coustics'08
Paris
June 29-July 4, 2008
www.acoustics08-paris.org

Extraction of lumped clarinet reed model parameters from numerically synthesised sound

Vasileios Chatziioannou and Maarten Van Walstijn

Sonic Arts Research Centre, Queen's University Belfast, BT7 1NN Belfast, UK
vchatziioannou01@qub.ac.uk

Fluid dynamical analysis and time-domain modelling of a single reed-mouthpiece-lip system can be used to inform the formulation of a lumped model of the woodwind excitation mechanism. Coupling this lumped model to a model of the instrument bore enables computationally efficient generation of sustained oscillations, using a small number of physical parameters that define the instrument and the way the player controls them. As such, the embouchure of the player as well as the geometry of the system is taken into account. In this paper, an attempt is carried out to use the numerically generated sound as an input to an inversion algorithm for the reed-mouthpiece-lip system. Assuming that the reed motion is proportional to the pressure difference across it, a relationship can be established between the pressure and the total flow inside the mouthpiece that allows a first estimation of the physical parameters using standard optimisation techniques. Currently we are undertaking efforts to apply the inversion to data measured under real playing conditions, i.e. effectively capturing player gesture information in the form of physical control parameters.

1 Introduction

Physical modelling of musical instruments simulates the sound production mechanism, given a set of physical parameters that govern the instrument oscillations. Concerning single reed woodwind instruments, this can be achieved by a lumped model of the reed-mouthpiece system coupled to a model for the wave propagation in the bore of the instrument. This constitutes a forward model of the instrument, that takes as input the parameters that govern the oscillation of the reed, and produces the signals of the pressure and flow inside the mouthpiece (see Figure 1). Inverting the above process means estimating the parameters from naturally performed sounds. Namely, an inverse model takes as input the signals of the pressure and flow inside the mouthpiece and gives an estimation of the lumped model parameters, as can be visualised in Figure 1. As such, realistic estimations can be made of the lumped model parameters, which are difficult if not impossible to be measured directly. In addition, our understanding of the physics of the instrument – which normally is employed for the purposes of investigating the instrument itself or for sound synthesis – can now be applied to capture and study the actions of the player. Such an inverse modelling procedure is presented in the current paper. In the initial developement of the algorithm, the input sound for this inverse model is obtained by a time-domain forward model of the clarinet.

Modelling single reed woodwind instruments using a time-domain approach has been introduced by Schumacher [1] and was then extended and built upon by several authors. Time-domain calculations can deal with non-linear oscillations and are able to model both the transient and the steady state behaviour of the system. The oscillation of the reed is mostly simulated using a one-mass (lumped) model. Originally formulated as a linear model [1, 2, 3, 4], subsequent studies introduced models with non-constant parameters [5] and methods for estimating parameters from distributed models of the reed [6] such that the vibrational behaviour of the lumped model is similar to that of the distributed model. Modelling the mechanical response of the reed-mouthpiece-lip system using a two dimensional vibrating plate simulation [7] enables a numerical estimation of the lumped model parameters, that inherently takes into account any torsional modes of the reed, that have been proved to affect the sound quality [8, 9]. Furthermore, the effect of the player's lips to the system has also been modelled; the stiffness and the exact position of the player's lips result in different values for the overall, "effective" stiffness of the system. In the present paper these parameters are used for a lumped model simulation, in order to produce the pressure and flow in the mouthpiece, that are later used as an input to the optimisation routine. Once realistic parameters have been estimated, it is possible to use them as an input to a lumped (forward) model simulating the oscillations of the instrument, to resynthsise the original sound.

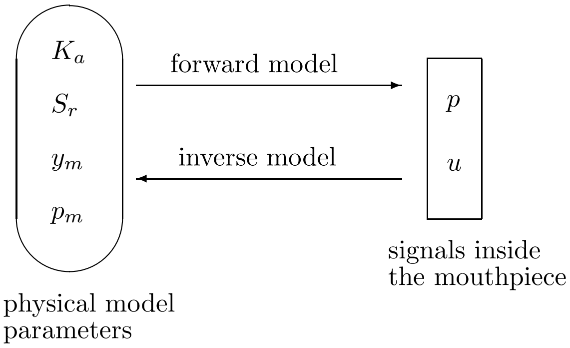

Figure 1: Forward and inverse modelling of the reed-mouthpiece system, where K_a is the effective stiffness, S_r the effective reed surface, y_m the closing position of the reed, p_m the blowing pressure and p and u the pressure and flow inside the mouthpiece.

