \qquad (3)$$

where ρ is the density of the elastic medium.

Substituting the relation $u_z = (\nabla\varphi)_z$ into Eq.(1) and using Eq.(3), we obtain that $\tau_{zz} = -\omega^2\rho\varphi$. It is known [4] that, in the case of a direct contact of the piezoelectric transducer with an elastic medium, its action can be represented in the form of a distribution of elastic stresses preset on the contact surface. The boundary conditions consist in the zero values of the stress tensor components over the entire surface of a solid, except for the radiator aperture. Under the above assumptions, a similar boundary condition will obviously be satisfied for the scalar potential.

Eq.(2) can be written as

$$\varphi(r,z) = \Phi_0(r,z)\exp[-i\omega t] =$$

$$= A(r,z)\exp\left[i\left(\psi(r,z)-\omega t\right)\right], \qquad (4)$$

where $\Phi_0(r,z)$ is the complex amplitude of the scalar potential and $\Phi_0(r,z) = A(r,z)\exp\left[i\psi(r,z)\right]$. Here

$A(r,z)$ is the scalar potential amplitude and $\psi(r,z)$ is phase coefficient depending on the spatial coordinates.

In accordance with Eq.(4), the phase coefficient is determined by the relationship $\psi(r,z) = arctg\left[\frac{\mathrm{Im}[\varphi_0(r,z)]}{\mathrm{Re}[\varphi_0(r,z)]}\right]$. The gradient of the phase coefficient of wave and wave vector $\mathbf{k}$ are interrelated by the eikonal equation $\nabla\psi(r,z) = \mathbf{k}$, whence is follows that $k_r(r,z) = \frac{\partial\psi(r,z)}{\partial r}$ and $k_z(r,z) = \frac{\partial\psi(r,z)}{\partial z}$. The vector $\mathbf{k}$ is collinear with the vector of normal $\mathbf{n}$ to the wavefront Ξ that is the locus of points having the coordinates (r^*, z^*) at which, at a certain instant $t = t^*$, the oscillations of the wave process have the same phase $\psi(r^*, z^*) = \psi^*$, where ψ^* is a constant. Thus, on the two-dimensional coordinate system, the wavefronts belong to the one-parameter set of curves defined by the equation

$$\Psi(r,z;t) = \psi(r,z) - \omega t = 0. \qquad (5)$$

Through every point of the field one can pass the ray, i.e., the line which is orthogonal to the wavefront passing through this point. It is obvious that the normal passed through any point of a wavefront is the tangent to ray passing through this point. Let us suppose that we know the coordinates of points possessed by the wavefronts Ξ_0 and Ξ corresponding to two instants of time: t_0 and t. We choose some point A, having the coordinates (r_0^*, z_0^*), possessed by Ξ_0. Let the ray Γ passing through the point A at the instant t_0, cross, at the instant t, the point B, possessed by the front Ξ, having the coordinates (r^*, z^*). If the phase in the point A is known, we can write the next expression to find the phase in the point B:

$$\psi(r^*, z^*) = \psi(r_0^*, z_0^*) + \int_\Gamma \mathbf{k}\cdot\mathbf{dR}, \qquad (6)$$

where $\mathbf{R}$ is the radius-vector.

3 Numerical results

We perform the field calculation using several models of the scalar potential distribution $\varphi(r,0)$ over the radiator aperture. The simplest example of distribution is a uniform one: the potential $\varphi(r,0)$ is taken to be equal to a constant value φ_0 for $r \le r_a$. As a smoother dependence, we take the Gaussian distribution: $\varphi(r,0) = \varphi_0 \exp(-r^2/a^2)$, where $a = r_a/\sqrt{\ln 2}$ is the characteristic width of the beam in the plane z = 0. The pictures of rays and wavefronts are shown in Fig. 1. The calculating parameters which have been accepted equal $r_a = 2.5$ mm; $f = 5$ MHz; $c_l = 5790$ m/s.

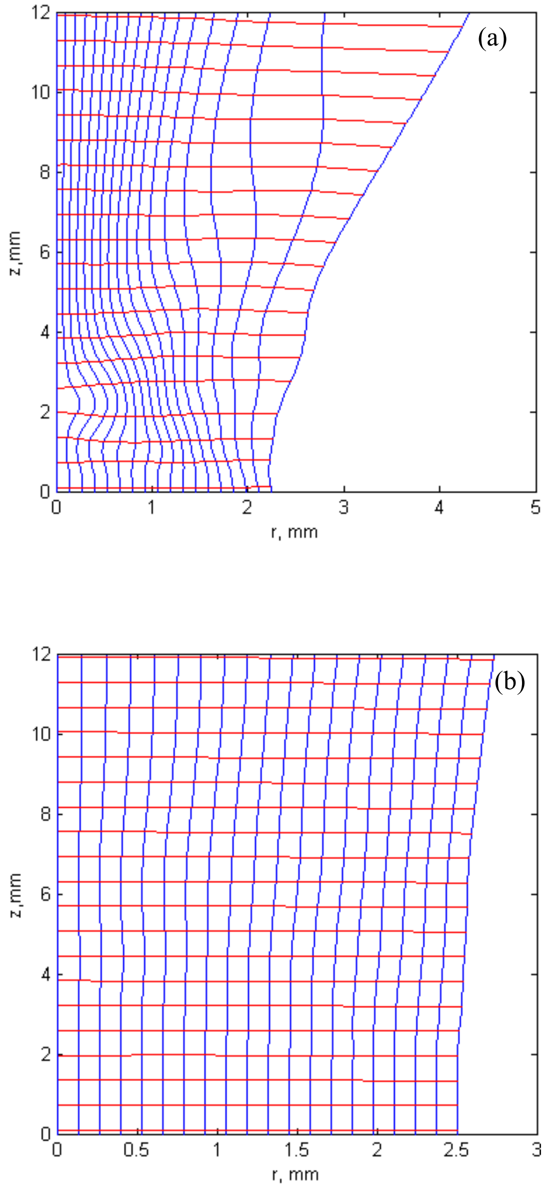
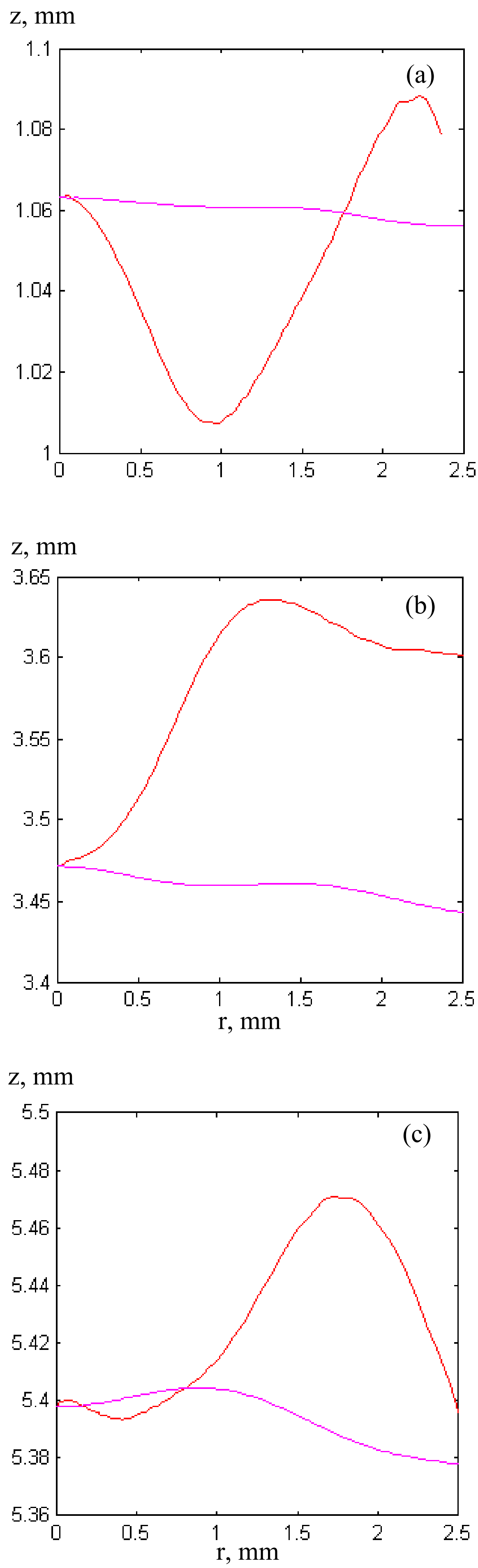


Fig. 1. The rays (blue) and wavefronts (red) corresponding to the uniform (a) and Gaussian (b) distributions of the scalar potential over the radiator aperture.

In Fig. 2, the wavefronts corresponding to the uniform and Gaussian distributions of the scalar potential over the radiator aperture are shown for different values of the Fresnel parameter [3] that is defined by the expression $F = \frac{c_l z}{f r_a^2}$. It is evident that the wavefront is not flat even in the near-field region. It is fairly, in particular, for the uniform distributions of the scalar potential.



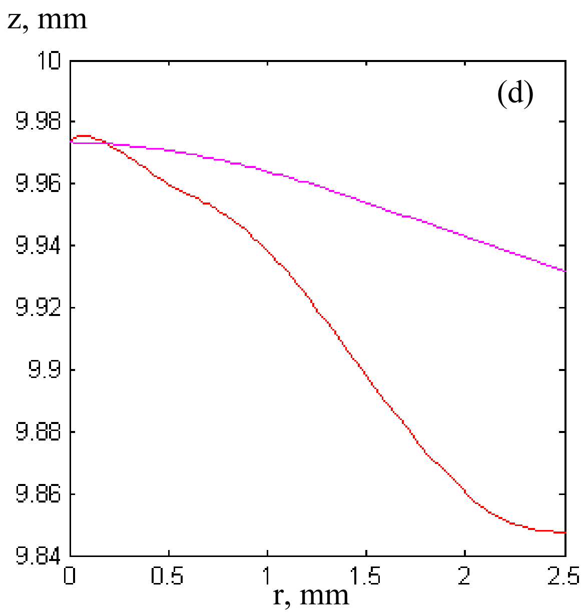


Fig. 2. The wavefronts corresponding to the uniform (red) and Gaussian (magenta) distributions of the scalar potential over the radiator aperture for different values of the Fresnel parameter: 0.20 (a), 0.65 (b), 1.00 (c) and 1.85 (d).

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