In this paper a way is presented to estimate these parameters, by establishing an analytical relationship between the pressure and the flow inside the mouthpiece. Section 2 explains the assumptions that have to be taken in order to extract a closed form expression for the pressure-flow interaction. The estimation process is described in Section 3 , followed by numerical results for an ideal (linear) and a more realistic (non-linear) case.

2 Assumptions

Although in the forward model the mass and the damping of the reed are included in the equation of motion for the oscillation of the reed, for the inverse model it is assumed that the reed displacement y is proportional to the pressure difference ΔP across it:

$$y = C\Delta P = C(p_m - p), \tag{1}$$

where C is the compliance of the reed [10], p_m the blowing pressure and p the pressure inside the mouthpiece. The reed opening h can be related to y as

$$h = y_m - y \tag{2}$$

with y_m the closing position of the reed. Under this assumption the effects of inertia forces due to the mass of the reed and frictional forces due to internal damping are neglected. It can be argued that even though these forces might dominate the transient behaviour of the system, their effect almost vanishes at steady state (see Figures 4 and 5 in [11]). Bearing in mind that the input data will be confined to the steady state of the sustained sound, the corresponding error will be small, as can be deduced by the numerical results of Section 3.1.

Concerning the fluid dynamics, we have developed a refined formulation based on various previous studies. Regarding the effective opening surface S_f, the two-dimensional simulation of the reed-mouthpiece system [7] provides a mapping between the lumped reed opening and a curved opening surface. In addition, the side openings have been taken into account, based on experimental results [12]. For the flow inside the reed channel things get more complicated though. Earlier studies assume a basic formulation of the flow, using either a Bernoulli flow occupying the whole reed channel [3, 6, 13, 14] or considering the formation of an air jet in the reed channel with a constant height [4, 15]. In this paper a more analytical formulation for the flow in the reed channel is adopted, using a variable air jet height predicted by boundary layer flow theory [11, 16, 17]. This height can be shown to behave as a function of the reed opening h. That is, if α is the vena contracta coefficient (nondimensional scaling factor of the reed channel height) then

$$\alpha(h) = 0.39 \ln(h + 16 \cdot 10^{-6}) - 993.92 \sin(h) + 4.27\ . \tag{3}$$

A similar flow behaviour has been predicted in [18] using a dynamical Lattice-Boltzman simulation. This dynamical data obtained for the vena contracta factor displays a qualitative resemblance to boundary layer flow theory, which suggests that using a theory-based variable-α formulation might provide a useful refinement of the lumped reed model. The motivation for the current authors to use this refined model is that it generates oscillation data that is more complex and different from that produced by the simple flow model, and as such it is more suited to the development of the inverse method, which will be applied to experimental data. That is, although it has not been experimentally verified that the parameter α will vary during oscillation according to equation (3), it is likely to not be completely constant. The key-point is that an inverse model that can deal with data generated based on the refined model using equation (3) will also be able to deal with any data in which α varies in another way.

The effect of the vena contracta α to the overall flow can be visualised by plotting the scaled-down opening surface αS_f as a function of the lumped reed opening h. As can be seen in Figure 2 the resulting function is not far from linear.
Thus, by introducing an extra parameter λ the flow

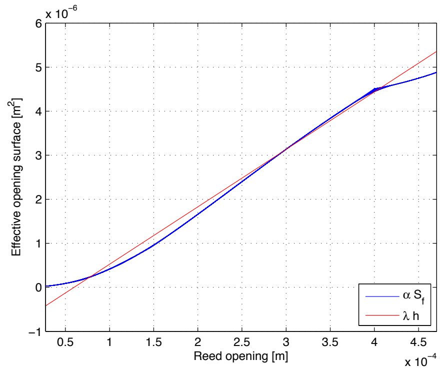

Figure 2: Effective opening surface as a function of the reed opening, plus a linear approximation to the corresponding curve.

through the reed channel can be modelled as

$$u_f = G\sqrt{\frac{2(p_m - p)}{\rho}}, \tag{4}$$

with $G = \alpha S_f \approx \lambda h$ being the effective opening surface, and since h is the reed opening, then λ can be termed as the "effective width" of the reed. Writing everything as a function of the reed displacement y to be consistent with equation (1) yields

$$G(y) = \lambda(y_m - y) = -\lambda y + \lambda y_m. \tag{5}$$

The reed induced flow can be easily calculated as

$$u_r = S_r \frac{dy}{dt} \overset{(1)}{=} -CS_r \frac{dp}{dt}, \tag{6}$$

which gives for the total flow into the mouthpiece the following non-linear differential equation.

$$\begin{aligned} u &= u_f + u_r \\ &= (-\lambda y + \lambda y_m)\sqrt{\frac{2(p_m - p)}{\rho}} - CS_r\frac{dp}{dt} \\ &= -C\lambda\sqrt{\frac{2}{\rho}}(p_m - p)^{3/2} + y_m\lambda\sqrt{\frac{2}{\rho}}(p_m - p)^{1/2} - CS_r\frac{dp}{dt} \\ &= c_1\sqrt{\frac{2}{\rho}}(p_m - p)^{3/2} + c_2\sqrt{\frac{2}{\rho}}(p_m - p)^{1/2} + c_3\frac{dp}{dt} \end{aligned} \tag{7}$$

$$\text{with} \quad \left\{\begin{array}{l} c_1 = -C\lambda \\ c_2 = y_m\lambda \\ c_3 = -CS_r \end{array}\right. \Rightarrow \left\{\begin{array}{l} K_a = -\lambda/c_1 \\ y_m = c_2/\lambda \\ S_r = \lambda c_3/c_1 \end{array}\right. ,$$

since the effective stiffness K_a is the reciprocal of the compliance of the reed. Note that there is no intention to solve equation (7). Instead it is used to formulate the objective function of the optimisation routine, as will be explained in the next section.

Including the derivative of the pressure with respect to time ($\frac{dp}{dt}$) in the above equation, which is introduced by the reed induced flow, allows the system to distinguish between the opening and closing state of the reed's motion. Even though u_r is very small compared to the flow through the reed channel ($u_f/u_r \approx 10^2$), it still has a significant effect on the total flow. Namely, if the flow is plotted as a function of the pressure difference across the reed, then two different branches appear, corresponding to the two different states of the reed's motion, as can be seen on Figures 3 and 6 in Section 3. In order to include the effect of u_r to the inverse model, $\frac{dp}{dt}$ has been estimated during the forward model simulation, using a centered difference approximation.

3 Parameter Estimation

Equation (7) involves pressure and flow inside the instrument, which can be obtained from measurements, plus three mixed parameters (c_1, c_2, c_3) and the blowing pressure p_m. An optimisation process can be carried out, matching the measured value of u with the right hand part of equation (7) allowing the estimation of p_m, c_1, c_2 and c_3. However, the forward model requires the full set of the physical parameters, namely λ, K_a, y_m and S_r to be extracted from the estimated values of c_1, c_2 and c_3. Including them in the form of the second last line of equation (7) during the optimisation process could result in non-physical values for the estimated parameters, since the optimisation routine has to cope with four different parameters, building up three products. To deal with that, the value of λ has been set, observing the data obtained from the forward model simulation, by matching the slope of the curve in Figure 2. This allows the estimation of K_a, y_m and S_r by formulating an objective function that equates the two parts of equation (7) and using the "Nelder-Mead" optimisation technique [19, 20].

As will be explained at the end of this paper, the estimated values of the physical parameters will be used as an input to a second non-linear optimisation routine. So, the value of λ will be fine tuned at that stage. For the time being, and since λ has been termed as the effective width of the system, it seems reasonable to use the width of the reed as an initial guess. As expected, this is in agreement with the value predicted by the slope of the curve in Figure 2.

Two different cases have been examined in this study. First the mechanical parameters that govern the oscillation of the reed are assumed to be constant, so that the behaviour of the reed-mouthpiece system is confined to a linear nature. In this case (Section 3.1) the estimated parameters from the inverse model can be directly compared to the constant values used during the forward model simulation. Then, in Section 3.2, a non-linear model of the reed oscillation is used, which is closer to the real behaviour of the system.

3.1 The Linear Case

In order to ensure the accuracy of the presented technique, it is necessary to consider the error introduced by the assumptions that need to be taken during the analysis of the system. Since the input data for the pressure and flow signals is currently available only by numerical simulations, the forward model can been used, for this first simplified case, to produce these signals, based on a set of constant values for K_a and S_r, so that any error introduced will be due to the simplifications made by neglecting inertia and internal friction, as well as those related to the opening surface.

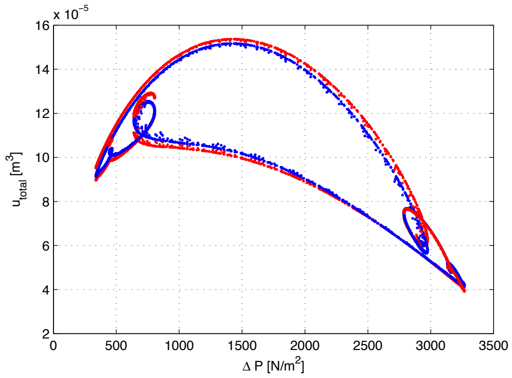

Figure 3: Flow into the mouthpiece over pressure difference, simulated by the forward model (blue) and calculated using the estimated parameters (red).

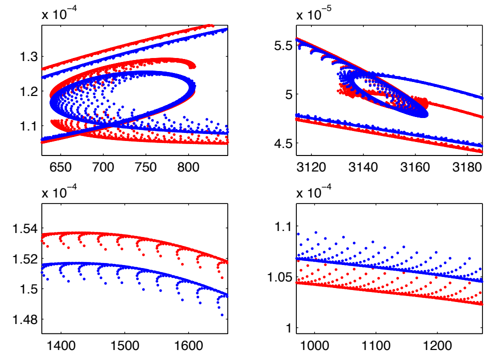

Figure 4: Several zoomed-in areas of the original plot in Figure 3.

To evaluate the results, the set of the estimated parameters is used to calculate the flow from the simulated pressure data using equation (7). This estimated flow is compared to the one produced by the forward model in Figure 3. In this case K_a has been estimated with a relevant error of -6.39%, and S_r with a relevant error of 0.43%. Figure 4 zooms in at certain areas of the graph to show how well the flow is approximated, with the estimated flow being able to follow the exact shape of all the curves that appear at the actual signal. The external blowing pressure used for all the simulations was 1800 N/m^2.

In order to achieve that sort of approximation it is necessary to distinguish between the two branches that build up the flow, as explained in the previous section. The derivative of the pressure in the mouthpiece, with respect to time, suggests that each branch is treated separately, with the upper branch corresponding to the closing state and the lower branch to the opening state of the reed. Thus computing the derivative of the pressure during the forward model simulation enables this successful branch separation, which provides the displayed results.

3.2 The Non-linear Case

Since the described technique seems to be working for a lumped model with constant parameters, the next step is to apply it to a more realistic case. In this case both K_a and S_r are changing throughout the simulation, being functions of the pressure difference across the reed

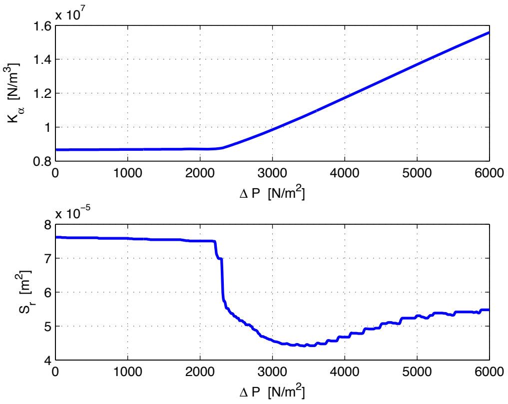

Figure 5: K_a and S_r as functions of the pressure difference across the reed.

(ΔP), as can be seen in Figure 5.

The estimated values are still constants, so it is not possible to define a relative error in the same sense as in the linear case. However it is possible to evaluate the results, by plotting the flow produced by the forward model and the one calculated by equation (7) using the estimated constant values of K_a, S_r and y_m. The same process can be carried out even with noisy data for both the pressure p and the flow u inside the mouthpiece. The results for the estimated flow in these two cases are depicted in Figure 6.

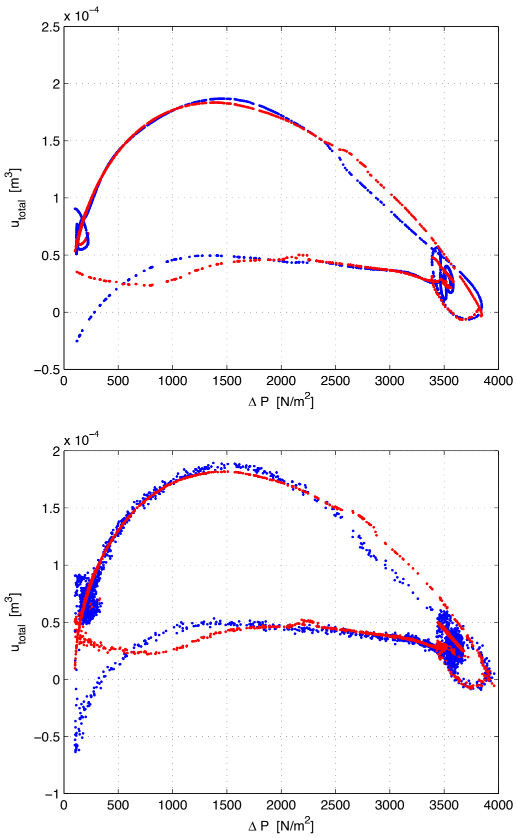

Figure 6: Flow into the mouthpiece over pressure difference, simulated by the forward model (blue) and calculated using the estimated parameters (red) based on signals generated with non-constant parameters (top) and based on noisy signals (bottom).

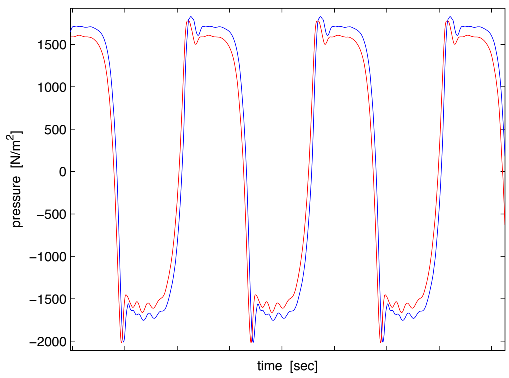

Figure 7: Comparison of the pressure signals in the mouthpiece for the forward model (blue) and the resynthesised sound (red).

The estimation of the flow seems to fail where the effects of mass (inertia) and damping are significant, i.e. at the end points of the oscillation of the reed. On the other hand, for pressure difference values in the range $[500, 3000]$ N/m^2 the estimation is good, even for the case of the noisy signals. Since the largest part of the cycle of the pressure difference lies within this range, it is reasonable to assume that the error, which occurs during the parameter estimation is small in relevant terms. As will be explained in the concluding section, this error will be further reduced in a future study, using alternative optimisation tools.

Having estimated the parameters involved, it is also possible to use them as an input to the forward model, in order to resynthesise the pressure and flow in the mouthpiece, that was used as the input to the optimisation algorithm. The pressure signals of the originally generated sound and the resynthesised sound are compared in Figure 7.

4 Conclusion

An inverse model has been proposed, that can estimate lumped model parameters of the clarinet reed mouthpiece system, based on the signals of the pressure and flow in the mouthpiece. The inversion is based on an analytic, closed-form expression that relates the pressure and the flow and is used to create the objective function for a non-linear optimisation routine.

The direct usefulness of the presented results, in terms of estimating the parameters and reproducing the flow, as well as resynthesising the produced sound, can be questioned due to the errors introduced by a series of assumptions and simplifications. However, the intention of the authors is to use the results of the current paper as an input to a future, second stage optimisation routine, which will be much less limited by simplifying assumptions, therefore enabling the optimisation algorithm to converge to a more realistic set of physical parameters.

The interaction of the player and the instrument is incorporated in the parameter extraction process in two ways. First, the formulation of the lumped model is based on a distributed model of the reed-mouthpiece-lip system that includes the effects of the player's embouchure. Second, the output of the inversion procedure, i.e. the lumped model parameters, directly represent how players adjust their embouchure in order to shape the sound.

References

[1] R.T. Schumacher. Ab intio calculations of the oscillations of a clarinet. *Acustica*, 48(2):71–85, 1981.

[2] T.A. Wilson and G.S. Beavers. Operating modes of the clarinet. *Journal of the Acoustical Society of America*, 56(2):653–658, 1974.

[3] J. Gilbert, J. Kergomard, and E. Ngoya. Calculation of the steady-state oscillations of a clarinet using the harmonic balance technique. *Journal of the Acoustical Society of America*, 86(1):35–41, 1989.

[4] A. Hirschberg, R.W.A. van der Laar, J.P. Marrou-Maurieres, A.P.J. Wijnands, H.J. Dane, S.G. Kruijswijk, and A.J.M. Houtsma. A quasi-stationery model of air flow in the reed channel of single-reed woodwind instruments. *Acustica*, 70:146–154, 1990.

[5] J.M. Adrien, R. Caussé, and E. Ducasse:. Dynamic modeling of stringed and wind instruments sound synthesis by physical models. In *International Computer Music Conference*, 1988.

[6] M. van Walstijn and F. Avanzini. Modelling the mechanical response of the reed-mouthpiece-lip system of a clarinet. Part II. A lumped model appproximation. *Acustica*, 93(1):435–446, 2007.

[7] V. Chatziioannou and M. van Walstijn. Reed vibration modelling for woodwind instruments using a two-dimensional finite difference method approach. In *International Symposium on Musical Acoustics*, Barcelona, 2007.

[8] F. Pinard, B. Laine, and H. Vach. Musical quality assessment of clarinet reeds using optical holography. *Journal of the Acoustical Society of America*, 113(3):1736–1742, 2003.

[9] M.L. Facchinetti, X. Boutillon, and A. Constantinescu. Numerical and experimental modal analysis of the reed and pipe of a clarinet. *Journal of the Acoustical Society of America*, 113(5):2874–2883, 2003.

[10] C.J. Nederveen. *Acoustical Aspects of Woodwind Instruments*. Frits Knuf, Amsterdam, 1969.

[11] V. Chatziioannou and M. van Walstijn. A refined physical model of the clarinet using a variable air jet height. In *The 3rd International Symposium on Communications, Control and Signal Processing (ISCCSP)*, Malta, 2008.

[12] A.M.C. Valkering. Characterization of a clarinet mouthpiece. Technical Report N. R-1219-S, Vakgroep Transportfysica, TUE, Eindhoven, 1993.

[13] B. Gazengel, J. Gilbert, and N. Amir. Time domain simulation of single reed wind instrument. From the measured input impedance to the synthesis signal. Where are the traps? *Acta Acustica*, 3:445–472, 1995.

[14] E. Ducasse. A physical model of a single-reed wind instrument, including actions of the player. *Computer Music Journal*, 27(1):59–70, 2003.

[15] J. P. Dalmont, J. Gilbert, and S. Ollivier. Nonlinear characteristics of single-reed instruments: Quasistatic volume flow and reed opening measurements. *Journal of the Acoustical Society of America*, 114(4):2253–2262, 2003.

[16] J.P.C. van Zon. Stromingsgeinduceerde klepinstabiliteiten. Technical Report N. R-1024-A, Vakgroep Transportfysica, TUE, Eindhoven, 1989.

[17] J. van Zon, A. Hirschberg, J. Gilbert, and A.P.J. Wijnands. Flow through the reed channel of a single reed music instrument. *Colloque de Phusique*, pages 821–824, 1990.

[18] A. R. da Silva, G. P. Scavone, and M. van Walstijn. Numerical simulations of fluid-structure interactions in single-reed mouthpieces. *Journal of the Acoustical Society of America*, 120:1798–1810, 2007.

[19] J.A. Nelder and R. Mead. A simplex method for function minimization. *The Computer Journal*, 7:308–313, 1964.

[20] M. Avriel. *Nonlinear programming: analysis and methods*. Mineola, NY: Dover Publications, 2nd edition, 2003.